„Système Interactif de Conception – the new tool of inverse analysis applied in geotechnics"

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LIST OF SYMBOLS

$\underline{\underline{A}}$ - matrix of sensibility
$c_{ij}=\underline{\underline{C}}$ – inversed matrix $\underline{\underline{M}}$
$\underline{f}$ – nodal forces vector
$f(\underline{u},\underline{x})$ – relation between vector $\underline{u}$, obtained from the calculation and *a priori* known vector $\underline{x}$
$\underline{\underline{K}}$ – global stiffness matrix
k – constant value
$\underline{\underline{M}}$ - matrix of covariance between $\underline{x}_i$ and $\underline{x}_j$
m - length of vector $\underline{x}_i$
n - number of vectors $\underline{x}_i$
p_k - probability of x_k
S, S_1 – objective functions
$\underline{u}$ – displacement vector
$\underline{X} = F(\underline{x}) = x_k\, p_k$ - vector of mean values
$\underline{x}$ - Ko or/and E
$\underline{x}_i$ – vector of measured variables
$\underline{X}_i = \underline{\alpha}_i$, α- known values - *a priori*
$\Delta\underline{x}$ – increment of vector $\underline{x}$
$\rho_{i,j}$ - coefficients of correlation between the variables $\underline{x}_i$ and $\underline{x}_j$
σ_{xi}, σ_{xi} – standard deviation for $\underline{x}_i$ and $\underline{x}_j$
σ - standard deviation of $\underline{u}$

STRESZCZENIE

W artykule przedstawiono przykład zastosowania analizy wstecznej w środowisku programowania Système Interactif de Conception (SIC). Struktura tej aplikacji oparta jest na procedurze Gensa-Ledesmy-Alonsa [1]. W artykule ograniczono się do zagadnień związanych z wyznaczaniem wartości modułu liniowej sprężystości dla ciągłego ośrodka odkształcalnego, wykorzystując algorytm minimalizujący funkcję celu, którą zdefiniowano w oparciu o formułę wielozmiennego rozkładu Gaussa. Procedurę zaprogramowano w języku fortran 77 i zainstalowano w systemie SIC 3.6. Zaprezentowano również przykład obliczeniowy przedstawiający poszukiwanie parametrów ciągłego ośrodka sprężystego obciążonego punktowo pojedynczą siłą skupioną.

SUMMARY

In the paper the example of simple application of the inverse analysis in the computer environment called Système Interactif de Conception (SIC) is presented. The structure of this application is based on Gens-Ledesma-Alonso procedure [1]. This paper is limited on problems relevant to the determination of linear deformation modulus value for a continuous deformable medium, exploiting the algorithm for minimisation of the objective function based on a formulae of multivariable Gauss distribution. The whole procedure was prepared in fortran 77 code and was installed in SIC 3.6. An example of calculation in a form of an investigation of the parameters' values for a continuous elastic medium, charged pointwise by a single force is also presented.

INTRODUCTION

The main aim of the inverse analysis is to determine values of the parameters describing the constitutive model, which govern the behaviour of studied phenomenon or evenement, exercising values obtained from measurements e.g. *in situ*. The presented application serves to achieve the following goals:

- determination of the modulus of linear elastic deformation E and optionally Poisson's ratio ν also, utilising known information e.g. values of settlements of a structure based on elastic continuous medium;
- application of some algorithms for minimisation the value of an objective function, which describes the differences between the measured values (e.g. settlements) and the values obtained from numerical calculations;
- exploitation in these algorithms *a priori* known values of module of linear deformation of continuous material E, and/or Poisson's ratio ν also.

OBJECTIVE FUNCTION

In the analysis the main assumption, that all values of the settlement, ν and E (measured) have a Gauss distribution, was taken into consideration (approach of Gens et al., [1]). The formulation of multivariable Gauss distribution for discretised variables has the well-known form:

$$f\left(\underline{x}_1 \ldots \underline{x}_n\right) = \left(2\pi\right)^{-n/2} * \left|\underline{\underline{M}}\right|^{-1/2} * e^{\left[-1/2 * \sum_1^n \sum_1^n \left(c_{ij} * \left(\underline{x}_i - \underline{X}_i\right) * \left(\underline{x}_j - \underline{X}_j\right)\right)\right]} \tag{1}$$

where:

$\underline{x}_i$ – vector of measured variables,
n - number of vectors $\underline{x}_i$,
$\underline{X} = F(\underline{x}) = x_k p_k$ vector of mean values,
p_k - probability of x_k,
m - length of vector $\underline{x}_i$,
$\underline{X}_i = \underline{\alpha}_i$ known values - *a priori*,
$\underline{\underline{M}}$ - matrix of covariance between x_i and x_j

$$m_{ij} = cov(\underline{x}_i, \underline{x}_j) = F[(\underline{x}_i - \underline{X}_i) * (\underline{x}_j - \underline{X}_j)] \tag{2a}$$

$$\underline{\underline{M}} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \\ & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 \\ Sym & & \sigma_3^2 \end{bmatrix} \tag{2b}$$

$\underline{\underline{C}} = \underline{\underline{M}}^{-1}$ inversed matrix $\underline{\underline{M}}$

$\rho_{i,j}$ - coefficients of correlation between the variables x_i and x_j, generally unknowns, but if the experimental results are available, it is possible to calculate all correlation coefficients utilising the formulae:

$$\rho_{ij} = \frac{\mathrm{cov}(x_i, x_j)}{\sigma_{xi}\sigma_{xj}} \tag{3}$$

where:

σ_{xi}, σ_{xi} – standard deviation for x_i and x_j (eq.5)

For the condition $n=1$ – if only one value (vector) *a priori* α (e.g. settlement, model parameter) is known, formulae (1) is simplified to:

$$f(\underline{u}) = \frac{1}{\sqrt{2\pi} * \sigma} * e^{\left[-\frac{(u-\alpha)^2}{2\sigma^2}\right]} \tag{4}$$

where: σ - standard deviation of $\underline{u}$:

$$\sigma = \sqrt{\sum_1^m (u_k - \alpha)^2 p_k} \tag{5}$$

p_k – probability of u_k, here assumed equal 1 for all components of known values.

In Gens' approach [1] the objective function S is determined in the form:

$$S = -2 * \ln(k * f(\underline{u}) * f(\underline{x})) \tag{6}$$

where:

$\underline{x}$ - Ko or/and E

$\underline{u}$ – displacement vector

k – constant value

Taking the assumption, that all parameters are independent (it means that there is no correlation between them) the sensibility matrix is simplified and takes the following form:

$$\underline{\underline{M}} = \sigma^2 \underline{\underline{I}} \tag{7}$$

Taking into consideration that standard deviation of all variables has constant value:

σ = const

and the information *a priori* X does not exist, the objective function is strongly simplified and takes the following form:

$$S = \Sigma(u_i - \alpha)^2 \tag{8}$$

which presents the least squares method formulae.

In presented investigation, another form of the objective function was assumed:

$$S_1 = -2 * \ln(f(\underline{u},\underline{x})) - k \qquad (9)$$

where:

$f(\underline{u},\underline{x})$ – relation between vector $\underline{u}$, obtained from the calculation (e.g. FEM) and *a priori* known vector $\underline{x}$,

k – constant value dependent on an accuracy of calculation, (e.g. $k = ln(a)$, a – value interpreted by machine as "zero", for FORTRAN 77 code on PC : $a = 2,225E\text{-}308$ and $k = -708.3965$).

Another form of the objective function S_1 was taken into calculation because of two inconveniences associated with function S:

- for big differences between the results obtained from FEM calculation and known real values, the objective function based on the least squares method gives high values very often, what generally is not a desirable effect in a numerical analysis,
- approaching the criterion of satisfactory solution, for big changes of the arguments u_i the objective function S gives small differences between all values corresponding to different arguments, what makes in lots of cases impossible to increase the accuracy of a calculation.

SYSTEME INTERACTIF DE CONCEPTION (SIC)

Among huge variety of software applications regarding geotechnical problems, the most popular and powerful are such versatile codes as ADINA or ABAQUS being used in other research areas. Some other such as PLAXIS, DIRTMOD, K2SOIL, TALREN or FONDOF are typically geotechnically-oriented codes, which gained high popularity in engineering practice. Additionally, there exist a variety of codes based on finite element method implementing individual constitutive laws and being developed in various research centres. Most of them can be available to other researchers for its verification and validation. Some of them are: PECPLAS, PEC3D developed in Laboratoire de Mechanique de Lille, SIC3.6 from Laboratoire Codiciel de Compiegne, MSHEET from Delft Geotechnics or DIANA from TNO Building Research.

For own numerical simulations the authors made use of SIC 3.6 for the analysis in plane strain state. The code is based on the finite element method. For the application of the inverse analysis problem the linearly elastic constitutive model has been assumed.

An additional information about computer application of the finite element method in SIC environment could be found on the internet site www.codiciel.fr. Introducing SIC the following topics are presented below (taken from quoted internet site).

SIC is the programming environment for performing research in the area of numerical modelling, specially designed for engineering and science. The main aims of its application are:

- solid mechanics,
- structures mechanics,
- thermique,
- and all disciplines associated with listed above.

The main characteristics of SIC are:

- library of compatible modules, with full possibility of communication between all modules and specialised external procedures and programs,
- adaptation in parallel and distributed calculations,

- huge variety of numerical approaches adopted in a wide range of problems, multileveled, working with multimodels in multimode.

SIC has a very clear form as a set of clearly described libraries and precompiled modules, specially designed for easy handling and modifying all available routines and easy for advanced developing. It makes a great possibility of exchanging the experiences in the net of scientific and industrial laboratories.

BASIC ALGORITHM

In the investigation two kinds of the algorithms were adopted taking into consideration linear elastic behaviour of analysed deformable medium:

- with constant length of step; after changing a direction of the calculation - diminution of current step:

$\Delta \underline{x}_i = \Delta \underline{x}_0 = const$ during the calculations progressing in one direction (e.g. forward)
$\Delta \underline{x}_{i+1} = 0.45 * \Delta \underline{x}_i$ after changing a direction of the calculation process (e.g. backward)
(“zero” index indicates starting value)

- with the calculated length of step during every iteration process:

a simple expectation:

$$\Delta \underline{x} = \Delta \underline{x}_0 \frac{|\underline{u}_0 - \underline{\alpha}|}{\Delta \underline{u}_0} \tag{10}$$

Gauss-Newton, Marquardt approach:

$$\Delta \underline{x} = \Delta \underline{x}_0 + \left(\underline{\underline{A}}^t \underline{\underline{M}}^{-1} \underline{\underline{A}} \right)^{-1} \underline{\underline{A}}^t \underline{\underline{M}}^{-1} \left(\Delta \underline{u} - \underline{\underline{A}} \Delta \underline{x}_0 \right) \tag{11}$$

where $\underline{\underline{A}}$ - matrix of sensibility calculated directly in FEM, the components of sensibility matrix could be calculated using the formulae (for linear elasticity only) [1]:

$$\frac{\partial \underline{u}}{\partial \underline{x}} = \underline{\underline{K}}^{-1} \left[\frac{\partial \underline{f}}{\partial \underline{x}} - \frac{\partial \underline{\underline{K}}}{\partial \underline{x}} \underline{u} \right] \tag{12}$$

where:
$\underline{\underline{K}}$ – global stiffness matrix,
$\underline{f}$ – nodal forces vector.

All tests with algorithms and different kinds of objective functions were made using a special feature applied in SIC. There exists the library UTI of procedures UTIXXX.F specially created for users, who want to experiment with their own ideas, when utilising FEM. On the basis of general ideas of algorithm construction, a simple example of an algorithm (fig.1) for searching-determining values of the parameters of linear elastic relation between stress and strains was presented. The objective function applied in this algorithm is based on the Gauss multivariable distribution and the constant step case.

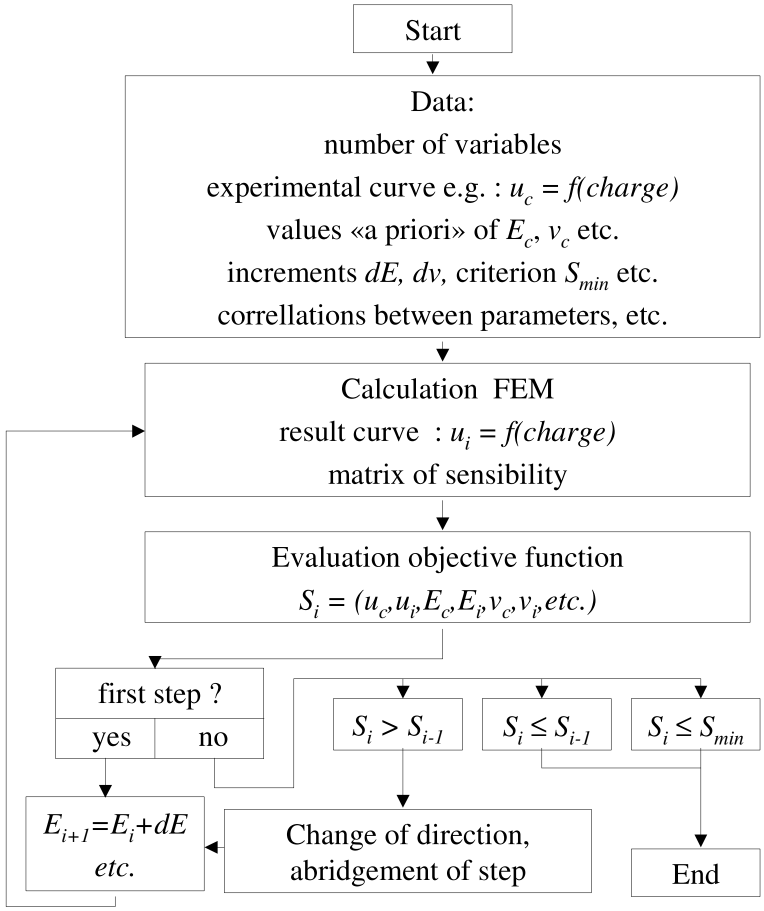

Fig. 1. The example of an algorithm with constant length of iteration step (installed in SIC as UTI104.F).

As a simple example of application of the algorithm structure (fig.1), the following problem is solved:

- continuous linear elastic base is charged pointwise by single force F=100 kN (fig.2)
- settlement in the point of applied force is known and is equal 0.1mm
- Poisson's ratio is known and is equal 0.3
- it is necessary to find a value of the linear elasticity modulus E, assuming a starting value of E=1155 MPa and its starting increment ΔE=10 MPa.

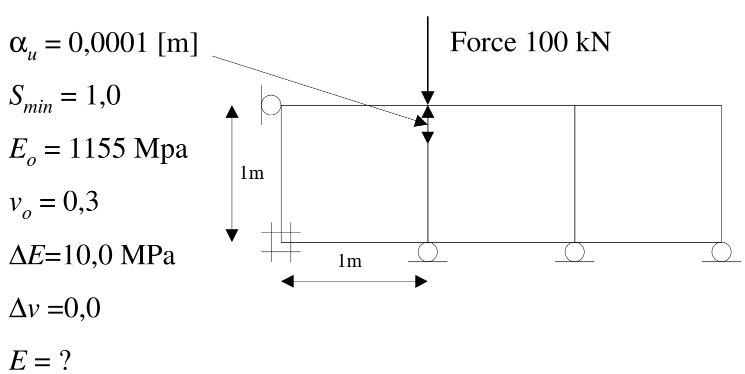

Fig.2. The example of solved problem.

On fig.3 the result of calculation, exploiting created in SIC macro-command "inverse", is presented.

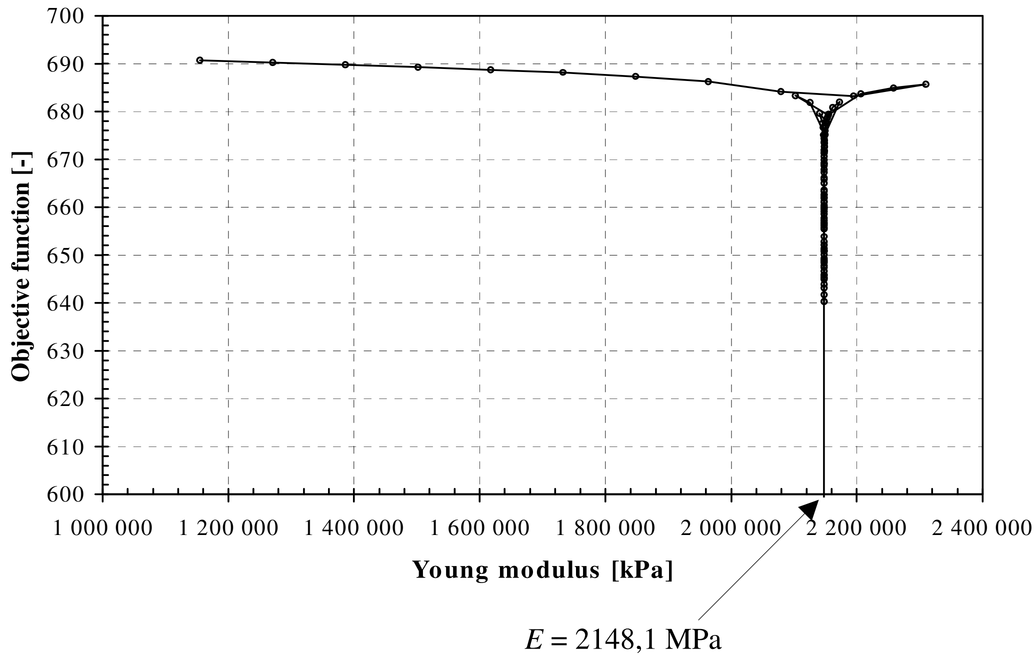

Fig.3. Results of the calculation

CONCLUSIONS

The example of a simple determination of unknown values of variables, which are not apparently relevant with known parameters (e.g. measured), was presented. Very complicated problems are usually faced in the engineer practice. Lots of difficult assignments of different tasks extort the solutions of multivariable problems, taking into consideration highly complicated constitutive models also. In such cases simple numerical methods for minimisation of multivariable objective function, such as Hooke-Jeeves, Powell, Gauss-Newton with all modifications (Marquardt), fail very often. These approaches give the solutions, which depend from starting conditions e.g. starting values of investigated parameters.

On fig.4. the simulation of the objective function behaviour for multivariable investigation case is presented. As it is shown, five unknown values of an abstract parameter e.g. x =32, 33, 34, 35 and 48, could not be found using simple minimisation algorithms without detailed analysis of starting conditions. This detailed analysis takes much more effort than the main investigation process. Besides most of unknown parameters, which build a numerical model, are not completely independent. There usually exists correlation between them. This changes the behaviour of simple algorithms additionally. On fig.5 the simulations of the objective function behaviour for two-variable case (x_1=35, x_2=31) with correlation coefficients equal respectively 0,2 and 0,8 are presented. These more complicated and rather "natural" problems could be solved utilising for example the genetic algorithms, which can govern the optimisation process for very intricate cases.

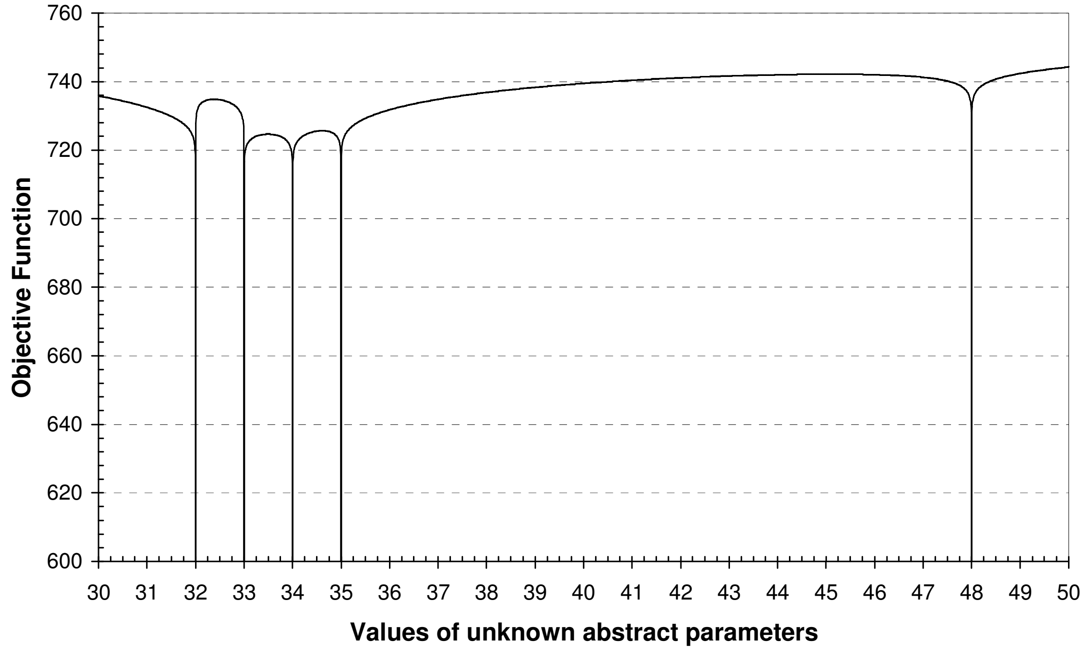

Fig.4. The simulation of the objective function behaviour for multivariable case

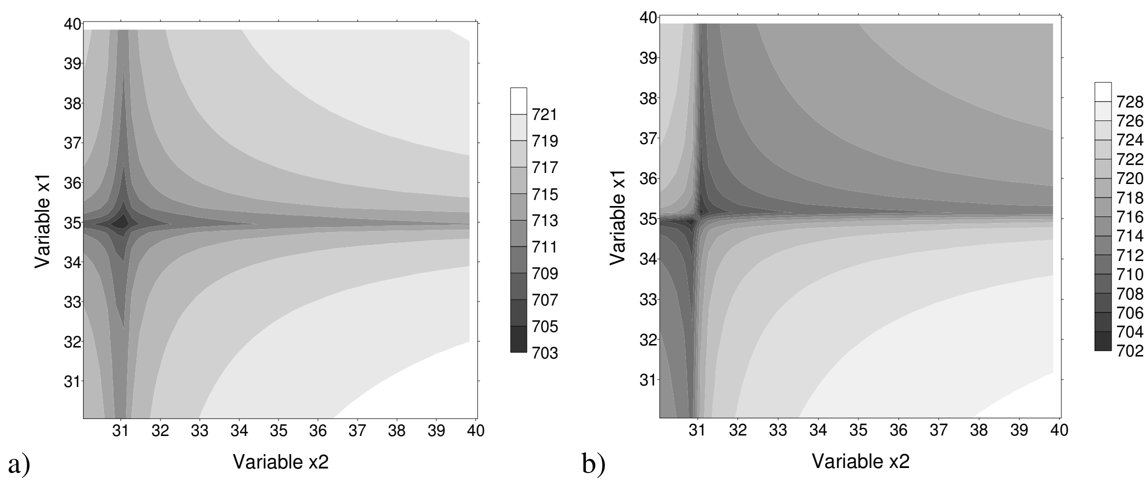

Fig.5. The simulation of the objective function behaviour for two-variable case

a) with correlation coefficient $\rho = 0{,}20$

b) with correlation coefficient $\rho = 0{,}80$

REFERENCES

1. Gens A., Ledesma A., Alonso E.E., *Estimation of parameters in geotechnical backanalysis -- I. Maximum Likelihood Approach*, Computers and Geotechnics 18, 1996, pp. 1-27,
2. Internet site with set of information about SIC : www.codiciel.fr