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METHODS OF CALCULATING CONCRETE STRAIN TAKING INTO ACCOUNT THE NONLINEAR CREEP

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K e y w o r d s: nonlinear creep of concrete, reinforced concrete, strain, stress.

A b s t r a c t

In the following article we present the law of nonlinear creep and simplified methods of calculating strains after *t* time. Next, we propose a constitutive rule for concrete, which may be used in analyzing strains of construction elements, where stress exceeds the limit of linear creep.

METODY OBLICZANIA ODKSZTAŁCEŃ BETONU Z UWZGLĘDNIENIEM NIELINIOWEGO PEŁZANIA

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S ł o w a k l u c z o w e: nieliniowe pełzanie betonu, konstrukcje żelbetowe, odkształcenia, naprężenia.

A b s t r a k t

W artykule omówiono prawo nieliniowego pełzania oraz uproszczone metody obliczania odkształceń po czasie *t*. Zaproponowano prawo konstytutywne dla betonu. Może być ono wykorzystane w analizach odkształceń elementów konstrukcji, w których naprężenia przekraczają granicę liniowego pełzania.

Introduction

According to the current regulations of the Polish norm PN-B-03264 based on the Eurocode EN 1992-1-1 (or EC2), it is necessary to take into account the nonlinear creep and to do so the creep coefficient $\phi(t,t_0)$ is substituted with coefficient $\phi_k(t,t_0)$obtained from the following formula:

$$\phi_k(t,t_0) = \phi(t,t_0)e^{1,5(k_\sigma - 0,45)} \tag{1}$$

where:
k_σ is a ratio of stress in concrete σ to the mean compressive strength at the time of applied loading $f_{cm}(t_0)$. The limit of linear creep is assumed to be the value of stress equal to 0,45 $f_{cm}(t_0)$. The current version of the Eurocode has been corrected by changing that value of the limit to 0,45 f_{ck}. It means that there is now a necessity to take into account the nonlinear creep even with smaller stresses. The rules and regulations of the Eurocode that are going to be binding in Poland as of 2010 include the new guidelines regarding the nonlinear creep. This is why we have to find new calculation methods for long term strains of selected construction elements (e.g. slender columns, pre-stressed concrete beams) with the use of the nonlinear theory, then comparing the results obtained by using these new methods with the results obtained by using the simplified method based on applying formula (1). Further in the paper we present the nonlinear creep law and the simplified methods for calculating strains after t time, as well as the constitutive law for concrete formulae (5) and (14) which may be used in numerical analyses of constructions.

The nonlinear creep law

The strain in the time function t is expressed by formula (2) (presented in ARUTIUNIAN'S paper from 1952)

$$\varepsilon(t)\ \frac{\sigma(t_0)}{E(t_0)} + F[\sigma(t_0)]C(t,t_0) + \int_{t_0}^{t}\left\{\frac{d\sigma(\tau)}{d\tau}\,\frac{1}{E(\tau)} + \frac{dF[\sigma(\tau)]}{d\tau}\,C(t,\tau)\right\}d\tau \tag{2}$$

where:

$C(t,\tau) = \dfrac{\varphi(t,\tau)}{E(\tau)}$ – coefficient described as specific creep,
$\varphi(t,\tau)$ – creep coefficient,
$E(\tau)$ – instantaneous elastic modulus in time τ,
$\sigma(t_0)$ – initial stress at the time t_0,
$E(t_0)$ – modulus of initial deformation at time t_0,

$F[\sigma(t_0)]$	– the value of nonlinear stress function at time t_0,
$F[\sigma(\tau)]$	– nonlinear stress function dependent on time τ.

Nonlinear stress function $F[\sigma(t)]$

ARUTIUNIAN (1952), JACENKA et al (2000), ULICKIJ (1967) all propose different expressions describing the nonlinear stress function $F[\sigma(t)]$. In our further analyses we use a very simple dependence described in Arutiunian's book from 1952 by the following relation (3).

$$F[\sigma(t)] = \sigma(t) + \beta\sigma^2(t) \tag{3}$$

where coefficient β is the stress function (Fig. 1).

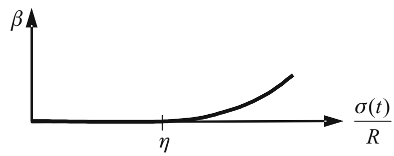

Fig.1 Dependence of coefficient β on stress $\sigma(t)/R$

The denomination of β is 1/MPa. According to [6] the value of β may be calculated from the following dependence

$$\beta = \begin{cases} 0 & \text{for } \sigma(t) \le \eta R \\ v\left[\dfrac{\sigma(t)}{R} - \eta\right]^n & \text{for } \sigma(t) > \eta R \end{cases} \tag{4}$$

where:

f_c – compressive strength of concrete (in ULICKIJ'S paper from 1967 compressive strength of concrete was marked by R),

η – the ratio of the value of the stress which causes linear creep to change into nonlinear creep to compressive strength of concrete f_c.

v,n – parameters assumed for the sake of experiment

According to EC2 the limit of the linear creep is assumed to have the value of stress equal to 0,45 f_{ck} (f_{ck} – characteristic compressive cylinder strength of concrete at 28 days). In this article we assume average values of compressive strength of concrete. Thus, further in the article the assumed value of the linear creep's limit is 0,45 f_c. Taking into account the mention assumption, dependence (4) may be expressed as follows

$$\beta = \begin{cases} 0 & \text{for } \sigma(t) \le 0{,}45\ f_c \\ \nu \left[\dfrac{\sigma(t)}{f_c} - 0{,}45\right]^n & \text{for } \sigma(t) > 0{,}45\ f_c \end{cases} \tag{5}$$

For the sake of further analyses we assumed the values of parameters $\nu = 0{,}15$ [1/MPa] and $n = 1{,}00$ from E.A. Jacenka's experiments described in ULICKIJ (1967).

Concrete strain under the constant stress

Creep in time (t_0, t) is taken into account. It is assumed that as a result of axial compression the stress in concrete at the initial moment t_0 is $\sigma(t_0)$ and does not change with time. The value of stress functions $F[\sigma(t_0)]$ is also constant in time (Fig. 2). The strain in time t is calculated from formula (2), which may be presented as formula (6) in case of constant stress

$$\varepsilon(t) = \varepsilon^{in}(t_0) + \varepsilon^{cr}(t) = \frac{\sigma(t_0)}{E(t_0)} + F[\sigma(t_0)]\ C(t,t_0) \tag{6}$$

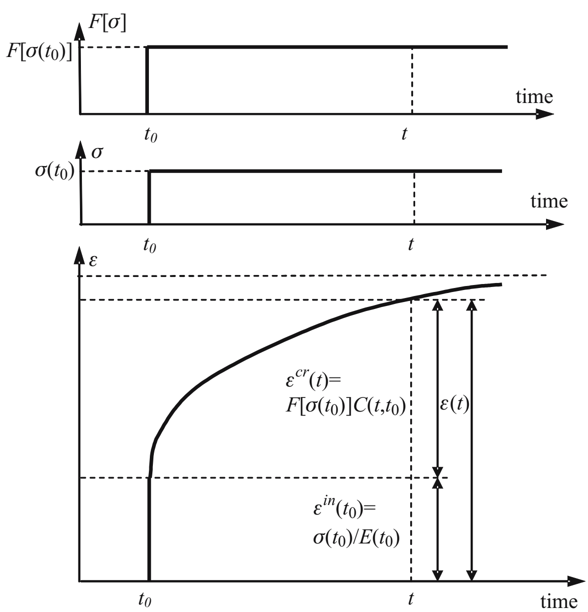

Fig. 2. Total strain $\varepsilon(t)$ at the moment t with constant stress; $\varepsilon^{in}(t_0)$ – elastic strain, $\varepsilon^{cr}(t_0)$ – creep strain

Concrete strain under changeable stress when stress is given in time function

Exact, numerical calculation of strain may be done by dividing the considered time into short periods and summing up the subsequent increments. Assuming that after time t_0 there are changes in stresses $\Delta\sigma(t_i)$, and increments of stress function $\Delta F[\sigma(t_i)]$ in subsequent time periods $\Delta t_i = t_i - t_{i-1}$, the integral in formula (2) is approximated by the following sum:

$$\int_{t_0}^{t}\left\{\frac{d\sigma(\tau)}{d\tau}\frac{1}{E(\tau)} + \frac{dF[\sigma(\tau)]}{d\tau}C(t,\tau)\right\}d\tau = \sum_{i=1}^{n}\frac{\Delta\sigma(t_i)}{E(t_i)} + \Delta F[\sigma(t_i)]\,C(t,t_i) \qquad (7)$$

After taking into account the relation (7), the dependence (2) may be expressed as the formula (8)

$$\varepsilon(t) = \frac{\sigma(t_0)}{E(t_0)} + F[\sigma(t_0)]\,C(t,t_0) + \sum_{i=1}^{n}\frac{\Delta\sigma(t_i)}{E(t_i)} + \Delta F[\sigma(t_i)]\,C(t,t_i) \qquad (8)$$

Formula (8) enables the exact calculation of strain. The method based on dependence (8) was named a ‘step by step’ method in the paper of CHIORINO et al. (1980). Depending on using different methods of approximating the integral in formula (2) we can distinguish the following simplified methods of calculating strain in time t:

1. Effective modulus method (EM)
2. Mean stress method (MS)
3. Age adjusted effective modulus method (AAEM)

The analysis of these methods in case of linear creep was presented in the paper by CHIORINO et al (1980). Below we present formulae for concrete strains in time t for nonlinear creep derived for these methods and based on the assumptions of CHIORINO et al (1980).

Effective modulus method (EM)

Assuming one time period with the use of the rectangular rule, the integral in dependence (2) may be approximated by the following expression:

$$\int_{t_0}^{t} \left\{ \frac{d\sigma(\tau)}{d\tau} \frac{1}{E(\tau)} + \frac{dF[\sigma(\tau)]}{d\tau} C(t,\tau) \right\} d\tau \cong (\sigma(t) - \sigma(t_0)) \frac{1}{E(t_0)} + (F[\sigma(t)] - - F[\sigma(t_0)])\, C(t,t_0) \tag{9}$$

After taking into account (9), dependence (2) may be expressed by formula (10)

$$\varepsilon(t) = \frac{\sigma(t)}{E(t_0)} + F[\sigma(t)]\, C(t,t_0) \tag{10}$$

Substituting (11) and (12)

$$E(t_0) = E_{cm} \tag{11}$$

$$C(t,t_0) = \varphi(t,t_0)/E_{cm} \tag{12}$$

in (10) we obtain dependence (13)

$$\varepsilon(t) = \frac{\sigma(t)}{E_{cm}} + \frac{F[\sigma(t)]}{E_{cm}} \varphi(t,t_0) \tag{13}$$

Substituting (3) in (13) we obtain a relationship between stresses and strains in concrete at the moment t:

$$\varepsilon(t) = \beta \frac{\sigma^2(t)}{E_{cm}} \varphi(t,t_0) + \frac{\sigma(t)}{E_{cm}} (\varphi(t,t_0) + 1) \tag{14}$$

With the stress growing in time, the strains calculated according to (14) come with excess. Using The method does not require calculating the value of initial stresses $\sigma(t_0)$.

In case of linear creep, the nonlinear stress function $F[\sigma(t)]$ equals the value of $\sigma(t)$, and thus dependence (14) may be expressed as this commonly known formula:

$$\varepsilon(t) = \frac{\sigma(t)}{E_{c,eff}} \tag{15}$$

where:

$$E_{c,eff} = \frac{E_{cm}}{1 + \varphi(t,t_0)} \tag{16}$$

Due to its simplicity, the EM method is used in the Eurocode and the Polish norm for calculating deflections of bent elements. Application of this method is based on substituting (in formulae of the linear theory of reinforced concrete) the modulus of concrete elasticity E_{cm} with the effective modulus $E_{c,eff}$.

Mean stress method (MS)

Assuming one period of time and using the trapezoidal rule, the integral in formula (2) is approximated by the following expression

$$\int^{t}\left\{\frac{d\sigma(\tau)}{d\tau}\frac{1}{E(\tau)} + \frac{dF[\sigma(\tau)]}{d\tau}C(t,\tau)\right\}d\tau \cong$$
$$\frac{(\sigma(t) - \sigma(t_0)}{2}\left(\frac{1}{E(t)} + \frac{1}{E(t_0)}\right) + (F[\sigma(t) - F[\sigma(t_0)]])\frac{C(t,t) + C(t,t_0)}{2} \tag{17}$$

Taking into account (17) and dependence (2) may be expressed as follows:

$$\varepsilon(t) = \frac{\sigma(t_0)}{E(t_0)} + \frac{F[\sigma(t) + F[\sigma(t_0)]}{2}C(t,t_0) + \frac{(\sigma(t) - \sigma(t_0)}{2}\left(\frac{1}{E(t)} + \frac{1}{E(t_0)}\right) \tag{18}$$

In case of linear creep, relations (3) and (5) result in dependences (19) and (20)

$$F[\sigma(t)] = \sigma(t) \tag{19}$$

$$F[\sigma(t_0)] = \sigma(t_0) \tag{20}$$

Taking into account the relation of (19) and (20), dependence (18) may be expressed as formula (21)

$$\varepsilon(t) = \frac{\sigma(t_0)}{E(t_0)} + \frac{\sigma(t) + \sigma(t_0)}{2} C(t,t_0) + \frac{(\sigma(t) - \sigma(t_0))}{2} \left(\frac{1}{E(t)} + \frac{1}{E(t_0)} \right) \tag{21}$$

Using the relation of (11) and (12), dependence (21) may be expressed as (22) derived in the paper by CHIORINO et al. (1980).

$$\varepsilon(t) = \frac{\sigma(t)}{E_{cm}} + \frac{\sigma(t) + \sigma(t_0)}{E_{cm}} \varphi(t,t_0) \tag{22}$$

Age adjusted effective modulus method (AAEM)

Using one period of time and the aging coefficient χ, the integral in formula (2) may be substituted with the following expression

$$\int_{t_0}^{t} \left\{ \frac{d\sigma(\tau)}{d\tau} \frac{1}{E(\tau)} + \frac{dF[\sigma(\tau)]}{d\tau} C(t,\tau) \right\} d\tau \cong (\sigma(t) - \sigma(t_0)) \frac{1}{E(t_0)} + (F[\sigma(t)] - $$

$$- F[\sigma(t_0)])\chi \, C(t,t_0) \tag{23}$$

Taking into account (23), dependence (2) may be expressed as formula (24)

$$\varepsilon(t) = \frac{\sigma(t_0)}{E(t_0)} + F[\sigma(t_0)] \, C(t,t_0) + (\sigma(t) - \sigma(t_0)) \frac{1}{E(t_0)} + (F[\sigma(t)] - $$

$$- F[\sigma(t_0)])\chi \, C(t,t_0) \tag{24}$$

Using dependences (11), (12) as well as (19) and (20), and taking into account linear creep, formula (24) may be expressed as (25)

$$\varepsilon(t) = \sigma(t_0)\left[\frac{1}{E_{cm}} + \frac{\varphi(t,t_0)}{E_{cm}}\right] + [\sigma(t) - \sigma(t_0)]\left[\frac{1}{E_{cm}} + \chi\,\frac{\varphi(t,t_0)}{E_{cm}}\right] \quad (25)$$

Based on analyses presented in CHORINO'S et al. (1980), this theory gives good results in cases where stresses do not change quickly in a given time.

Relation between stresses and strains in time *t* function – effective modulus method EM

A relation between stresses and strains in the time t function has been presented in figure 3. Assuming that $t = t_0$, $\varphi(t,t_0) = 0$, formula (14) is transformed into a linear dependence (26) (straight line I, figure 3)

$$\varepsilon(t_0) = \frac{\sigma_c(t_0)}{E_{cm}} \quad (26)$$

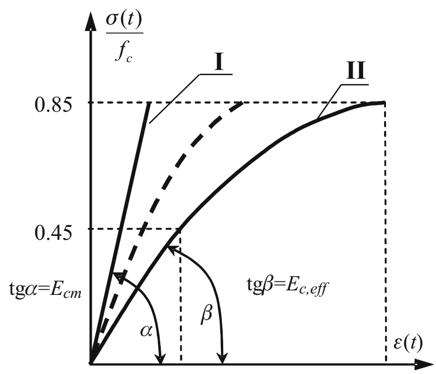

Fig. 3. Dependence $\frac{\sigma(t)}{f_c} - \varepsilon(t)$; I – linear dependence for $t = t_0$ for a short-term load, II – nonlinear *dependence for t* $\rightarrow \infty$ (boundary curve) for long-term loads

If $t \rightarrow \infty$, then the relation between stresses and strains is linear only for stresses $\sigma \leq 0{,}45\, f_c$ and can be expressed with formula (15). For stresses greater than $0{,}45\, f_c$ the dependence is nonlinear (curve II, figure 3). The curve marked with a broken line reflects dependence $\sigma(\tau) - \varepsilon(t)$ defined for any moment t from the interval (t_0, t_∞). Because compressive strength of concrete decreases in case of long-term loading, the range of stresses considered here has been limited to $0{,}85\, f_c$.

Straight line I shows a dependence resulting from immediate loading of only one concrete sample (strain is defined when loading is increasing). Dependences presented by curve II cannot be determined with only one experiment, but with a series of them. To run just one experiment, we would have to apply to the sample an axial load which causes stress $\sigma(t)$, and then, after an infinite time ($t \to \infty$) of being loaded, define strain $\varepsilon(t)$. Successive experiments would have to be carried out in a similar way, but using different samples, each one loaded with increasing force. Such a series of experiments would result in obtaining the dependence presented by curve II.

Conclusion

1. We have presented in this article three simplified methods which enable to calculate strains resulting from concrete creep after t time under the influence of great stresses ($\sigma > 0,45\ f_c$).

2. Approximating the integral in formula (2) with expression (9) (effective modulus method) and using expression (3) which describes the stress function we obtained dependence (14). Dependence $\sigma(t) - \varepsilon(t)$ presented by formulae (5) and (14) (curve II, figure 3) may be used while analysing strain in construction elements where stresses exceed the limit of linear creep.

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