TECHNICAL SCIENCES
Abbrev.: Techn. Sc., No 11, Y 2008

DOI 10.2478/v10022-008-0014-z

MODELLING OF THE SILTING UP OF NAVIGATION CHANNELS

Leszek M. Kaczmarek

Institute of Hydroengineering of the Polish Academy of Sciences (IBW PAN) in Gdańsk
Department of Civil Engineering and Building Structures
University of Warmia and Mazury in Olsztyn

Key words: transport of bedload and suspended sediments, grain size distribution, surface waving, sea currents, silting up of water routes.

Abstract

A mathematical model describing transport of non-uniformly graded sediments has been applied to analyzing the silting up of approach routes (navigation channels) leading to ports. This model distinguishes three layers in the movement of sediments, assuming that the vertical sorting occurs only in the process of picking up grains in the contact layer. It is also assumed that along the widnward edge of the route sediments are transported in the bedload and contact layers during the wave crest phase and – as suspended sediments – in the outer region under the influence of the resultant current. On the leeward side sediments are transported only during the wave trough phase in the bedload and contact layer. The computations have demonstrated that the above model can be a useful tool for predicting both the rate and volume of sediments silting up navigation channels as well as grain-size distribution of sediments which fill up a water route.

MODELOWANIE PROCESÓW ZAPIASZCZANIA KANAŁÓW NAWIGACYJNYCH

Leszek M. Kaczmarek

Instytut Budownictwa Wodnego PAN w Gdańsku
Katedra Budownictwa i Konstrukcji Budowlanych
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: transport osadów wleczonych i zawieszonych, rozkład granulometryczny, falowanie powierzchniowe, prądy morskie, zapiaszczanie torów wodnych.

Abstrakt

Model matematyczny transportu osadów niejednorodnych granulometrycznie zastosowano do analizy zapiaszczania torów podejściowych (kanałów nawigacyjnych) do portów. Wyszczególnia on trzy warstwy ruchu osadów, przy czym założono, że pionowe sortowanie odbywa się tylko w procesie podrywania ziaren w warstwie kontaktowej. Zakłada się, że na krawędzi nawietrznej toru osady transportowane są w fazie grzbietu fali w warstwie wleczenia i kontaktowej oraz zewnętrznej – w formie zawieszonej – pod wpływem wypadkowego prądu. Na krawędzi zawietrznej osady transportowane są tylko w fazie doliny fali w warstwie wleczenia i kontaktowej. Przeprowadzone obliczenia pokazują, że zastosowany model może być użytecznym narzędziem w predykcji zarówno wielkości i tempa zapiaszczania, jak i określaniu rozkładów granulometrycznych osadów wypełniających tor wodny.

Introduction

The mathematical model of sediment transport, which for years has been developed at the Institute of Hydroengineering of the Polish Academy of Sciences (IBW PAN) in Gdańsk, Poland, has been applied to analyze the rate and extent of silting up in approach routes to ports. A three-layer theoretical model (KACZMAREK 1999, KACZMAREK, OSTROWSKI 2000, 2002) has been used. This model distinguishes between the bedload layer, contact load (transient) layer and the outer region (Fig. 1). The nature of interactions between water

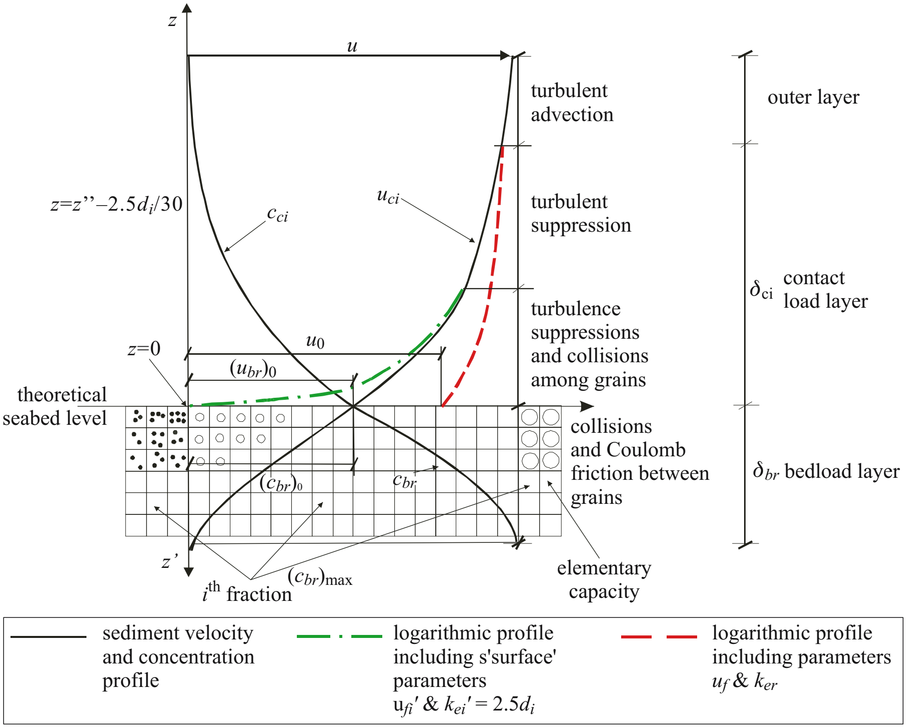

Fig. 1. Diagram of the three-layer momentum exchange model

and sediments is different in each of these layers, therefore they are described using different equations although at the contact of these layers the equations are folded, thus providing a complete theoretical description of the structure of sandy sediment transport.

In 1997, this model (in its simplified version albeit with a well-recognized bedload layer) was applied to analyze the silting-up of an approach route in Łeba port (Kaczmarek et al. 1996, Kaczmarek et al. 1997). The results were highly promising, although it needs to be recalled that at that stage of its development the model was constrained to descriptions of uniformly graded sediments.

However, in order to plan and carry out dredging and silting work, it seems necessary to have sound knowledge of mechanisms governing movements of non-uniformly graded sandy sediments and to be able to predict transport of each fraction as well as grain-size distribution of sediments which fill in a given approach route. Not infrequently, silt dredged from a navigation channel is used to strengthen shores near the harbour, in which case it is absolutely necessary to know the grain-size distribution of such silt, as it can be a futile and uneconomical effort to try and support the shores using sand which is too fine.

This paper relies on a three-layer mathematical model elaborated by Kaczmarek et al. (2004) and Kaczmarek and Biegowski (2006). This model enables description and prediction of non-uniformly graded shoreface debris movements.

Sediment transport model

For the mathematical modelling it has been assumed that all the fractions in the bedload layer move at identical velocity in the form of a dense mixture of water and soil, and that sediments in this layer are sorted. In addition, the model accounts for the fact that the most intensive vertical sorting occurs while grains are being picked up in the contact load layer above the sea bottom. Higher than this layer the grain-size distribution of sediments hardly changes.

Taking advantage of Sayed and Savage's dependencies (1983) for viscosity tensions and tensions caused by Coulomb friction, a set of equations has been obtained to describe the rate and concentration of sedimentation in the bedload layer, which can be presented as:

$$\alpha^0\left(\frac{c_b - c_0}{c_m - c_b}\right) \sin\varphi \sin 2\psi + \mu_1\left(\frac{\partial u_b}{\partial z'}\right)^2 = \rho u_f^2 \tag{1}$$

$$\alpha^0\left(\frac{c_b - c_0}{c_m - c_b}\right)(1 - \sin\varphi\sin 2\psi) + (\mu_0 + \mu_2)\left(\frac{\partial u_b}{\partial z'}\right)^2 = \\ = \left(\frac{\mu_0 + \mu_2}{\mu_1}\right)\Bigg|_{c=c_0}\rho u_f^2 + (\rho_s - \rho)g\int_0^{z'} c_b\,dz' \tag{2}$$

The coefficients μ_0, μ_1 and μ_2 are sediment concentration c functions, defined by the formulas

$$\frac{\mu_1}{\rho_s d^2} = \frac{0.03}{(c_m - c_b)^{1.5}} \tag{3}$$

$$\frac{\mu_0 + \mu_2}{\rho_s d^2} = \frac{0.02}{(c_m - c_b)^{1.75}} \tag{4}$$

where $c_m(= 0.53)$ is the maximum concentration of inert sediment, when grains closely adhere to one another; c_b is sediment concentration and u_b is the velocity of sediment motion in the bedload layer while c_0 is the sediment concentration at the theoretical seabed level; (c_0 = 0.32), $\alpha^0 / \rho_s g d = 1$, ϕ is a quasi-static inner friction angle $\psi = \frac{\pi}{4} - \frac{\phi}{2}$.

Because of strong interactions between grains of particular fractions in the bedload layer, it was assumed that all fraction moved at the same velocity $[u_{br}(z',t)]$ and had the same concentration $[c_{br}(z',t)]$ at a given level z'. From this assumption it follows that sediment is not sorted in the bedload layer and it is possible to describe sediment transport using the representative diameter ($d_r = d_{50}$). Thus, taking the representative diameter d_r the effective roughness $k_e = k_{er}$ is derived from the formula suggested by KACZMAREK (1999) and then, from the integral model developed by FREDSØE (1984), temporary friction velocities $u_f(t)$ on the sea bed surface are obtained. From equations (1) ÷ (4) temporary values of velocity u_{br} (z',t) and concentration $c_{br}(z',t)$ of sediments in the bedload layer of the thickness δb_r are obtained. Noteworthy is the fact that the above model removes from the bedload layer the largest simplification used so far when modelling non-uniformly graded sediment motions. The present model assumes that interactions between sediment fractions are so strong that as a result smaller fractions are retarded by larger ones and that means that all the fractions travel at identical speed. Thus, simple totalling of transport intensities of particular fractions, treated as uniform sediment, does not apply to sediment motions in the bedload layer This conclusion is in accord with many laboratory observations (cf. HASSAN et al. 2001).

In the contact load layer, numerous turbulent pulsations and chaotic collisions among grains cause large variation in the transport of particular sediment fractions. Very close to the sea bottom – in the sub-layer where the distribution of velocities of the i^{th} fraction of sediments evidently reveal the occurrence of sliding velocity u_{br} – there is very strong interaction between the fractions, evoked by mutual chaotic collisions. Further away from the sea bottom, these interactions between the fractions weaken. However, the concentration of the i^{th} fraction is big enough to suppress turbulences, and this effect is dependent on the grain diameter d_i. Thus, it can be expected that each i^{th} fraction, owing to mutual interactions, moves at its own speed $u_{ci}\,(z'',t)$ and is characterised by its own concentration $c_{ci}(z'',t)$. In this model, temporary vertical velocity and concentration distributions are derived from the equations suggested by KACZMAREK (1999), knowing the friction velocity $u_{fi}'(t)$, which is variable during the surface wave period:

$$\left[\frac{3}{2}\left(\alpha_s \frac{d}{w_s} \frac{\partial u_c}{\partial z''} \frac{2}{3} \frac{s + c_M}{c_D} + \beta_s\right)^2 d^2 c_c^2 (s + c_M) + l^2\right]\left(\frac{\partial u_c}{\partial z''}\right)^2 = u'^2_f\,(t) \tag{5}$$

$$\left[3\left(\alpha_s \frac{d}{w_s} \frac{\partial u_c}{\partial z''} \frac{2}{3} \frac{s + c_M}{c_D} + \beta_s\right)^2 d^2 \frac{du_c}{dz''} c_c + l^2 \frac{du_c}{dz''}\right]\frac{dc_c}{dz''} = -w_s \mathrm{c}_\mathrm{c} \tag{6}$$

where c_D is the resistance coefficient and c_M is the added mass coefficient. DEIGAARD (1993) assumed that $(s + c_M) = 3.0$, and $c_D = 1.0$. The proportionality coefficients α_s and β_s remain unknown and have to determined by the model calibration; l is the mixing length expressed as $l = \kappa z'' = 0.4z''$.

The value $u_{fi}'(t)$ is calculated from FREDSØE'S integral model (1984), assuming that the surface height of roughness k'_{ei} is determined from the equation $k'_{ei} = 2.5d_i/30$. The boundary conditions for all the fractions are the same:

$$u_{ci}(z'' = 2.5d_i\ /\ 30,t) = u_{br}\ (z' = 0,t) \tag{7}$$

$$c_{ci}(z'' = 2.5d_i\ /\ 30,t) = c_{br}\ (z' = 0,t) = 0.32 \tag{8}$$

The topmost limit of the contact load layer – common for all the fractions (δ_{cr}) – is determined using FREDSØE'S integral model (1984) including the effective surface roughness determined for the representative diameter $k'_{er} = 2.5d_r = 2.5d_{50}$. As has been demonstrated by KACZMAREK (1999) this thickness can be expressed by the dependence $\delta_{cr} = \delta_1'/2$, where δ_1' represents the wall adjacent thickness (found from the solution suggested by FREDSØE (1994) for $k'_{er} = 2.5d_r = 2.5d_{50}$), at the moment of the maximum (during the wave period) orbital velocity at the sea bottom.

It should be noticed here that in the suggested model the velocities and concentrations of rougher fractions are larger than the values these fractions would obtain should the sea bed be uniform, consisting of just one corresponding fraction. This increase in velocities in a mixture results from mutual interactions between the fractions, where rougher fractions are accelerated by finer grains. This finding agrees with laboratory observations (cf. DE MEIJER et al. 2002), which prove that larger fractions of sediments are transported more intensively in a non-uniform mixture than in a uniformly graded sediment.

Recapitulating, it needs to be stressed that simple aggregation of stresses occurring in the transport of all fractions, treated as uniform sediment, is not applicable to the contact load layer model (same as the bedload layer). Thus, the contribution of the i^{th} fraction to the transport of mixture $n_i q_{ci}$ does not equal an 'independent' transport $n_i q_i$, that is $n_i q_i \neq n_i q_{ci}$.

In the outer layer, due to the adopted transport model, the grain-size distribution of the transported sediment does not alter. The grain-size distribution of suspended sediment

$$c(z) = c_z(z = \delta_{cr})\left(\frac{z}{\delta_{cr}}\right)^{-\alpha_1} \tag{9}$$

is dependent exclusively on the grain-size distribution at the reference level, i.e.

$$c_z(z = \delta_{cr}) = \bar{c}_z\ (z = \delta_{cr}) = \sum_{i=1}^{N}[\bar{c}_{ci}(z = \delta_{cr})n_i] \tag{10}$$

where:

$$\bar{c}_{ci}\ (z = \delta_{cr}) = \frac{1}{T}\int_0^T c_{ci}(z = \delta_{cr}, t)dt \tag{11}$$

BIEGOWSKI (2006) demonstrated that the power function exponent is $\alpha_1 = 0.6$.

Therefore, including the assumptions pertaining to flows of sediments, the following equations can be suggested to describe the resultant (over the wave period) transport of non-uniformly graded sediments:

$$
\begin{aligned}
q_{net} &= \sum_{i=1}^{N} n_i \bar{q}_{1i} - \sum_{i=1}^{N} n_i \bar{q}_{2i} = \\
&= \sum_{i=n}^{N} n_i \frac{1}{T} \int_0^T \left(\int_0^{\delta_{br}} u_{br}(z',t) c_{br}(z',t) dz' \right) dt + \\
&+ \sum_{i=n}^{N} n_i \frac{1}{T} \int_0^T \left(\int_0^{\delta'_1/2+2.5d_i/30} u_{ci}(z'',t) c_{ci}(z'',t) dz'' \right) dt - \\
&- \sum_{i=1}^{N} n_i \int_{\delta'_1/2}^{h_t} u_z(z) c_z(z) dz = \\
&= \frac{1}{T} \int_0^T \left(\int_0^{\delta_{br}} u_{br}(z',t) c_{br}(z',t) dz' \right) dt + \\
&+ \sum_{i=n}^{N} n_i \frac{1}{T} \int_0^T \left(\int_0^{\delta'_1/2+2.5d_i/30} u_{ci}(z'',t) c_{ci}(z'',t) dz'' \right) dt - \\
&- \int_{\delta'_1/2}^{h_t} u_z(z) c_z(z) dz
\end{aligned}
\qquad (12)
$$

where h_t is the height from the sea bed to the wave trough level while u_z is the value of the resultant of the cross-shore and longshore currents.

By including to the first two components of equation (12) temporary values of velocities and concentrations of sediments for the crest and trough wave respectively, the average intensity of sediment transport in the bedload layer during the wave period and in the contact load layer during the wave crest and trough can be described.

Computation procedure

For calculating the transformations of wave formation and currents, a procedure based on the CROSMMOR software was suggested. The software package KLEPSYDRA, which uses the three-layer model of non-uniformly graded sediment motions and is described by equations (1) ÷ (12) was applied to calculate sediment transport values.

CROSMOR, which has been elaborated at Utrecht University by a team of researchers supervised by professor Leo van Rijn and dr Bart Grasmeijer, allows the determination of surface wave transformation in a shore cross-section profile. The transformation is described by several parameters, of which the major ones are: the mean-square average height (H_{rms}), incident wave angle (α) (Fig. 2) and depth-averaged cross-shore and longshore velocities.

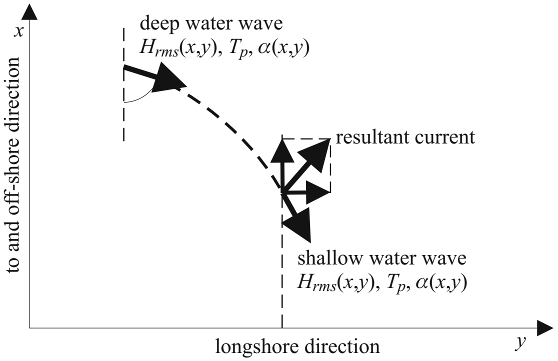

Fig. 2. Wave transformation pattern

The parameters for wave and current formation obtained from CROSMOR are then fed to the software programme KLEPSYDRA, elaborated at the Institute of Hydroengineering of the Polish Academy of Sciences (IBW PAN) in Gdańsk by a team headed by Leszek Kaczmarek. Based on the equations (1) ÷ (12) this programme performs calculations of the transport intensity for each fraction in the three layers, i.e. bedload, contact load and outer layers. The calculations are carried out along the two edges of an approach route, i.e. the windward and leeward sides of the channel (cf. Fig. 3). It is assumed that each wave characterised by the parameters H_{rms} and T_p can be described by Stokes second approximation, with a heightened crest and flattened trough. Near the sea shorelines, sediment transport is most often described using the above description of waves (cf. KACZMAREK 1999).

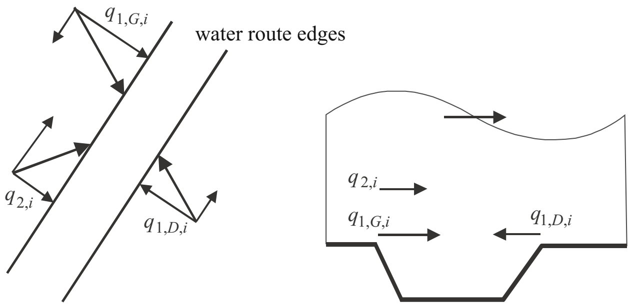

Fig. 3. Diagram of the *ith* sediment fraction transport intensity at the edges of a water route in the crest (G) and trough (D) wave phase

Having performed the calculations, the following are obtained:

- transport intensity of the i^{th} fraction in the bedload layer during the wave crest phase ($q_{b,G,i}$),
- transport intensity of the i^{th} fraction in the bedload layer during the wave trough phase ($q_{b,D,i}$),
- transport intensity the i^{th} fraction in the contact load layer during the wave crest phase ($q_{c,G,i}$),
- transport intensity the i^{th} fraction in the contact load layer during the wave trough phase ($q_{c,D,i}$),
- transport intensity of the i^{th} fraction during the wave crest phase ($q_{1,G,i}$)

$$q_{1,G,i} = q_{b,G,i} + q_{c,G,i} \tag{13}$$

- transport intensity of the i^{th} fraction during the wave trough phase ($q_{1,D,i}$)

$$q_{1,D,i} = q_{b,D,i} + q_{c,D,i} \tag{14}$$

- transport intensity of the i^{th} fraction of sediments suspended under the influence of the resultant (cross-shore and longshore) current in the outer layer ($q_{2,i}$).

As illustrated in Figure 3, at the windward (updrift) edge of the approach route, the sediments are conveyed during the wave crest phase in the bedload and contact load layers and in the outer layer they are transported by the resultant current. On the leeward (downdrift) edge, the sediments are transported only during the wave trough phase in the bedload and contact load layers.

The above results are applied to calculations of volumes of sediment transported over periods of time and determinations of grain-size composition of the sediment captured in a navigation channel. Knowing these results, i.e. the values of transport intensity for each sediment fraction both in the bedload, contact load and outer layers, with the sediment being suspended in the latter layer, it is possible to determine quantities of material carried along both edges of a water route. The calculations should include such factors as the mutual location of the approach route, the shore cross-section profile and the incident wave angle to the shoreline along the edge of the water route.

Changes in the grain-size composition of sediment halted in a water route on its windward side are expressed by the equation:

$$n_{i,naw.} = \frac{q_{1,G,i} + q_{2,i}}{\sum\limits_{i} (q_{1,G,i} + q_{2,i})} = \frac{q_{1,G,i} + q_{2,i}}{q_{1,G} + q_{2}} \tag{15}$$

By analogy, for the leeward side, the following equation is valid;

$$n_{i,zaw.} = \frac{q_{1,D,i}}{\sum_i q_{1,D,i}} \tag{16}$$

The quantities and grain-size distribution of sediments which silt up both sides of a water route at a length of L over a time interval t can be calculated from the formulas:

$$\begin{gathered} Q = \sum_i (q_{1,G,i}\, q_{2,i} + q_{1,D,i}) \cdot t \cdot L \\ n_i = \frac{Q_i}{Q} \end{gathered} \tag{17}$$

Computations: a case study

In order to analyze the effect of grain-size distribution on the silting up of a water route, case computations have been performed for the approach route to Łeba port. In the initial stage, the effect of one parameter only, i.e. d_{50}, was analyzed. It was therefore assumed that the bottom sediments are perfectly sorted out and consist of just one fraction measuring d_{50} in diameter. The calculations of the mean annual silting up were done for two grain diameters: d_{50} = 0.2 mm and d_{50} = 0.25 mm. The following volumes of silting were obtained: for d_{50} = 0.2 mm → Q = 74 thousand m^3/year and for d_{50}= 0.25 mm → Q = 40 thousand m^3/year.

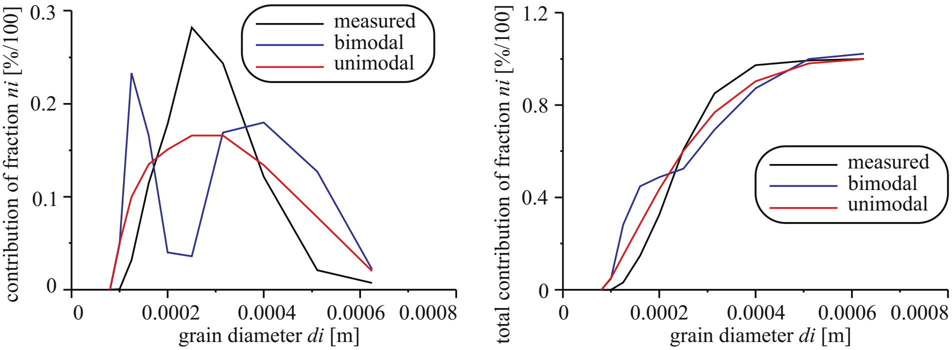

Fig. 4. Grain-size distributions taken for the calculations

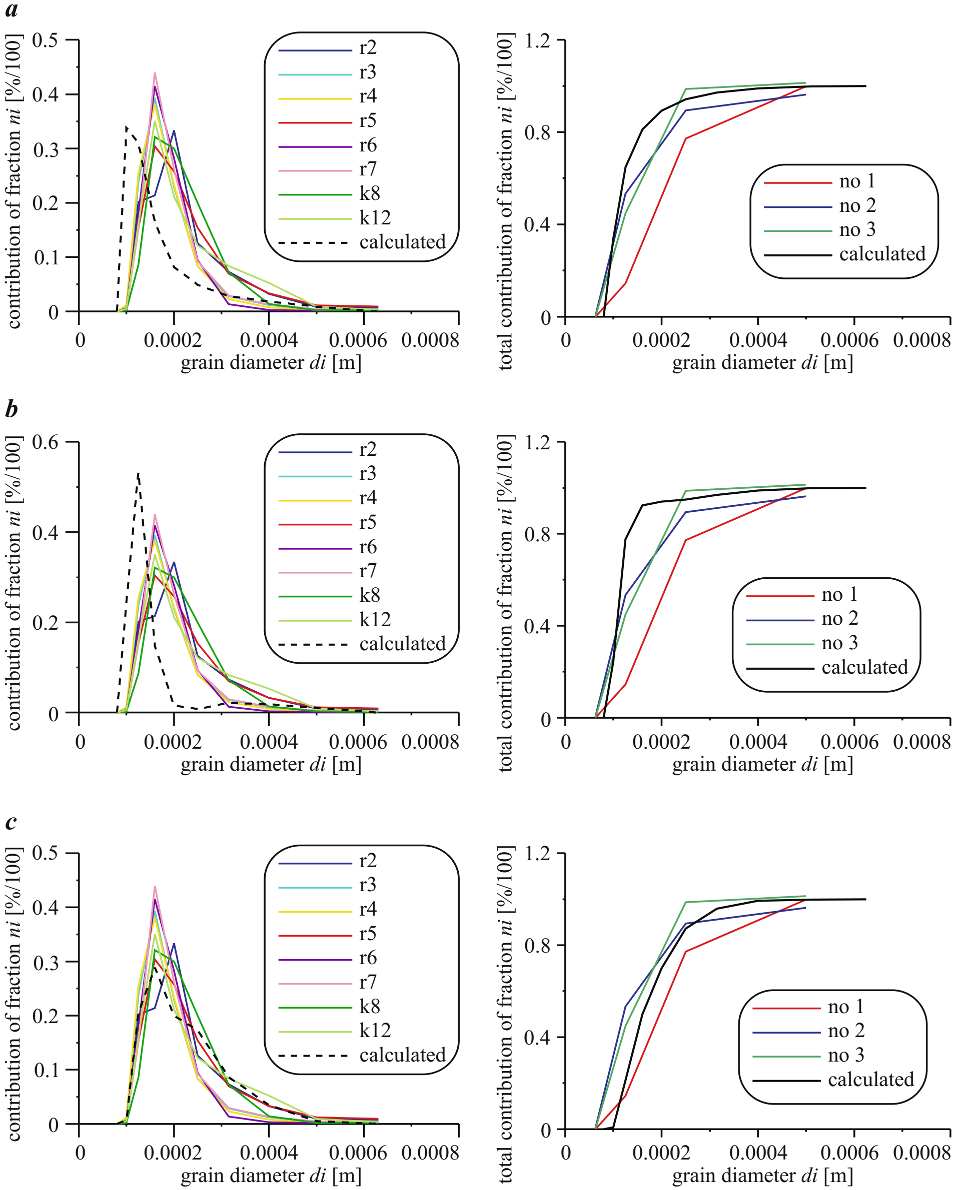

Fig. 5. Comparison of the measurements with the modelling of grain-size distribution of sediments silting up a water route for a) unimodal, b) bimodal, c) measured distribution (r2-r7, k8, k12 – designations of surface sediment samples taken from the approach route in Łeba)

The second stage comprised an analysis of the influence of the shape of grain-size distribution on the volume of mean annual silting of this approach route. The calculations involved the grain-size distribution patterns which differed in the shape but had a constant value of d_{50}. The grain-size distributions taken for these calculations are shown in Figure 4 against the background of the grain-size distribution measured along the edge of the route and used for the calculations presented in this paper. The following values of the mean annual silting up of the water route have been obtained: for a bimodal distribution with a significant mode within the fine-grained fractions $Q = 170$ thousand m^3/year; for a unimodal distribution $Q = 126$ thousand m^3/year; for the distribution measured along the edge of the route $Q = 63{,}609$ m^3/year. For comparison, the dredging and silting work in 2005 removed 69,300 m^3/year (the data given by the Maritime Office in Słupsk).

As seen from the above, increased contribution of fine fractions, even at the constant value d_{50}, has a considerable influence on the silting up of a water route. Figure 5 illustrates a comparison between the computed results for grain-size distribution of sediments silting up the route as uni- and bimodal calculations and these measured along the edge of the water route. Very good agreement was obtained between the computations and the measurements of the grain-size distribution measured at the edge of the water route. It is evident that the calculated grain-size distributions for uni- and bimodal distribution models are considerably different from those measured in the water route.

To recapitulate, it should be emphasized that knowledge of only one parameter, i.e. d_{50} may not suffice for a reliable evaluation of sediment transport and analysis of silting up of water routes. Apart from the value of d_{50}, the shape of grain-size distribution is another important component. The meaning of this element becomes ever more important as the contribution of fine-grained fractions increases.

Summary

The aim of this paper has been to describe a model of non-uniformly graded sediment transport and to present how this model can be helpful in describing the process of silting up of navigation channels.

The mathematical model has been presented in the form of a set of numerical algorithms. The calculations that have been executed show that it is possible to predict the amounts of transported sediments of different grain size and to assess the transport intensity of particular fractions. Consequently, it is possible to detect the grain-size distributions of the sediments which fill up an approach route to a port. Such information can be used to plan and carry out dredging and silting work, which is associated with artificial maintaining of seashores. The material picked up from water routes as a result of dredging is used to support the shores near ports, which means that it is essential to have good knowledge of its grain-size distribution.

The analysis of the effect of grain-size distribution on the silting up of a water route has revealed that in order to be able to successfully assess the rate of silting up it is not enough to know the parameter d_{50}. Another essential factor is the grain-size distribution shape. Knowing this component becomes even more important when sediments contain large proportions of fine sands.

The research results presented in this paper are a stage in the execution of a research and development grant number R0401703 on the hydrodynamic aspects of the plan to built a canal to the Vistula Lagoon.

References

BIEGOWSKI J. 2006. *Dynamika osadów morskich o niejednorodnym uziarnieniu w świetle teorii i eksperymentu.* Praca doktorska, IBW PAN.

DEIGAARD R. 1993. *Modelling of sheet flow: dispersion stresses vs. the diffusion concept.* Prog. Rep., 74, Inst. Hydrodyn. and Hydraulic Eng., Tech. Univ. Denmark, pp. 65-81.

DE MEIJER R.J., BOSBOOM J., CLOIN B., KATOPODI I., KITOU N., KOOMANS R.L., MANSO F. 2002. *Gradation effects in sediment transport.* Coastal Engineering, 47: 179-210.

FREDSØE J. 1984. *Turbulent boundary layer in combined wave-current motion.* J. Hydraulic Eng., ASCE, Vol. 110, No. HY8: 1103-1120.

HASSAN W.N., KROEKENSTOEL D.K., RIBBERINK J.S. 2001. *Size-gradation effect on sand transport rates under oscillatory sheet-flows.* Proc. Coastal Dynamics '01, ASCE, Reston VA, pp. 928-937.

KACZMAREK L.M, MIELCZARSKI A., NAGUSZEWSKI A., OSTROWSKI R., PRUSZAK Z., SKAJA M., SZMYTKIEWICZ M. 1996. *Badanie ruchu rumowiska w Porcie Łeba. Ocena przewidywanego wpływu projektowanej przebudowy układu falochronów na warunki zapiaszczania portu i na kształtowanie się brzegu.* Opracowanie wewnętrzne, IBW PAN, Gdańsk.

KACZMAREK L.M., MIELCZARSKI A., OSTROWSKI R. 1997. *Zastosowanie nowego modelu transportu osadów do oceny zapiaszczania portowych torów podejściowych*. Inżynieria Morska i Geotechnika, 21: 113-117.

KACZMAREK L.M. 1999. *Moveable Sea Bed Boundary Layer and Mechanics of Sediment Transport.* IBW PAN, Gdańsk.

KACZMAREK L.M., OSTROWSKI R. 2000. *Wzdłużbrzegowy transport osadów morskich w świetle modelu mieszaniny wodno-gruntowej i danych terenowych.* Inżynieria Morska i Geotechnika, 3:. 128-133.

KACZMAREK L.M., R. OSTROWSKI 2002. *Modelling intensive near-bed sand transport under wave-current flow versus laboratory and field data.* Coastal Engineering, 45(1): 1-18.

KACZMAREK L.M., BIEGOWSKI J., OSTROWSKI R. 2004. *Modelling cross-shore intensive sand transport and changes of grain size distribution versus field data.* Coastal Engineering, 51: 501-529.

KACZMAREK L.M., BIEGOWSKI J. 2006. *Modelowanie segregacji osadów wzdłuż profilu poprzecznego brzegu w porównaniu z pomiarami w naturze*. Inżynieria Morska i Geotechnika, 2: 79-87.

SAYED M., SAVAGE S.B. 1983. *Rapid gravity flow of cohesionless granular materials down inclined chutes*. J. Applied Mathematics and Physics (ZAMP), 34: 84-100.

ZDUNEK A. 2006. *Analiza hydrodynamiczna zapiaszczania toru podejściowego do portu Łeba w aspekcie projektowania budowli i optymalizacji prac pogłębiarskich*. Praca magisterska, Uniwersytet Warmińsko-Mazurski, Olsztyn.

Accepted for print 4.07.2008 r.