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SELECTED PROBLEMS OF MEASUREMENT UNCERTAINTY – PART 2

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K e y w o r d s: measurement uncertainty, A-type and B-type uncertainty evaluation, normal (Gaussian) and rectangular distribution, uncertainty budget.

A b s t r a c t

The rules for evaluation of measurement uncertainty are presented in this paper. It includes the list of definitions for basic terms that are associated with the issue and explains how to evaluate the measurement uncertainty for well-defined physical parameters that are considered as the measurands (part 1). Next, the methods for evaluation of measurement uncertainty are exhibited on the examples when measurement uncertainty is estimated for verification of a micrometer as well as evaluated for basic reliability parameters (part 2).

WYBRANE ZAGADNIENIA NIEPEWNOŚCI POMIARU – CZ. 2

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S ł o w a k l u c z o w e: niepewność pomiaru, niepewność typu A i typu B, błąd pomiaru, rozkład normalny i prostokątny, budżet niepewności.

A b s t r a k t

W pracy podano zasady postępowania podczas wyznaczania niepewności pomiaru. Przedstawiono podstawowe pojęcia dotyczące tego zagadnienia oraz sposób wyznaczania niepewności pomiaru dobrze określonej wielkości fizycznej stanowiącej wielkość mierzoną (cz. 1). Przedstawiono sposób szacowania niepewności pomiaru na przykładzie szacowania niepewności podczas sprawdzania mikrometru i szacowania niepewności podstawowych parametrów wytrzymałościowych (cz. 2).

Estimation of uncertainty for verification of a micrometer

The error of micrometer readout can be calculated with use of the formula (ARENDALSKI 2003):

$$E_x = (L - W + P_t) \pm U(E_x) \tag{1}$$

where:

L – the maximum readout from the micrometer for three measure-ments of dimensions,

W – rated length of the standardized plate (W_n) with account for the correction factor – deviations of the plate length,

P_t – correction factor for temperature conditions,

$U(\mathrm{E_x})$ – the expanded uncertainty at the confidence level of $1_\alpha = 0.95$.

The value of the temperature correction factor is neglected.

Uncertainty equation

Due to the fact that input parameters are uncorrelated, the standard uncertainty connected with the already determined absolute deviation for the micrometer readout can be expressed by the formula:

$$u_c(E_x) = \sqrt{\sum (c_i \cdot u_i)^2} \tag{2}$$

where:

c_i – sensitivity coefficients, i.e. the partial derivatives of the measurement function for the function components.

In this case $c_i = 1$ or $c_i = -1$. The standard uncertainty for the limit error can be then expressed in the following form:

$$u_c(E_x) = \sqrt{u^2(L) + u^2(W) + u^2(P_t)} \tag{3}$$

Determining of component standard uncertainties

The standard uncertainty for the micrometer readout is determined on the basis of the instrument resolution r, that for this case is 0.002 (readout with use a magnifying glass), by means of the B-type method and with the assumption of the rectangular distribution of the uncertainty:

$$u(L) = \frac{r}{2\sqrt{3}} = 0.00058 \text{ mm} \tag{4}$$

The standard uncertainty for the standardized plate length is determined on the basis of the expanded uncertainty U for determining of the deviation of the plate length from its rated length as provided by the calibration certificate. The B-type method is applied with assumption of the Gaussian distribution.

$$u(W) = \frac{U}{2} \tag{5}$$

The standard uncertainty for the temperature correction factor $u(P_t)$ is determined under the assumption that coefficients of thermal expansion for the standardized plate and for the micrometer are the same: $\alpha_W = \alpha_L = \alpha_t = 11{,}5 \cdot 10^{-6}\ {}^{o}C^{-1}$ and $\Delta t = t_W - t_L$ (where: $t_W = 20^{o}C$, t_L – ambient temperature in the laboratory room $T_I = 20 \pm 10^{o}C$). The uncertainty $u(\Delta t)$, as determined with use of the B-type method (rectangular distribution) amounts to $\frac{\Delta t}{2\sqrt{3}}$. Thus:

$$u(P_t) = W \cdot \alpha_t \cdot \frac{\Delta t}{2\sqrt{3}} \tag{6}$$

Expanded uncertainty

If all values of component uncertainties $u(E_x)$ are comparable, the expanded uncertainty at the confidence level of $1 - \alpha = 0.95$ can be calculated with the formula:

$$U(E_x) = 2 \cdot u_c(E_x) \tag{7}$$

If the uncertainty $u_c(E_x)$ has one dominating component (e.g. $u(L)$) and the rectangular distribution is assumed, then the distribution can be considered as the distribution for readout errors and then the expanded uncertainty at the level of confidence of $1 - \alpha = 0.95$ can be calculated with the formula:

$$U(E_x) = 1.65 \cdot u_c(E_x) \tag{8}$$

If the uncertainty $u_c(E_x)$ has two dominating components (e.g. $u(L)$ and $u(W)$) and the rectangular distribution is assumed for the both components with the respective spans $R_1 = 2a_1$ and $R_2 = 2a_2$, the components can be superposed to make up the trapezoidal evenarmed distribution with the span of $R = 2a = 2(a_1 + a_2)$ and upper base of $b = 2a\beta$, where $\beta = (a_2 - a_1)/(a_2 + a_1)$.

Then the composed uncertainty amounts to

$$u_c(E_x) = \sqrt{u^2(x_1) + u^2(x_2)} = \frac{\sqrt{a_1^2 + a_2^2}}{\sqrt{3}} \tag{9}$$

The expanded uncertainty at the confidence level of $P = 1 - \alpha = 0.95$ can be calculated by the formula:

$$U(E_x) = \frac{1 - \sqrt{(1-P)(1-\beta^2)}}{\sqrt{\frac{1+\beta^2}{6}}} \cdot u_c(E_x) \tag{10}$$

Finally, the indication error value shall be:

$$E_x = max\ \{d - U(E_x),\ d + U(E_x)\} \tag{11}$$

where:

d – the maximum deviation from the W dimension, $d = L - W$.

Note: uncertainty components $u_c(E_x)$ can be considered as insignificant and then neglected if

$$u_c(E_x) - u_c^*(E_x) \leq 0{,}05\ u_c(E_x)$$

(where $u_c^*(E_x)$ – the composed uncertainty with one or two components ignored), i.e. when omission of one or two components in the formula for the uncertainty calculation results in alteration of such uncertainty by not more than 5%.

Uncertainty budget

All the informations for analysis of uncertainty are brought together in the table 1.

Table 1

Uncertainty budget for calculation of the composed uncertainty for verification of a micrometer

Parameter symbol X_i	Parameter estimation x_i	Standard uncertainty $u(x_i)$	Contribution of $u(x_i)$ into the standard uncertainty for each individual standardized plate (of n)			
L	–	Eq. (4)	$c_i\ u_1(L_1)$	$c_i\ u_2(L_2)$	$c_i\ u_3(L_3)$	$c_i\ u_n(L_n)$
W	–	Eq. (5)	$c_i\ u_1(W_1)$	$c_i\ u_2(W_2)$	$c_i\ u_3(W_3)$	$c_i\ u_n(W_{3n})$
P_t	0	Eq. (6)	$c_i\ u_1(P_{t1})$	$c_i\ u_2(P_{t2})$	$c_i\ u_3(P_{t3})$	$c_i\ u_n(P_{tn})$
E_x	$L - W$	–	Eq. (3)	Eq. (3)	Eq. (3)	Eq. (3)

Estimation of uncertainty for determination of the tensile strength

Tensile strength (CWA 15261-2. 2005)

$$R_m = f(F_m, \bar{d}_0) = \frac{F_m}{S_0} = \frac{4F_m}{\pi \bar{d}_0^2} \tag{12}$$

where:
F_m – maximum force recorded during the tensile test,
S_0 – initial cross-section of the specimen,
$\bar{d}_0$ – initial average diameter of the specimen.

Uncertainty equation

Due to the fact that input parameters are uncorrelated, the standard uncertainty connected with the determined tensile strength of the specimen is defined by the following formula:

$$u(R_m) = \sqrt{\Sigma(c_i \cdot u_i)^2} \tag{13}$$

where:
c_i – sensitivity coefficients, i.e. the partial derivatives of the measurement function for the i^{th} function components,
u_i – standard uncertainties for individual components.

In this case $c_{Fm} = \frac{4}{\pi \bar{d}_0^2}$ and $c_{do} = -\frac{8F_m}{\pi \bar{d}_0^3}$, thus:

$$u(R_m) = \sqrt{\left(\frac{4}{\pi \bar{d}_0^2}\right)^2 u^2(F_m) + \left(-\frac{8F_m}{\pi \bar{d}_0^3}\right) u^2(\bar{d}_0)} \tag{14}$$

Determination of component standard uncertainties

The uncertainty for measurement of the specimen diameter is calculated by means of:

a) on the basic of the arithmeticcal means for the series of six measurements (the A-type method), with the t-Student distribution assigned (for $p_\alpha =$ $= 68.27\%$):

$$u(\bar{d}_{0s}) = 1.11 \cdot \sqrt{\frac{\sum_{k=1}^{n} (d_{0k} - \bar{d}_0)^2}{n(n-1)}} \tag{15}$$

where:
n – number of measurements, or

b) on the basic of the micrometer resolution, with use of the formula:

$$u(d_{0m}) = \frac{0.01}{2\sqrt{3}} \tag{16}$$

where:
$u(d_{0m})$ amounts to 0.00289 mm.

The value which is higher is adopted for further calculations.

Major factors that affect total uncertainty of measurement of the F force include:

– uncertainty of the measurement of the force

$$u_w(F_m) = \frac{U_{Fm} \cdot F_m}{200} \tag{17}$$

where:
U_{Fm} – the uncertainty of measurement (in percents), attributable to the dynamometer of the machine, as read from the calibration certificate for the force that is the closest to the force value F_m measured during the tensile test and for the selected range of the applied measuring head,

– zero adjustment in the forcemeasuring path,
– possible misalignment of the force applied,
– ambient temperature during test and rate of the load application.

The error that results from the aforementioned factors was evaluated to ±1%. Therefore, the uncertainty for measurement of the maximum force shall be calculated with the formula:

$$u(F_m) = \frac{0.01 \cdot F_m}{\sqrt{3}} \tag{18}$$

Uncertainty budget

The information for further analysis of uncertainty is presented in the Table 2.

Table 2

Uncertainty budget for composed uncertainty for tensile strength

Parameter symbol X_i	Parameter estimation x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Contribution into the composed standard uncertainty $u(x_i)$
Fm	–	Eq. (18)	$\frac{4}{\pi \bar{d}_0^2}$	$c_i \cdot u(F_m)$
$\bar{d}_0$	–	Eq. (15) or (16)	$-\frac{8F_m}{\pi \bar{d}_0^3}$	$c_i \cdot u(\bar{d}_0)$
R_m	$\frac{4F_m}{\pi \bar{d}_0^2}$	–	–	Eq. (14)

Determination of the expanded uncertainty

The relative expanded uncertainty with the expansion coefficient $k_\alpha = 2$ at the confidence level of $1 - \alpha = 0.95$ is calculated as:

$$U(R_m) = \frac{2 \cdot u(R_m)}{R_m} \cdot 100\% \tag{19}$$

Estimation of the uncertainty for determination of the proof stress

Proof stress (at non proportional elongation)

$$R_{p0.2} = f\,(F_{0.2}, \bar{d}_0) = \frac{F_{0.2}}{S_0} = \frac{4F_{0.2}}{\pi \bar{d}_0^2} \tag{20}$$

where:

$F_{0.2}$ – tension force that is applied during the test and then brings to permanent elongation of the specimen equal to 0.2% of the measurement length that corresponds to the extensometer base,

S_0 – initial cross-section of the specimen,

$\bar{d}_0$ – initial average diameter of the specimen.

Uncertainty equation

Due to the fact that input parameters are uncorrelated, the standard uncertainty connected with the determined proof stress is defined as:

$$u(R_{p0.2}) = \sqrt{\sum (c_i \cdot u_i)^2} \tag{21}$$

where:

c_i – sensitivity coefficients, i.e. the partial derivatives of the measurement function for the i^{th} function component,

u_i – standard uncertainties for individual components.

In this case $c_{F0.2} = \frac{4}{\pi \bar{d}_0^2}$ and $c_{do} = -\frac{8f_{0.2}}{\pi \bar{d}_0^3}$, thus:

$$u(R_{p0.2}) = \sqrt{\left(\frac{4}{\pi \bar{d}_0^2}\right)^2 u^2(F_{0.2}) + \left(-\frac{8F_{0.2}}{\pi \bar{d}_0^3}\right)^2 u^2(\bar{d}_0)} \tag{22}$$

Determination of component standard uncertainties

The uncertainty for measurement of the specimen diameter – equations (15 ÷ 17).

Overall uncertainty for measurement of the force $F_{0.2}$:

$$u_c(F_{0.2}) = \sqrt{u^2(F_{0.2}) + u^2(\Delta F_{0.2}) + u^2(F_{0.2E})} \tag{23}$$

where subsequent components result from:
– uncertainty of the force measurement [see formula (18)]

$$u(F_{0.2}) = \frac{0.01 \cdot F_{0.2}}{\sqrt{3}}$$

– recording frequency during automatic measurement

$$u(\Delta F_{0.2}) = \frac{F_{0.2(1)} - F_{0.2(2)}}{2\sqrt{3}}$$

where:
$F_{0.2(1)}$ – the closest value of force that was higher than $F_{0.2}$,
$F_{0.2(2)}$ – the closest value of force that was lower than $F_{0.2}$;

– inclination of the straight line that is parallel to the linear section of the σ-ε curve that is described by the formulas: $\sigma_{0.2E} = E(\varepsilon - 0.002)$ or

$$F_{0.2E} = \frac{\Delta F}{\Delta \varepsilon}(\varepsilon - 0.002)$$

$$u(F_{0.2E}) = \sqrt{\left(\frac{\varepsilon - 0.002}{\Delta\varepsilon}\right)^2 \cdot u^2(\Delta F) + \left(-\frac{\Delta F(\varepsilon - 0.002)}{(\Delta\varepsilon)^2}\right)^2 \cdot u^2(\Delta\varepsilon) + \left(\frac{\Delta F}{\Delta\varepsilon}\right)^2}$$

where:

$$u(\Delta F) = \sqrt{u^2(F_{max}) + u^2(F_{min})}$$

where:

$$u(F_{max}) = \frac{0.01 \cdot F_{max}}{\sqrt{3}} \quad \text{and} \quad u(F_{min}) = \frac{0.01 \cdot F_{min}}{\sqrt{3}}$$

$$u(\Delta\varepsilon) = \sqrt{u^2(\varepsilon_{max}) + u^2(\varepsilon_{min})}$$

where:

$$u(\varepsilon_{max}) = \frac{K_\varepsilon \cdot \varepsilon_{max}}{\sqrt{3}} \quad \text{and} \quad u(\varepsilon_{min}) = \frac{K_\varepsilon \cdot \varepsilon_{min}}{\sqrt{3}}$$

$$U(\varepsilon) = \frac{K_\varepsilon \cdot \varepsilon}{\sqrt{3}}$$

where:
K_ε – accuracy class of the applied extensometer.

Uncertainty budget

The informations for further analysis of uncertainty are presented in the Table 3.

Table 3
Uncertainty budget for composed uncertainty for proof stress

Parameter symbol X_i	Parameter estimation x_i	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Contribution into the composed standard uncertainty $u(x_i)$
$F_{0.2}$	–	Eq. (23)	$\frac{4}{\pi \bar{d}_0^2}$	$c_i \cdot u_c(F_{0.2})$
$\bar{d}_0$	–	Eq. (15) or (16)	$-\frac{8F_{0.2}}{\pi \bar{d}_0^3}$	$c_i \cdot u(\bar{d}_0)$
$R_{p0.2}$	$\frac{4F_{0.2}}{\pi \bar{d}_0^2}$	–	–	Eq. (22)

Determination of the expanded uncertainty

The relative expanded uncertainty with the expansion coefficient $k_\alpha = 2$ at the confidence level of $1 - \alpha = 0.95$ is calculated with the following formula:

$$U(R_{p0.2}) = \frac{2 \cdot u(R_{p0.2})}{R_{p0.2}} \cdot 100\% \tag{24}$$

Recapitulation

On the basic of presented above methods for determination of measurement error and uncertainties the analysis of micrometer verification results and test proficiency results (in with accredited testing laboratory of Air Force Institute of Technology have been participated) were done.

The research covered verification of the analogue micrometer with its measurement range 0 ÷ 25 mm, serial No 102-217-9157246, manufactured by MITUTOYO company. For reference the set of gauge plates was used, No 714390, with its calibration certificate No M11-419-578.3/2004.

The verification procedure was carried out at the ambient temperature of 20 ± 0.2°C, in accordance with the instruction from the metrological surveillance IW-31-11-L5, for the full measurement range of the micrometer when the gauge plates with rated sizes of W_n = 1; 1.05; 1.5; 2; 5; 8; 10; 15; 20; 25 mm were subsequently used. Verification results are collected in Table 6.

Table 6

Results of the micrometer verification, (mm)

Length of the standardized plate W_n	Readout / established value					Indication error	
	readout I L_1	readout II L_2	readout III L_3	maximum deviation from the W dimension d	measurement uncertainty $U(E_x)$	$d+U(E_x)$	$d-U(E_x)$
1	1.000	1.000	1.001	0.001054	0.0009	2.0	0.1
1.05	1.051	1.050	1.051	0.000921	0.0009	1.9	0.0
1.5	1.500	1.500	1.500	0.000008	0.0009	0.9	-0.9
2	2.000	2.001	2.000	0.001060	0.0009	2.0	0.1
5	5.000	5.001	5.001	0.001048	0.0009	2.0	0.1
8	8.000	8.001	8.001	0.001006	0.0009	1.9	0.1
10	10.0011	0.0011	0.001	0.000948	0.0009	1.9	0.1
15	15.0011	5.0011	5.001	0.000920	0.0009	1.8	0.0
20	20.002	20.001	20.001	0.001911	0.0009	2.8	1.0
25	25.002	25.002	25.001	0.001798	0.0009	2.7	0.9

Verification result was accepted as passing one, because the indication error equal 2.8 μm was less than the permissible limit error for micrometric instruments E_g = ±4 μm (PN-82/M-53200).

In 2005 the Laboratory for Material Strength Testing (LMST) participated in the survey of competence – the proficiency test (PT) (scope of strength tests for round steel bars under room temperature). The survey was organized by the Institut für Eignungsprüfung (Germany). The survey brought together research laboratories from 29 countries and 78 of the participants had the accreditation in accordance with the standard EN ISO/IEC 17025.

The examination results along with uncertainties values, calculated in accordance with the foregoing formulas, are presented in Table 5.

Based on comparison of data covered by the report (*Proficiency Test*... 1999) submitted to the Institut für Eignungsprüfung with the information presented in Table 5 the conclusions about general matching of the results can be made. The LMST laboratory has fulfilled the requirements related to the competence survey and was granted with the certificate.

Table 5

Measurement results obtained by the LMST laboratory during the competence survey in 2005

No of specimens	$R_{p0,2}$ (MPa)	$U(R_{po,2})$ (%)	R_m (MPa)	$U(R_m)$ (%)	A (%)	$U(A)$ (%)	E (MPa)	$U(E)$ (%)
1	719	± 3.06	796	± 1.24	13.9	± 1.18	185 000	± 2.13
2	721	± 3.01	795	± 1.21	14.3	± 1.17	186 400	± 2.13
3	718	± 3.03	792	± 1.28	13.5	± 1.17	187 200	± 2.14
4	714	± 3.01	784	± 1.18	16.1	± 1.17	185 300	± 2.11
5	704	± 3.12	781	± 1.24	14.8	± 1.17	186 200	± 2.18
6	712	± 3.01	792	± 1.20	14.1	± 1.18	188 700	± 2.11
Average LMST	715	± 3.04	790	± 1.23	14.6	± 1.17	186 500	± 2.13
Participants	712	± 2.00	790	± 1.30	16.2	± 2.00	186 500	± 4.00
Organizers	691	± 2.30	786	± 1.36	15.7	± 1.20	n/a	n/a

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