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ALGORITHMS APPLIED IN TURBOMACHINE MODELING WITH VARIABLE INPUT DATA

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K e y w o r d s: modeling, heuristic, rotor, bearing.

A b s t r a c t

The idea of the allowance method for the input data uncertainties in the turbo-machines rotor-bearings systems modelling are contained in the paper. The author presents selected calculation results, when two different generation algorithms of the random variability course of the shear force, that imposes the rotor shaft loading, have been applied. The initial numerical research results displayed in this paper allow for acknowledge of such tools' kind usability, however they also signal the difficulties occuring in the interpretation of the alanysed systems answers, received this way.

ALGORYTMY STOSOWANE W MODELOWANIU MASZYN WIRNIKOWYCH Z UWZGLĘDNIENIEM ZMIENNOŚCI DANYCH WEJŚCIOWYCH

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S ł o w a k l u c z o w e: modelowanie, heurystyka, wirnik, łożyskowanie.

A b s t r a k t

Artykuł opisuje ideę uwzględniania niepewności danych wejściowych w modelowaniu układów wirnik-łożyska w maszynach wirnikowych. Przedstawiono wybrane rezultaty obliczeń z zastosowaniem dwóch różnych algorytmów generowania losowego przebiegu zmienności siły poprzecznej obciążającej wał wirnika. Wyniki wstępnych badań numerycznych pozwalają na potwierdzenie przydatności wykorzystywanych narzędzi, jednak sygnalizują także występujące trudności w interpretacji otrzymywanych tą drogą odpowiedzi analizowanych układów.

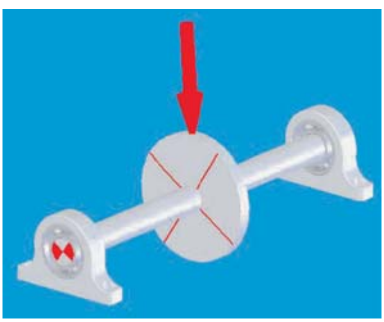
Introduction

In view of the ongoing progress in calculation technology, numerical calculations are popularly applied in various types of scientific research. Despite a very high level of advancement, further development is needed due to the following reasons:
- difficulties are encountered in the process of building mathematical models that describe physical phenomena with sufficient accuracy,
- the selection of an adequate model from the range of the existing models and the models proposed by engineering software designers is a difficult process,
- as a calculating machine, a computer operates based on deterministic algorithms, and every time the performed calculations are replicated, a single, identical result is produced for the same set of input data,
- as regards various models and numerical tools, the results are unknown when input data are modified during calculations.

In view of the above problems, the results of numerical calculations are often difficult or impossible to compare with the outcome of experiments carried out on a real object where the results of successive replications often differ not only due to the inaccuracy of measurements but also due to the variability of experimental conditions. This paper discusses the existing algorithms for generating stochastic variable input data used in numerical calculations. The objective of this long-term study was to validate the suitability of the analyzed algorithms for turbomachine modeling and to bring the results of numerical calculations closer to experimental data.

The analyzed model and the scope of the study

Figure 1 presents the model analyzed in this paper. It is a single-mass, inboard rotor with a journal bearing. Concentrated mass is found in the mid-length of the rotor in the form of a disc. The rotor is subjected to a single



shaft length:	1.4 m
shaft diameter:	0.1 m
disc diameter:	0.4 m
disc thickness:	0.1 m
rotor weight:	179 kg

Fig. 1. The analyzed model of a rotor subjected to rotating transverse force

variable transverse force which rotates at a set rotational speed when applied at the mid-length of the rotor. The known ratio of the rotational speed of that force to the rotor's rotational speed is marked with the symbol x. This study presents the results for $x = 1$.

The objective of the performed calculations was to determine the effect of changes in transverse force during the calculations on the trajectory of the rotor shaft axis and its properties, mostly the direction of rotation, shape, angle of inclination and the effect on the frequency and amplitude of rotor axis vibrations. The observed trajectories will apply to the rotor axis in the cross-section of the applicable transverse force.

As regards physical phenomena applicable to turbomachines, the effect of changes in force value on resonance and the loss of rotor-bearing stability had to be determined. The objective of the experiment was to:

– determine the effect of stochastic variability in transverse force on the trajectory of the rotor shaft axis (PIETKIEWICZ 2007),
– investigate the response of the rotor-bearing system in the form of rotor vibrations, subject to the shaft's rotational speed, at variable transverse force, and to evaluate the compatibility of the resulting functions with the functions mapped for constant force values (PIETKIEWICZ 2007),
– compare resonance phenomena occurring at constant and variable transverse force (PIETKIEWICZ 2007),
– compare the phenomena accompanying the loss of rotor-bearing stability (PIETKIEWICZ 2008),
– model the location where the rotor's moving parts rub against the fixed components of the modeled object (PIETKIEWICZ 2009) (in an actual object, e.g. against the housing).

The applied algorithms for generating stochastic processes

Numerical calculations were performed with the use of tools developed and applied by the Szewalski Institute of Fluid-Flow Machinery of the Polish Academy of Sciences in Gdańsk. The solver was the NLDW software described in many publications on turbomachine modeling (KICIŃSKI 2005, PIETKIEWICZ et al. 2006, PIETKIEWICZ 2007), adapted to account for changes in external load applied to a moving rotor. The post-processor were Matlab applications for processing and visualizing calculation results. To account for the stochastic variation of transverse force during rotor operation, a pre-processor had to be build to generate successive values of the load applied to the rotor in accordance with various algorithms. The program generated a series of numbers corresponding to the stochastic variation of the transverse force applied to the rotor at preset rotor operating parameters:

– rotational speed n,
– nominal force P,
– scope of possible changes in transverse force $|\Delta P|$,
– maximum change in force value in the successive stage of generating the force function $|\delta_{max}P|$,
– ratio of the rotational speed of the applied force to rotor speed x,
– angle of shaft rotation $\Delta\alpha$ which leads to a change in force value P.

Transverse force functions were generated according to two types of algorithms. The first algorithm concept is presented in Figure 2. The algorithm generating stochastic variations "without limitations" accounts for random changes in value in the preset range of $-\Delta P$ to ΔP in each calculation step. Figure 2 presents a situation in which nominal force value $P = 500$ N changes in the range of ± 20%, starting at rotational angle $\alpha = 7.5^o$. According to the described algorithm, a change in force value, e.g. from maximum to minimum, is possible (but not necessary) within one step (a section of one shaft revolution) equal to $\Delta\alpha$. In Figure 2, the above can be observed when the angle of rotation changes from 15° to 17.5°.

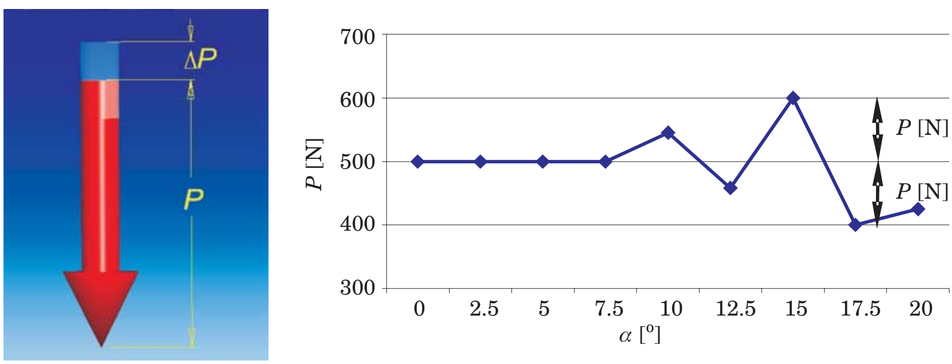


Fig.2. Conceptual algorithm generating a transverse force function "without limitations"

Figure 3 illustrates an algorithm generating a stochastic process "with additional limitations". The additional value $\delta_{max}P$ denotes the maximum scope of the successive change in the transverse force value when the shaft is rotated by angle $\Delta\alpha$. Any changes in the function value, from the maximum value limited by $P + \Delta P$ to the minimum value determined by $P - \Delta P$, are impossible when the shaft rotates only by angle $\Delta\alpha$. In the example shown in Figure 3, the algorithm becomes applicable starting with angle $\alpha = 2.5^o$, and the value of the force changes from maximum to minimum within the range of 7.5° to 17.5°. Owing to its stochastic nature, the illustrated example is purely theoretical and improbable.

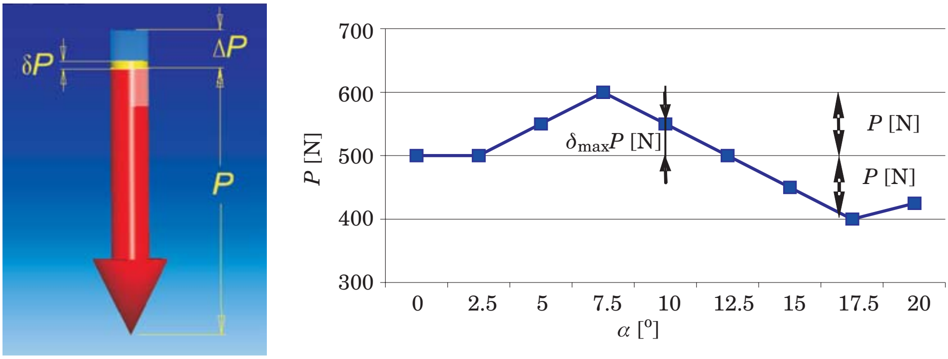


Fig. 3. Conceptual algorithm generating a transverse force function "with additional limitations"

A minimal number of steps in pre-processor operation, designed to change the transverse force value from maximum to minimum, is expressed by the following relationship (1):

$$k = \frac{2 \cdot \Delta P}{\delta_{max} P} \tag{1}$$

The limitation to a single change in the transverse force value was introduced to account for the physical possibility of change in the force value when the rotor moves by angle $\Delta\alpha$. In calculations applicable to high rotational speeds and small angles $\Delta\alpha$, which determine the frequency of change in the value of the force applied to the rotor, a physical explanation could be difficult to formulate for a model "without limitations".

A close correlation exists between the described algorithms. If the algorithm generating a stochastic process "with additional limitations" permits a change in the value of the transverse force from maximum to minimum in one step ($k = 1$), the relationship (1) may be transformed to that shown in (2), which corresponds to an algorithm "without limitations":

$$\delta_{max} P = 2 \cdot \Delta P \tag{2}$$

In view of relationship (2), the algorithm generating the transverse force function "without limitations", which allows for a single-step change in the value of force P from minimum to maximum (or the reverse), will be referred to as algorithm K-1 in subsequent parts of this study. Since the algorithm "with additional limitations" sets the m number of steps required to change the force value from minimum to maximum (or the reverse), this algorithm will be referred to as K-m, where m indicates the minimum number of generator steps needed to change the force value between its extremes, and m is an integer that satisfies the condition $m > 1$.

Results

The use of numerical calculations with deterministic algorithms (e.g. in the form of an NLDW program) for modeling a rotor subjected to a transverse force, as shown in Figure 1, produces results in the form illustrated in Figure 4.

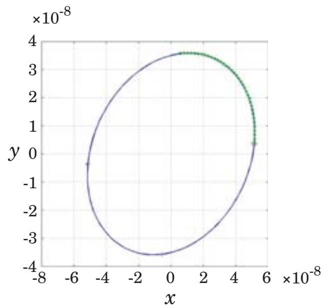


Fig. 4. Model trajectory of the rotor axis at the cross-section where the force is applied

The trajectory of the rotor's axis of symmetry in a horizontal plane is a clearly defined ellipse or, in special cases, a different plane figure (KICIŃSKI 2005, PIETKIEWICZ 2007). The effect of changes in the transverse force applied to the rotor may be described with the use of the existing numerical methods only as regards the determination of the modeled system's response in the form presented in Figure 5.

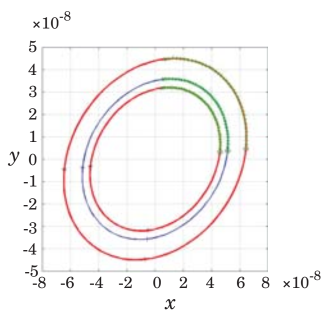


Fig. 5. A quasi-static approach to accounting for changes in the value of the transverse force applied to the rotor

Assuming that the system is subjected to a force in the range of F_{min} to F_{max}, when changes in the value of the force applied to the rotor are taken into account, the system's probable response to the load should be within the range marked by two red trajectories (Fig. 5). They correspond to the system's response to load which varies subject to a preset range of values. This hypothesis does not account for changes in load value during the calculations which correspond to the motion of the rotor shaft. The results accounting for changes in the value of the transverse force during the modeled rotation are presented below. The relevant calculations were performed on the following assumptions:

– the rotor is ideally balanced,
– the rotational speed of the shaft equals $n = 3000$ rpm,
– the value of the force applied at node 8 is $P = 500$ N,
– the maximum deviation of force P is $\Delta P = 100$ N (20%),
– the angle of the force change interval is $\Delta\alpha = 2.5^{\circ}$,
– the ratio of angular velocity of the transverse force to the shaft is $x = 1; 2; 0.5$,
– oil viscosity is constant.

Calculations using algorithm *K*-1 for generating variable value of force *P*

For the needs of the presented calculations, it has been assumed that a force with a nominal value of $P = 500$ N, variable in a range of $\Delta P = \pm 100$ N, rotates in accordance with the rotor as regards rotational speed. At the assumed speed of n, the rotor moves at the speed of 50 revolutions per second, producing vibration frequency of 50 Hz. This case will be regarded as the base case in subsequent calculations, and it is presented in Figure 6. In the generated figure drawings, the post-processor registers the results of the rotor's last 6 rotations for which the calculations have been performed.

A hodograph of the transverse load applied to the rotor at mid-length is presented in the top left corner of the table. Changes in the projection of the vector of force P, in one of the adopted directions of the coordinate system, are shown in the top right corner. The rotor-bearing system's response to load is presented in the center in the form of a trajectory of the rotor shaft axis in a transverse plane where force P was applied. Changes in shaft displacement projection in one of the directions of the coordinate system are also illustrated in the center of the table. The third row contains the results of a spectrum signal analysis (fft) of vibrations at a point shared by the rotor axis and the transverse plane where force P rotates. The system's response is marked by an

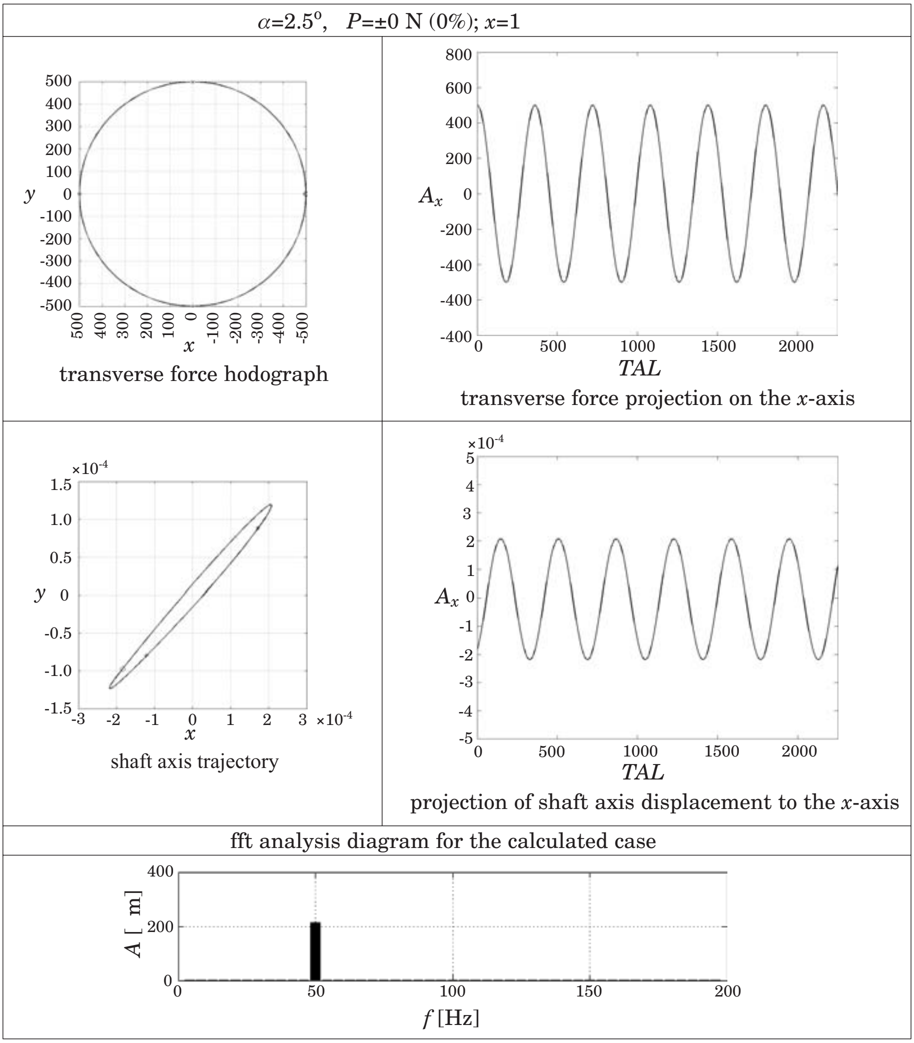


Fig. 6. Calculation results for the base case

ellipse with a clearly defined contour. The lines generated by the calculations of 6 rotor revolutions overlap, pointing to highly stable results, a correct model and its deterministic character.

Figure 7 presents the results of identical calculations performed on the assumption that force P changes its value within the range of 10% of its nominal value, as per algorithm K-1. The force vector hodograph in Figure 7 clearly indicates that the value of force P changes during rotation. The projection

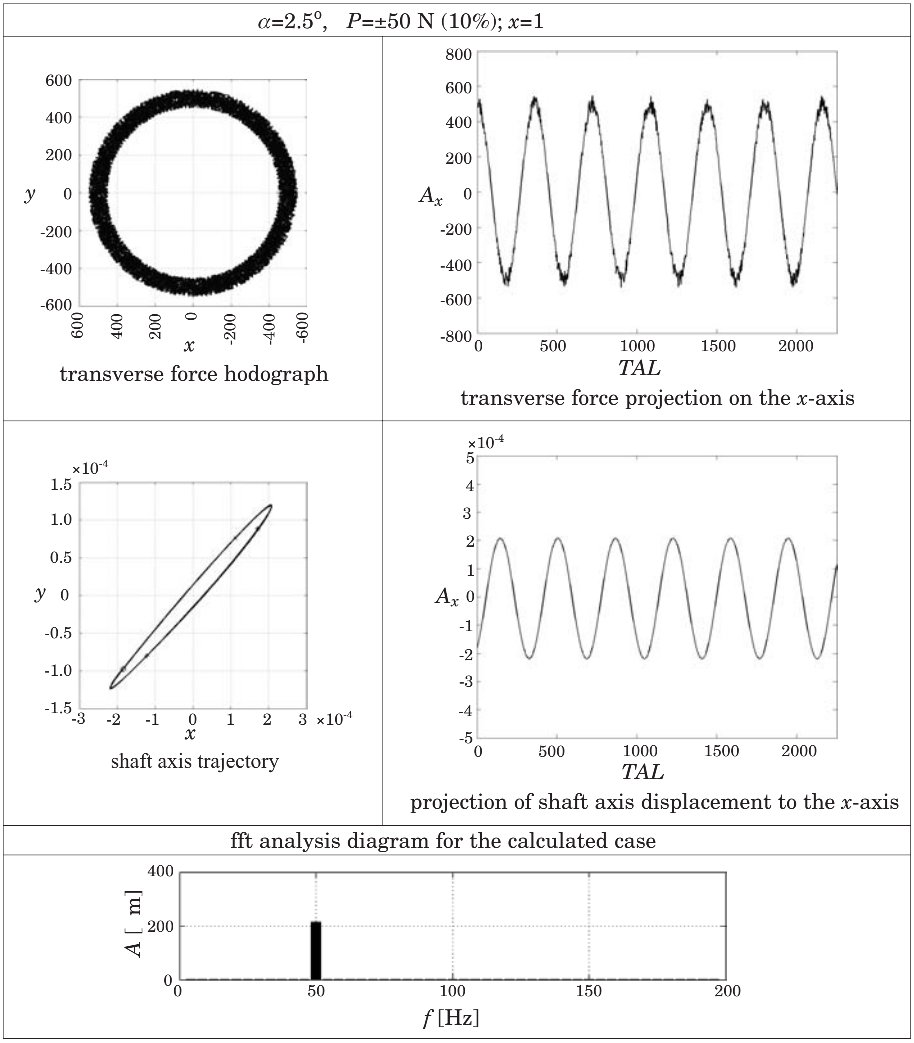


Fig. 7. Calculation results for ΔP = ± 50 N (10%)

of force on the x-axis shows "disturbances" in the sinusoidal pattern of the curve in the area of the relative extremum. The contour of the trajectory of the rotor shaft axis is visibly thicker in comparison with the base case. The above is due to the fact that the trajectory of the observed point is not repeated in the transverse plane in the successive 6 revolutions. The displacement between the ellipses is negligent, and the resulting changes are not noticeable in the diagram of vibration projections on the *x*-axis. The vibration component, drawn as the

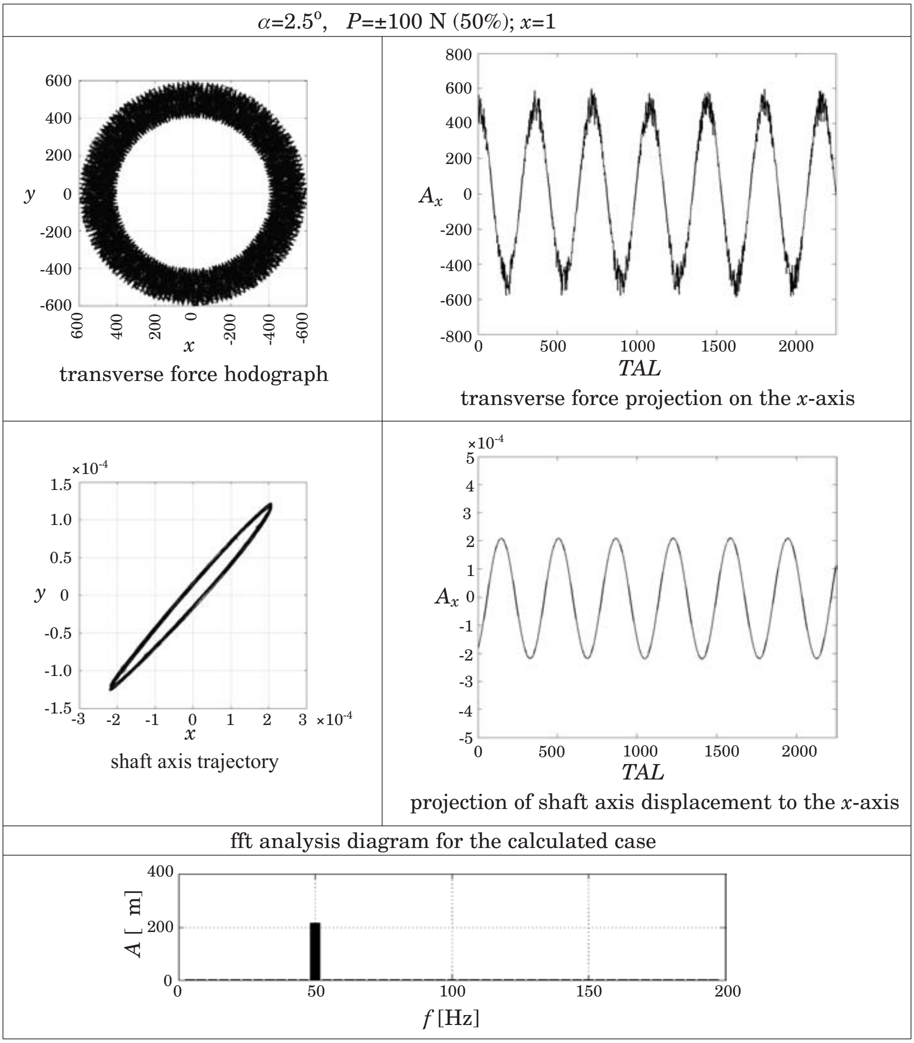


Fig. 8. Calculation results for ΔP = ± 100 N (20%)

spectrum of vibration signals, is maintained at a frequency of 50 Hz. A different frequency of rotor axis vibrations is not revealed.

In Figure 8, which lists calculation results for case ΔP = ± 100 N, the value of force P clearly varies during revolution in comparison with case ΔP = ± 50 N. In the projection on the x-axis, the disturbances in the sinusoidal pattern are visible not only in the area of the relative extremum, but also on the edges of the harmonic function. The above implies that the range of random changes in the value of force P in the direction of the x-axis is comparable with the change resulting from the harmonic function. The system's response is an ellipse with a clearly displaced contour. The projection of the displacement on the x-axis and the fft analysis are not sufficiently sensitive to deviations in the value of force *P* for the changes to become observable to the naked eye.

Calculations using algorithm *K-m* for generating variable force *P*

The experiment was carried out for algorithms *K*-2, *K*-4, *K*-10 and *K*-20. The applied symbols indicate the minimum number of steps required to change the value of force *P* from minimum to maximum, or the reverse (point 2).

Figure 9 presents sample hodographs of transverse force vectors obtained with the use of set algorithms for generating stochastic variations in the value of force *P*. The maximum value has been adopted at $|\Delta P| = 0.2P$. $|\delta_{\max} P|$ values were determined by the value of *m* in each algorithm.

As demonstrated by the hodographs of force *P*, the diagrams produced with the application of algorithms *K*-10 and *K*-20 differ in "structure". As regards the system's response to the applied variable load according to algorithm *K*-10, the trajectory of the rotor axis clearly differs from the remaining trajectories.

Additional calculations were performed to generate stochastic variations in the value of force *P* according to algorithms *K*-10 and *K*-20. Selected results are presented in Figures 10-11.

Every replication of the process generating stochastic variations in the value of force *P* produces completely different results. The hodographs of the transverse force applied to the rotor, as presented in Figure 10, are apparently identical, yet a spectrum analysis of the results produced by one of the calculations shows noise introduced by the random variability of the transverse force. The hodographs of force vectors generated in three successive replications, as presented in Figure 11, show a completely different pattern of variations in the value of the transverse force. In the first process generating the diagram of force *P* values, a cluster of values was noted in the area of the highest allowable value of force *P*, while the second replication produced a more event distribution of stochastic values. In the third replication, the contour of the force diagram was even more clearly defined in the area of both extreme values. The trajectories of the modeled system's response to load are completely different. Disturbances in the shape of the ellipse are visible in the third replication. The above could suggest that the variability of input data over time has a significant effect on calculation results.

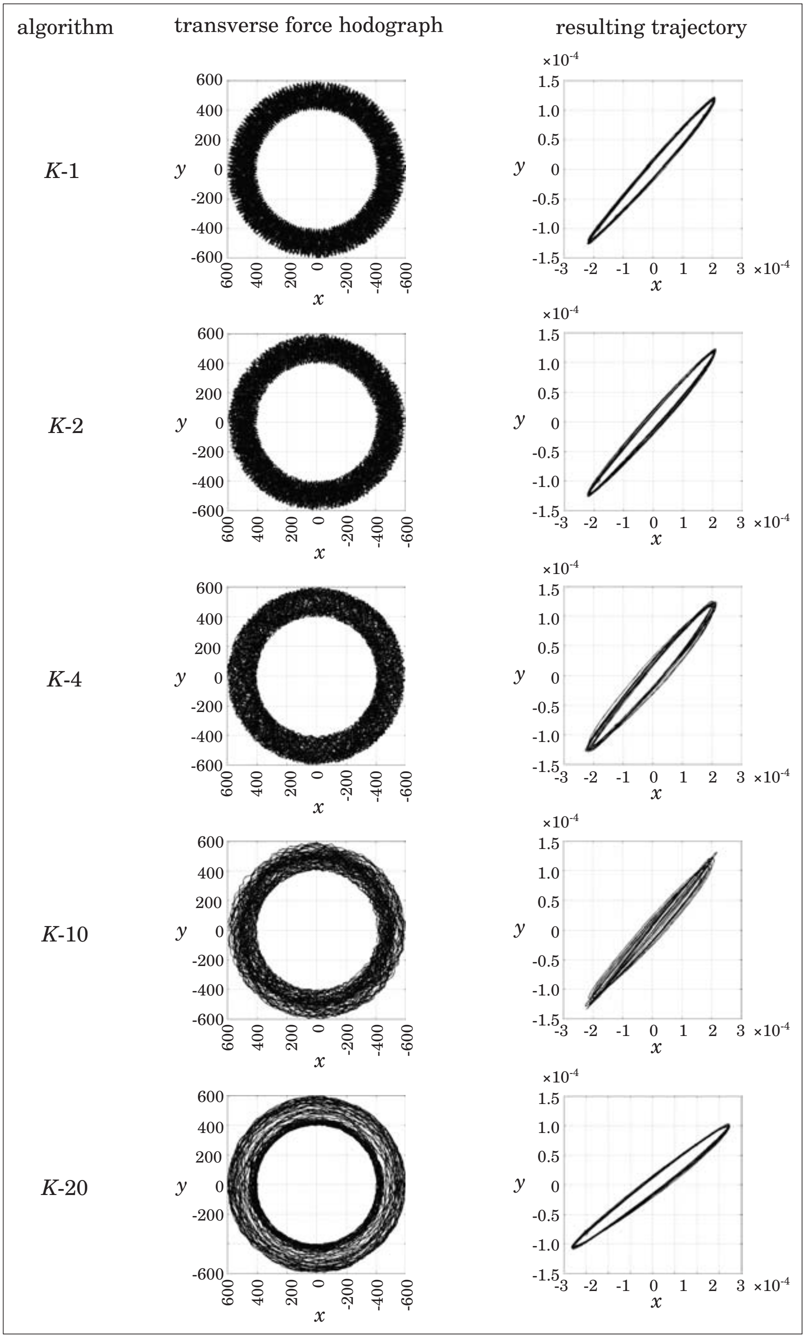


Fig. 9. A comparison of hodographs of stochastically varied transverse forces according to K-*m* algorithms at different values of n, $\Delta P = 0.2\ P$

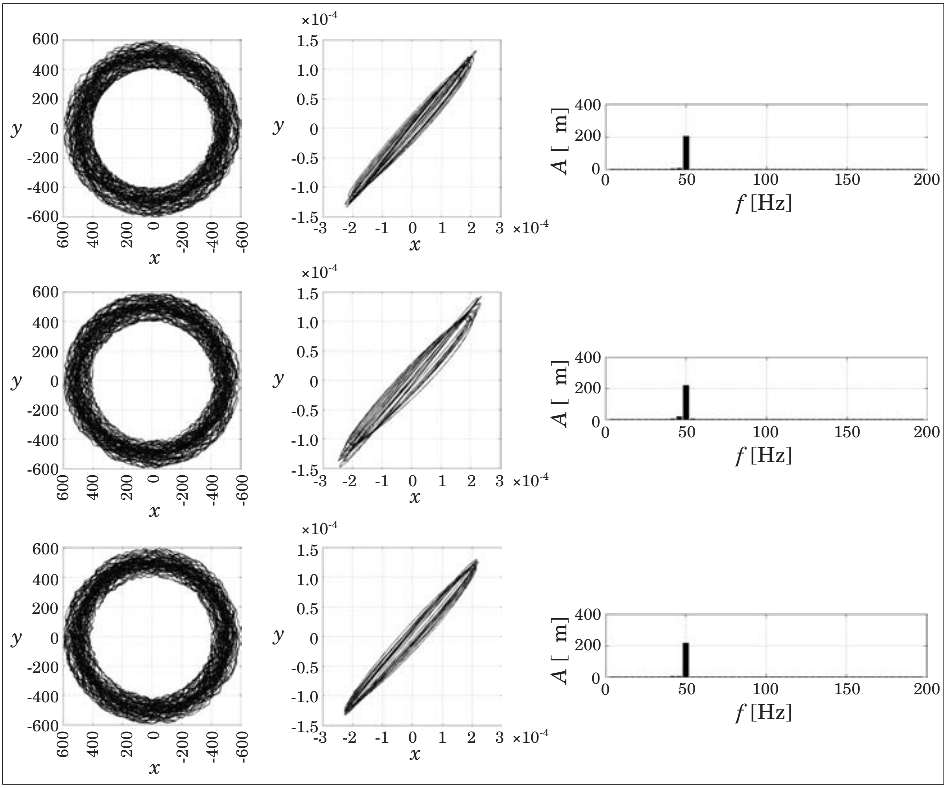


Fig. 10. Results of additional calculations using algorithm *K*-10

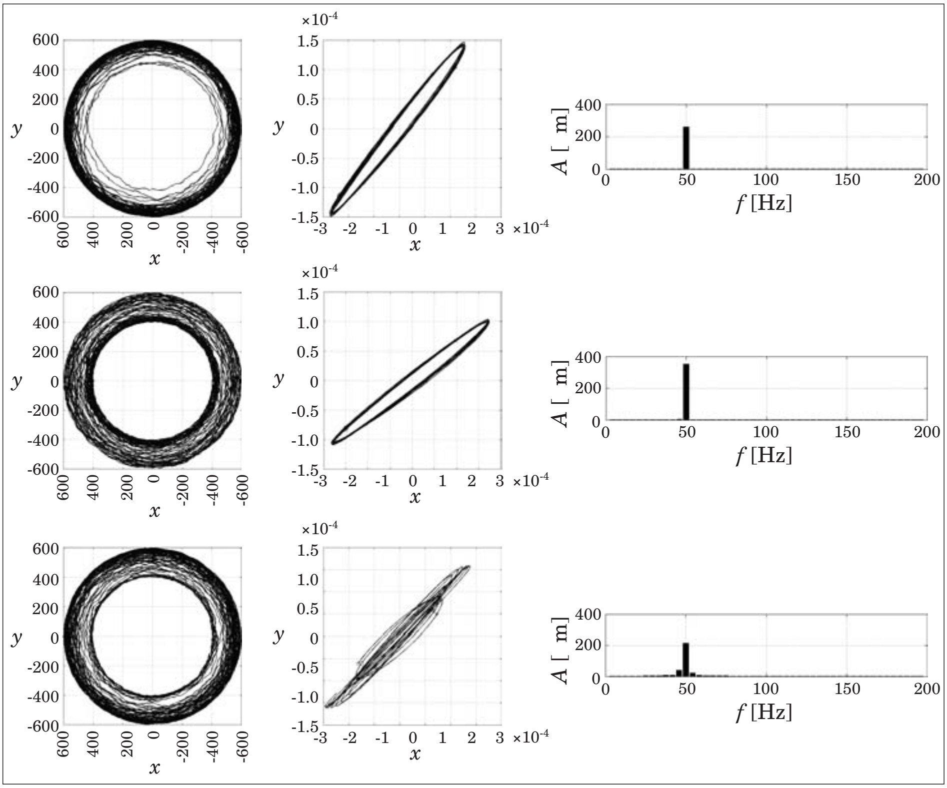


Fig. 11. Results of additional calculations using algorithm *K*-20

Conclusions

The findings of this study point to problems in the process of obtaining reliable results with the use of deterministic algorithms in engineering software. The use of variable input data in numerical calculations and simulations produces a number of probable results rather than a single repeatable result. The above leads to significant inconsistencies in the interpretation of the results.

The presented results concern a model of a rotor-bearing system to which a single force variable over time has been applied. If various forces and their variability are taken into account, the produced results would require a highly accurate and complex interpretation.

A quasi-static approach to accounting for variations in input data does not support the recognition of the non-linearity of correlations that describe the phenomena occurring in the rotor-bearing system. The results of calculations involving stochastic processes that account for variations in the value of the radial force applied to the rotor indicate that the use of the relevant algorithms in numerical calculations accurately reflects the dynamics of the phenomena observed in the studied systems.

According to the author, the absence of full repeatability of the obtained results as well as their form – sets rather than specific values or processes – are more characteristic of experiments replaced by numerical calculations. Similarly to the hitherto practice of interpreting the results of experiments performed on actual objects, the results of many numerical calculations are much more prone to error and difficult to interpret.

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