

## MATHEMATICAL MODELING OF BED GEOMETRY IN THE CULTIVATION OF SEED POTATOES

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Key words: potato, bed cultivation, theoretical analysis.

### Abstract

This paper investigates the effect of the working width of the bed forming unit, tractor wheel track and the inclination angle of ridger wing on the thickness of the soil layer tipped on a potato bed. The developed model was verified in a field experiment.

## MODELOWANIE MATEMATYCZNE GEOMETRII ZAGONU W TECHNOLOGII UPRAWY ZIEMNIAKÓW NA SADZENIAKI

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Słowa kluczowe: ziemniak, uprawa zagonowa, analiza teoretyczna.

### Abstract

Przeprowadzono analizę wpływu ustawienia głębokości roboczej zespołu kształtującego zagon, rozstawy kół ciągnika i kąta nachylenia skrzydeł obsypnika na grubość warstwy gleby usypanej na ukształtowanym zagonie. Oceniono teoretyczną głębokość przykrycia bulw. Opracowany model zweryfikowano.

### Key

$L$  – rear wheel track, mm,

$L_r$  – width of the working path, mm,

$s_0$  – width of the tractor's rear tire, mm,

$\Delta_s$  – deformation of the tractor's rear tire, mm,

$\Delta$  – difference between half the width of the working path and the width of the ridging body, mm,

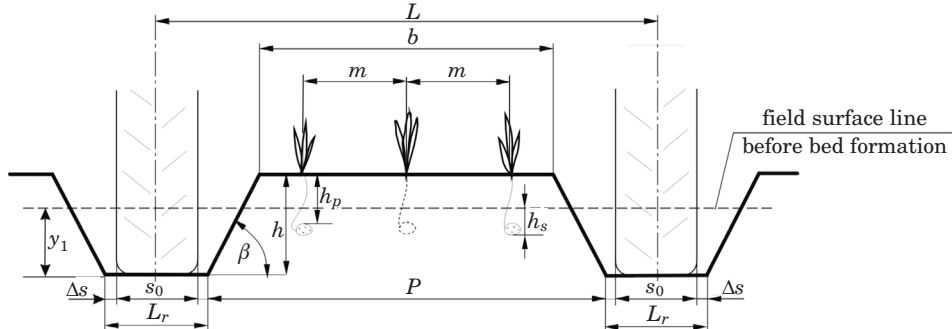
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- $P$  – width of bed base, mm,  
 $b$  – width of bed crest, mm,  
 $d$  – tuber width, mm,  
 $m$  – distance between seed potatoes in the bed, mm,  
 $h$  – height of formed bed, mm,  
 $h_p$  – thickness of soil layer covering potato tubers, mm,  
 $h_s$  – planting depth (coulters penetration depth), mm,  
 $s_p$  – working width of the ridging body, mm,  
 $\alpha$  – angle of ridging body wing, °,  
 $\beta$  – angle of inclination of the formed bed's side wall, °,  
 $y_1$  – working depth of the ridging body, mm,  
 $y_2$  – height of the soil layer tipped on the bed, mm,  
 $V_1$  – volume of soil lifted by the ridging body, m<sup>3</sup>,  
 $s_1$  – cross-sectional area of soil lifted by the ridging body, m<sup>2</sup>,  
 $V_2$  – volume of soil tipped on the bed, m<sup>3</sup>,  
 $s_2$  – cross-sectional area of soil tipped on the bed, m<sup>2</sup>,  
 $x_1$  – distance between the vertical wall of the working path and the side wall of the bed along the surface line of the field before bed formation, mm,  
 $x_1$  – distance between the vertical wall of the working path and the apex of the bed crest, mm,  
 $k_0$  – soil loosening coefficient,  
 $k_1$  – soil sliding coefficient.

## Introduction

The main aim of modern farming practices is to maximize the yield of crops characterized by specific qualitative parameters of tubers, in particular in selected areas of production, such as seed potatoes. Bed cultivation is one of such methods applied in potato farming. This technology offers a variety of advantages: the planting method improves the effectiveness of field utilization by the plant's root system, higher plant density does not lower total yield and it supports vegetation. In practice, the effectiveness of the bed cultivation technology in potato farming is determined by the performance of farming machines and working units. The three-row planter delivers optimal results in this potato cultivation technology. The setting of a planter's working units, including coulters and ridging bodies, affects bed formation.

The objective of this study was to develop a mathematical model describing the geometric parameters of a potato bed. The model has been developed based on the following simplified assumptions:

- the soil raised and shifted by the ridging body is spread across the entire bed area to ensure that all tuber rows are evenly covered;
- the width of the working path ( $L_r$ ) is equal to the nominal width of the tractor's rear tire ( $s_0$ ), increased by the value of tire deformation ( $\Delta_s$ ) (Fig. 1).



$\Delta_s$  – deformation of the tractor's rear tire,  $s_0$  – width of the tractor's rear tire,  $L_r$  – width of the working path,  $L$  – rear wheel track,  $b$  – width of bed crest,  $m$  – distance between seed potatoes in the bed,  $h$  – height of formed bed,  $h_p$  – thickness of soil layer covering potato tubers,  $h_s$  – planting depth,  $\beta$  – angle of inclination of the formed bed's side wall ( $\beta = 90 - \alpha$ ),  $P$  – width of bed base

Fig. 1. Diagram of a formed bed

## Theoretical analysis

Potatoes are planted in beds using a three-row planter equipped with special ridging bodies. The setting of the planter's working units determines the shape of the formed bed, it creates a supportive environment for plant growth and it contributes to higher potato yields. The depth of the soil layer covering seed potatoes should range from 60 mm to 90 mm, while the optimal planting depth is 40 mm to 60 mm.

The bed is formed when ridging bodies lift soil from depth ( $y_1$ ) and shifts it to the center of the formed bed. The geometry of the ridging body wing ensures that the side wall of the bed has the right profile, and the working width of the ridging body supports the formation of working paths.

The base of a correctly formed bed should have the optimal width ( $P$ ) and height ( $h$ ) to ensure the required planting depth ( $h_s$ ) and thickness of the soil layer covering tubers ( $h_p$ ) (Fig. 1).

The width of the working path ( $L_r$ ) has to be at least equal to the nominal width of the tractor's rear tire ( $s_0$ ), increased by the value of tire deformation ( $2\Delta_s$ ). This correlation is expressed by the following formula:

$$L_r = s_0 + 2\Delta_s \quad (1)$$

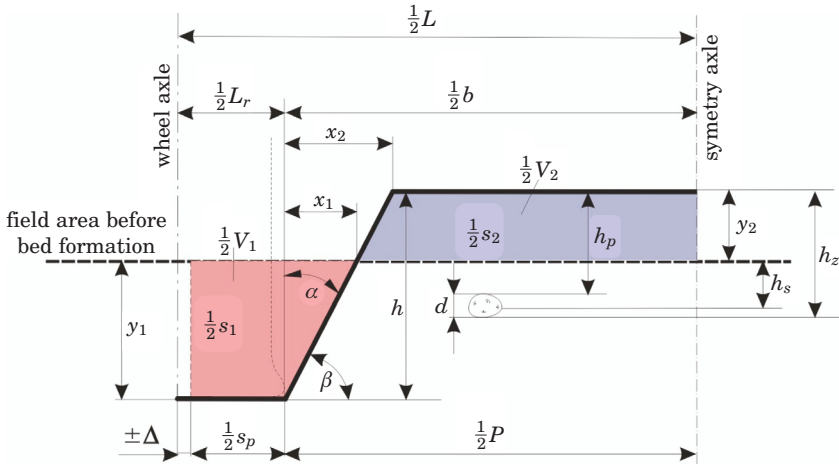
At different wheel track values ( $L = 1350; 1500; 1800$  mm), the width of the bed base ( $P$ ) will be equal to:

$$P = L - (s_0 + 2\Delta s) \quad (2)$$

At optimal pressure, the deformation of the rear tire will be minimal. Assuming that  $\Delta_s = 0$ , the width of the rear tire will be equal to the width of the working path  $s_0 = L_r$ , therefore:

$$P = L - L_r \quad (2a)$$

The geometric parameters of the bed in the mathematical model are presented in Figure 2.



$L$  – wheel track,  $L_r$  – width of the working path,  $\alpha$  – angle of ridging body wing,  $\beta$  – angle of inclination of the formed bed's side wall ( $\beta = 90 - \alpha$ ),  $y_1$  – working depth of the ridging body,  $y_2$  – height of the soil layer on the bed,  $h_s$  – working depth of coulters (planting depth),  $h$  – height of the formed bed,  $h_p$  – thickness of soil layer covering potato tubers,  $d$  – tuber width,  $V_1$  – volume of soil lifted by the ridging body,  $V_2$  – volume of soil tipped on the bed,  $s_1$  – cross-sectional area of soil lifted by the ridging body,  $s_2$  – cross-sectional area of soil tipped on the bed,  $s_p$  – working width of the ridging body,  $\Delta$  – difference between half the width of the working path and the width of the ridging body,  $x_1$  – distance between the vertical wall of the working path and the apex of the bed crest

Fig. 2. Geometric parameters of the bed

Ridging bodies working at depth ( $y_1$ ) lift soil from a trench with volume ( $V_1$ ) and spread it evenly on the bed surface to occupy volume ( $V_2$ ). Soil will be loosened, and the above can be described by the following formula:

$$V_2 = k_0 \cdot V_1 \quad (3)$$

where:  $k_0$  – soil loosening coefficient ( $k_0 = 1.00 \div 1.25$ ).

In very long beds, changes in the volume of soil cover resulting from soil loosening will be observed only in the perpendicular direction. To account for the above, cross-sections ( $s_1$ ) and ( $s_2$ ) have been adopted, and formula (3) will take on the following form:

$$s_2 = k_0 \cdot s_1 \quad (4)$$

The following can be extrapolated from Figure 2:

$$s_1 = \left( \frac{2s_p + y_1 \operatorname{tg} \alpha}{2} \right) \cdot y_1 \quad (5)$$

$$s_2 = \left( \frac{L - 2s_p - 2\Delta - x_2 - x_1}{2} \right) \cdot y_2 \quad (6)$$

and

$$x_1 = y_1 \operatorname{tg} \alpha \quad (7)$$

$$x_2 = (y_1 + y_2) \operatorname{tg} \alpha \quad (8)$$

When expressions (7) and (8) are substituted for (6), the result is:

$$s_2 = \left( \frac{L - 2s_p - 2\Delta - 2y_1 \operatorname{tg} \alpha - y_2 \operatorname{tg} \alpha}{2} \right) \cdot y_2 \quad (9)$$

When expressions (5) and (9) are inserted in equation (4) and transformed, the result is:

$$y_2^2 \operatorname{tg} \alpha - y_2(L - 2s_p - 2\Delta - 2y_1 \operatorname{tg} \alpha) + k_0 y_1 (2s_p + y_1 \operatorname{tg} \alpha) = 0 \quad (10)$$

The above equation describes the correlation between the working depth of the ridging body ( $y_1$ ), the angle of ridging body wing ( $\alpha$ ) and the thickness of the soil layer ( $y_2$ ) covering the bed. Based on the required thickness of the soil layer covering the tuber ( $h_p$ ), the height of the soil layer tipped on the bed has been determined for the set planting depth ( $h_s$ ):

$$y_2 = h_p - h_s + d \quad (11)$$

and bed height:

$$h = y_1 + y_2 \quad (12)$$

In a field environment, the soil lifted by ridging bodies is shifted upwards and spread evenly over the entire bed. Expression (4) was used to calculate the theoretical cross-sectional area ( $s_2$ ) and ( $s_1$ ). Some soil slides away from ridging bodies, and it is returned to the working path, therefore the layer of soil that is actually tipped on the bed is thinner than the calculated layer. For this reason,

coefficient ( $k_1$ ) has been introduced to determine the actual value of variable ( $y_2$ ) – the height of the soil layer on the bed. Coefficient  $k_1$  (soil sliding coefficient) is determined based on formula (13):

$$\frac{s_{1rz}}{s_t} = k_1 \quad (13)$$

where:

$s_{1rz}$  – actual cross-sectional area of soil lifted by the ridging body;

$s_t$  – theoretical cross-sectional area of soil lifted by the ridging body.

The actual height of the soil layer tipped on the bed will be calculated with the use of expression (14) for  $k_1 \leq 1$ :

$$y_{2rz} = y_2 \cdot k_1 \quad (14)$$

The mathematical model described by formulas (10), (11) and (12) describes the correlations between the geometric parameters of the bed. The above parameters constitute a basis for determining the working parameters of the planter ( $s_p, y^{31}, \alpha, h_s$ ) and the tractor ( $L, L_r$ ) that affect the thickness of the soil layer covering the planted tubers.

## Simulation analyses

Simulation analyses were performed on the assumption that a three-row planter will be operated with a  $9 \div 14$  kN tractor. The maximum rear tire width in tractors of the above class is  $241 \div 315$  mm. When formula (2a) is applied for wheel track  $L = 1500$  mm, the width of the bed base is  $P = 1200 \div 1300$  mm. This width of the bed base supports plant growth in line with the cultivation requirements for seed potatoes.

A computer-aided simulation of bed geometry was carried out based on the developed mathematical model. The following variable parameters were identified: working depth of the ridging body ( $y_1 = 0 \div 200$  mm), angle of the ridging body wing ( $\alpha = 40 \div 25^\circ$ ), soil loosening coefficient ( $k_0 = 1.0 \div 1.2$ ) and planting depth ( $h_s = 40 \div 70$  mm). The simulation experiment analyzed the above variables' effect on: total bed height ( $h$ ), height of the soil layer forming the bed crest ( $y_2$ ), height of the soil layer covering tubers ( $h_p$ ).

Diagrams illustrating the results of computer-aided simulations are presented in Figure 3. An analysis of the displayed data indicates that an increase in the working depth of the ridging body ( $y_1$ ) above 125 mm significantly affects the investigated parameters.

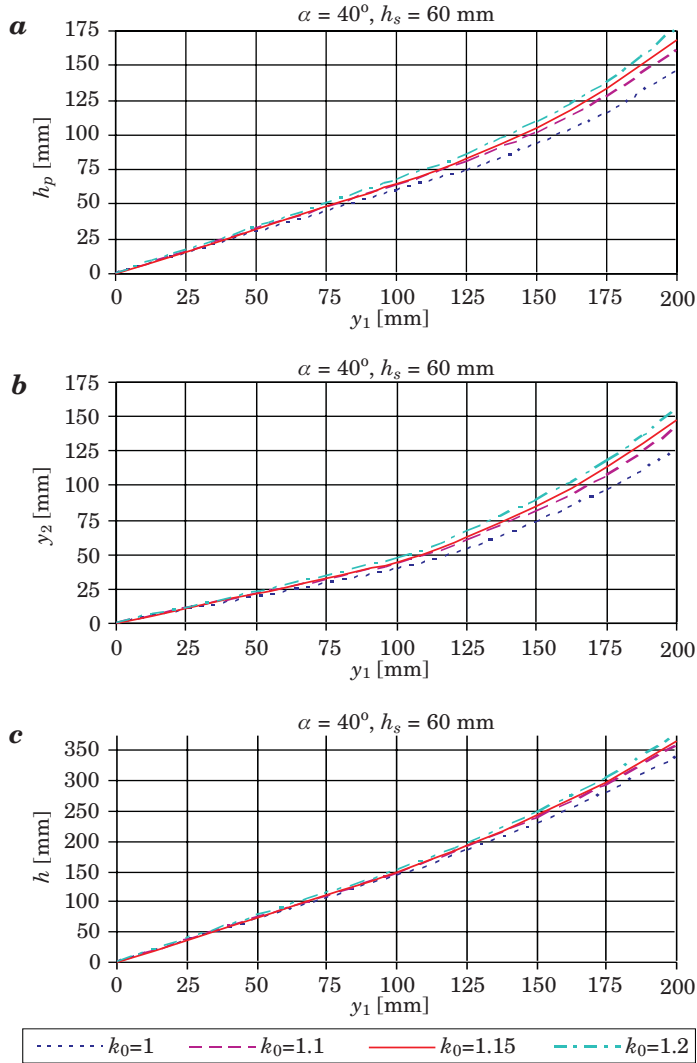


Fig. 3. Correlations between: a) thickness of the soil layer tipped on the bed ( $y_2$ ), b) height of the soil layer covering tubers ( $h_p$ ), c) bed height ( $h$ ) and the working depth of the ridging body ( $y_1$ ), angle of the ridging body wing ( $\alpha$ ) and planting depth for wheel track  $L = 1500 \text{ mm}$

A change in the working depth of the ridger ( $y_1$ ) from 125 mm to 175 mm with planting depth ( $h_s$ ) of 60 mm changes the height of the soil layer tipped on the bed ( $y_2$ ) from 60 mm to 125 mm, the height of the soil layer covering tubers ( $h_p$ ) from 75 mm to 140 mm, while the total increment in bed height increases from 200 mm to 300 mm.

The results of simulation analyses suggest that bed formation and tuber planting are affected by the following planter parameters: the working depth of the coulter and the working depth of the ridging body.

### Model verification

A field experiment verifying the theoretical analyses of bed geometry was carried out on sandy loam with relative humidity of 25.5%.

The experimental plot had the length of 15 m. A three-row planter for bed cultivation was operated with a C-360 tractor with wheel track of 1500 mm. The planter was equipped with ridging bodies comprising skim-coulters with moldboard slats. Operating speed was constant. Seed potatoes cv. Bryza with the size of 30 ÷ 40 mm were used.

The input parameter in the mathematical model was the working depth of the ridging body ( $y_1$ ) of 150 mm and planting depth ( $h_s$ ) of 40 mm and 60 mm. After the planter had traveled the reference section, a profilogram was used to record the cross-sectional profiles of potato beds. Bed height and tuber depth in the bed were measured. All measurements were performed in three replications.

Profilogram data was used to determine the cross-sectional area of the soil lifted by ridging bodies and the cross-sectional area of soil tipped on the bed crest. The results of measurements and calculations are presented in Table 1.

The obtained results were used to calculate the relative error with the use of the below formula:

$$\delta_w = \frac{|x_w - x_o|}{x_o} \cdot 100\% \quad (15)$$

where:  $x_w$  – measurement result,  $x_o$  – calculation result.

Table 1  
Geometric parameters of the formed bed at working depth of the ridging body  $y_1 = 1500$  mm

Bed parameters	Planting depth $h_s = 40$ mm	Planting depth $h_s = 60$ mm
Width of bed base ( $P$ ), mm	1350	1320
Height of formed bed ( $h$ ), mm	223	220
Soil sliding coefficient ( $k_1$ )	0.9	0.9
Height of soil layer tipped on bed ( $y_2$ ), mm	from 71 to 73	from 70 to 74
Thickness of soil layer covering potato tubers ( $h_p$ ), mm	from 75 to 80	from 91 to 94



A comparison of the height of the soil layer tipped on the bed determined in a field experiment with the theoretical height calculated from the mathematical model produced a relative error of 2.7%. A low value of the relative error indicates that the mathematical model determines the geometric parameters of the bed with a satisfactory degree of precision.

## Conclusions

The results of the study indicate that the geometric parameters of the bed are determined by the mathematical model with a satisfactory degree of precision for practical applications in the proposed potato planting technology. The height of the soil layer covering potato tubers is determined by the working depth of the ridging body and coulter penetration depth set in the planter (planting depth). In the bed cultivation system, the optimal conditions for potato farming are met when the working depth of ridging bodies is set in the range of 150 mm to 170 mm.

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