

## MODELING THE NATURAL CONVECTION HEATING OF LIQUID IN A VERTICAL CYLINDER

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Key words: natural convection, vertical cylinder, liquid food, modeling.

### Abstract

Natural convection heating within a can of liquid food was simulated by solving the governing equations for continuity, momentum and energy conservation in an axisymmetric case using a commercial COMSOL Multiphysics software (Comsol). A physical experiment and computer simulations using four different models were studied. The results allowed to suggest a method of defining the properties of the material and the boundary conditions applied and to determine the usefulness of COMSOL Multiphysics package in modeling the natural convection heating of liquid food.

## MODELOWANIE PROCESU NAGRZEWANIA CIECZY W PIONOWYM CYLINDRZE W WARUNKACH KONWEKCJI NATURALNEJ

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Słowa kluczowe: konwekcja naturalna, pionowy cylinder, żywność płynna, modelowanie.

### Abstract

W pracy przeprowadzono ocenę możliwości wykorzystania pakietu Comsol Multiphysics do modelowania konwekcyjnego ogrzewania cieczy w pionowym zbiorniku w kształcie cylindra. Opisano eksperyment fizyczny oraz symulacje komputerowe procesu oparte na czterech odmiennie zdefiniowanych modelach, zrealizowanych za pomocą pakietu Comsol. Otrzymane wyniki pozwoliły na określenie przydatności pakietu Comsol Multiphysics do modelowania zjawisk konwekcji naturalnej oraz definiowania właściwości modelowanej substancji oraz zastosowanych warunków brzegowych.

**Notation**

$$\text{Gr} = \frac{D^3 \rho^2 g \beta (T - T_w)}{\eta^2} - \text{Grashoff number}$$

$$\text{Nu} = \frac{hD}{\lambda} - \text{Nusselt number}$$

$$\text{Pr} = \frac{c_p \eta}{\lambda} - \text{Prandtl number}$$

$\beta$  – thermal expansion [1/K]

$\lambda$  – thermal conductivity [W/(m K)]

$\rho$  – actual density [kg m<sup>-3</sup>]

$\rho_0$  – initial density [kg m<sup>-3</sup>]

$\eta$  – viscosity [Pa s]

$c_p$  – thermal capacity [J/(kg K)]

$\mathbf{g}$  – gravity acceleration [m s<sup>-2</sup>]

$h$  – heat transfer coefficient [W/(m<sup>2</sup> K)]

$n$  – number of measurements

$p$  – pressure [Pa]

$I$  – unitary matrix

$Q$  – heat source [W]

$Q_s$  – heat flux [W/(m<sup>2</sup> K)]

$r$  – radial coordinate

RMSE – root mean square error [K]

$T$  – temperature [K]

$t$  – time [s]

$T_e, T_s$  – measured, simulated temperature [K]

$\mathbf{u}$  – velocity [m s<sup>-1</sup>]

$w$  – water

$z$  – vertical coordinate

**Introduction**

Conventional canning processes extend the shelf life of food products and make the food safe for human consumption by destroying the pathogenic microorganisms. Natural convection induced by thermal buoyancy effects in a gravitational force field is observed in many applications. In the food industry products are thermally processed before or after packing. In the second case it is important to establish the properties of food as affected by temperature, the characteristic process parameters and to know the behavior

of the processed product in enclosure. A computational fluid dynamics (CFD) approach is a very useful tool broadly applied in the research on the behavior of liquid foods during thermal processing. GHANI et al. (1999) studied natural convection heating of canned liquid food using CFD methods. They detected the slowest heating zone and identified the time required to reach the sterilization temperature in this zone for vertical cylinder filled with two different liquids. GHANI et al. (2002) focuses on the same problems, but for a horizontal cylinder. There are also studies on effect of different shapes of enclosure on a natural convection heating of liquid food (VARMA MAHESH et al. 2006). RABIEY et al. (2007) studied transient temperature and fluid flow during natural convection heating of a cylindrical can containing large food particles. In all the studies mentioned above such commercial software as FLUENT, CFX and PHOENIX was used. Many other commercial packages enable performing CFD simulation of fluids. One of the advantages of COMSOL MULTIPHYSICS package (Comsol Co.) is its ability to work integrated with MATLAB package (MathWorks Inc.). This feature of COMSOL is not found in other engineering software packages. Therefore, the aim of this research was to study the abilities of COMSOL MULTIPHYSICS package to perform numerical simulation of heat, momentum and mass transfer in food liquid during natural convection heating of a vertical cylindrical container. The sensitivity of the model to the different values of the properties of the material and to the boundary conditions setting was also studied.

## **Material and experimental stand**

10% water solution of sucrose was heated by natural convection in a steel can using the experimental stand shown in Figure 1. The container made of stainless steel brass with a thickness of 0.2 mm was 160 mm height and 150 mm diameter. Eight J-type thermocouples were placed on the bottom, lid, and on the wall surface of the container. Additional three thermocouples were placed inside the can: the first near the geometric centre of the can, the second and third near the bottom and the lid of the can, respectively, as can be seen in Fig. 1. Signals from thermocouples were registered by computer every 10 s. Before the beginning of the experiment, container was filled with the solution and was chilled to temperature close to 3°C. Chilled can with water was placed in steered water bath. The temperature of the water bath was maintained on the level of  $40 \pm 1^\circ\text{C}$  during the experiment. Experiment was repeated 3 times.

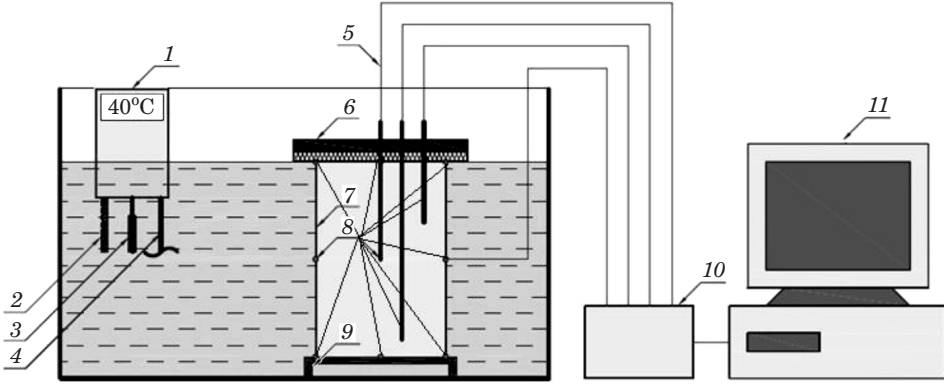


Fig. 1. The scheme of the experimental setup: 1 – thermostat, 2 – thermometer, 3 – heater, 4 – stirrer, 5 – thermocouples' wires, 6 – cover, 7 – container, 8 – thermocouples, 9 – stand, 10 – converter, 11 – PC

## Mathematical model

All models were defined as two-dimension problems, with axial symmetry. Balance equations of heat (1), momentum (2) and mass transfer (3) were used as follows:

$$\rho \cdot c_p \cdot \frac{\partial T}{\partial t} + \nabla \cdot (-\lambda \cdot \nabla T) = Q + q_s \cdot T - \rho \cdot c_p \cdot \mathbf{u} \cdot \nabla T \quad (1)$$

$$\rho \cdot \frac{\partial \mathbf{u}}{\partial t} + \rho \cdot (\mathbf{u} \cdot \nabla) \cdot \mathbf{u} = \nabla \cdot [-p \cdot \mathbf{I} + \eta \cdot (\nabla \mathbf{u} \cdot (\nabla \mathbf{u})^T)] - \frac{2}{3} \cdot \eta \cdot \nabla \mathbf{u} \cdot \mathbf{I} + (\rho - \rho_0) \cdot \mathbf{g} \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \cdot \mathbf{u}) = 0 \quad (3)$$

In order to perform the computer simulations behavior of density, viscosity, thermal conductivity, and thermal capacity of a liquid food should be known and the initial and boundary conditions should be formulated. In all the cases studied the initial temperature of the liquid was evaluated as uniformly distributed in the liquid and the initial condition was described with the following formula:

$$t = 0 \rightarrow T(r,z) = T_0 = \text{const} \quad (4)$$

Four different versions of boundary conditions, marked as M1, M2, M3 and M4, were studied. The details regarding boundary conditions applied were described underneath while the details regarding the physical properties of the liquid applied during computer simulation are set in Table 1.

Table 1  
Physical properties of the water solution of the sucrose applied during simulations

Model	Coefficient
M1	$\rho(T) = (-0.004) \cdot T^2 + 2.12 \cdot T + 763.43$ [kg m <sup>-3</sup> ] $c_p = 4\,183$ [J kg <sup>-1</sup> K] $\lambda = 0.599$ [W m <sup>-1</sup> K] $\eta = 1.004 \cdot 10^{-3}$ [Pa s]
M2, M3, M4	$\rho(T) = (-0.004) \cdot T^2 + 2.12 \cdot T + 763.43$ [kg m <sup>-3</sup> ] $c_p(T) = (-0.01367) \cdot T^2 + 8.83 \cdot T + 2535.78$ [J kg <sup>-1</sup> K] $\lambda(T) = 4.93 \cdot 10^{-8} \cdot T^4 + (-5.84) \cdot 10^{-5} \cdot T^3 + 0.026 \cdot T^2 + (-5.12) \cdot T + 378.43$ [W m <sup>-1</sup> K] $\eta(T) = (-52.08 + 0.21 \cdot T)^{-2.5}$

**M1.** In the first case the simplest model was applied. In this model viscosity, thermal conductivity and thermal capacity of the liquid were assumed to be constant, while density was assumed to be dependent on temperature of the liquid. The temperature of a given point  $P$  on the walls, under the lid and in the bottom of the container was assumed to be constant. The boundary conditions were described with following equation:

$$t > 0 \rightarrow T|_{P \in \Omega} = T_W = \text{const} \quad (5)$$

**M2.** In this model viscosity, thermal conductivity, thermal capacity and density of the water solution of the sucrose were temperature dependent while similarly to the model  $M1$  the boundary conditions were described with equation (5).

**M3.** The third model, similarly to the previous one was characterized by temperature dependent viscosity, thermal conductivity, thermal capacity and density of the water solution of the sucrose. It was assumed that Newton's law of cooling (6) can be applied to describe convection type heat transfer between surface of the cylinder and the liquid outside of the cylinder.

$$t > 0 \rightarrow -\lambda \cdot \text{grad}(T)|_{P \in \Omega} = h(t - T_W) \quad (6)$$

For this purpose the heat transfer coefficient,  $h$ , was calculated based on SERWIŃSKI (1971) on the assumption about the natural convection flow of the liquid round the vertical cylinder. Temperature dependency of the heat transfer coefficient was assumed after SERWINSKI (1971):

$$\text{Nu} = 1.18(\text{Gr} \cdot \text{Pr})^{1.125} \quad (7)$$

**M4.** The fourth model incorporated an artificial neural network (ANN) based on experimental data, generated by MATLAB R2009b (Mathworks Inc.). The General Regression Neural Networks (GRNN) type of ANN was used to form boundary conditions with temperature of the wall, lid and bottom varying in time. The ANN contained two layers. The number of the neurons in the first layer was equal to the size of the data set used for the teaching of ANN purpose. The number of the neurons in the second layer was 2. This approach seemed to be the most similar to real phenomenon that occurred during experiments. The boundary conditions were described with following formula:

$$t > 0 \rightarrow T|_{P \in \Omega_i} = T_i(t) \quad i = 1, 2, 3 \quad (8)$$

The values of  $T_i$  in equation (8) were generated using ANN. The values 1, 2, and 3 of index  $i$  denote wall, lid and bottom of a cylinder, respectively.

## Simulations

The aim of the next stage was to simulate the natural convection heating of liquid food and calculate the changes in temperature of the liquid as monitored at measurement points shown in Figure 1. The balance equations of mass, momentum and heat (1)-(3) were used during simulations. To perform the simulations the used algorithms were implemented in COMSOL MULTIPHYSICS v.3.5a and enhanced with the Heat Transfer module. The geometry of the container used in the physical experiment was used during simulation experiments. The time range of simulations was from 0 to 300s. Solver *Direct PARADISO* was used in all simulations.

A comparison of the courses of temperature changes received from computer simulations with the results of experiments was also performed. In order to rate the differences between those results of experiments and results of the simulation root mean square error (RMSE) coefficient was used. RMSE was calculated using following formula:

$$\text{RMSE} = \frac{\sqrt{\sum_{i=1}^n (T_e^i - T_s^i)^2}}{n} \quad (9)$$

## Results and discussion

The values of RMSE coefficients derived from the applied models are summarized in Table 2. The values shown in Table 2 are reflecting the correspondence of the results of the simulations to the experimental data. It can be seen in Table 2 that the results received from the models M1 and M2 are characterized by the lowest values of RMSE coefficients. The values of RMSE received from the models M3 and M4 were higher. However, the simulated values were received for the can filled only with the liquid with no thermocouple. In the case of experiment the thermocouples used for monitoring the changes in temperatures disturb the circulation of the liquid, thus temperatures and velocity of the liquid measured with and without the thermocouples poured into the liquid may be different. Figure 2 shows changes in temperature of the water solution of sucrose measured and simulated for the point near geometric centre of the cylinder during 300 s of heating. Figure 3 presents the simulated changes in average temperature of the liquid. Simulated courses of temperatures of the liquid were derived using Eqs (1)-(3) for all investigated cases of physical properties of the liquid and the boundary conditions applied. Temperature of the liquid averaged over its volume is presented in Figure 3. As can be seen in Figs 2 and 3 differences between the results received from different models are relatively small. Small values of RMSE received for all the tested models suggest that the results of simulation did not vary significantly from the experimental results. However, the least accurate results were obtained from the model M4.

Table 2

RMSE coefficients

Location of the thermocouples [m]		M1	M2	M3	M4
$r$	$z$				
0	0.08	0.111	0.111	0.270	0.340
0.025	0.05	0.221	0.221	0.282	0.312
0.005	0.11	0.078	0.078	0.124	0.164

Simulated distributions of the temperature of the liquid vs. time of heating and spatial coordinates, received for each of the applied case of the physical properties of the liquid and the boundary conditions, are shown in Figs 4–6. In case of all the courses of temperature of the liquid as derived using the method of computer simulation the period of the oscillations of the temperatures and velocities of the liquid was observed at the initial phase of the process (Fig. 2).

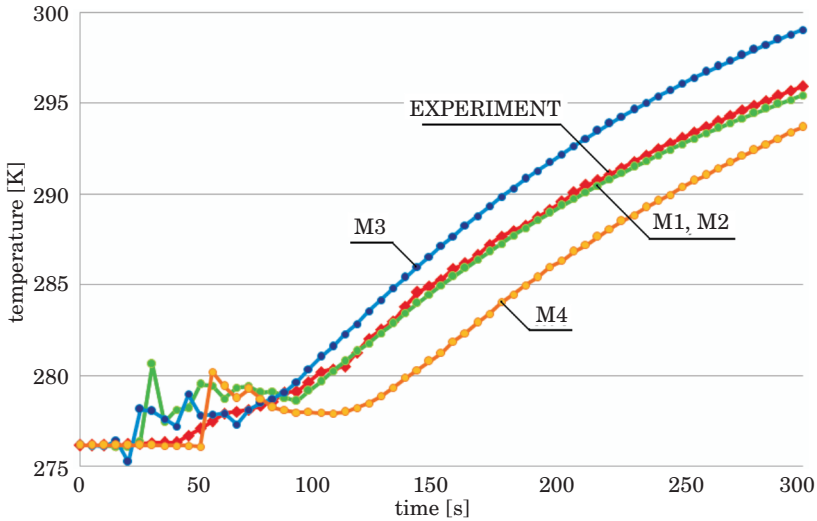


Fig. 2. Changes in the temperature of the liquid simulated at the center of the container

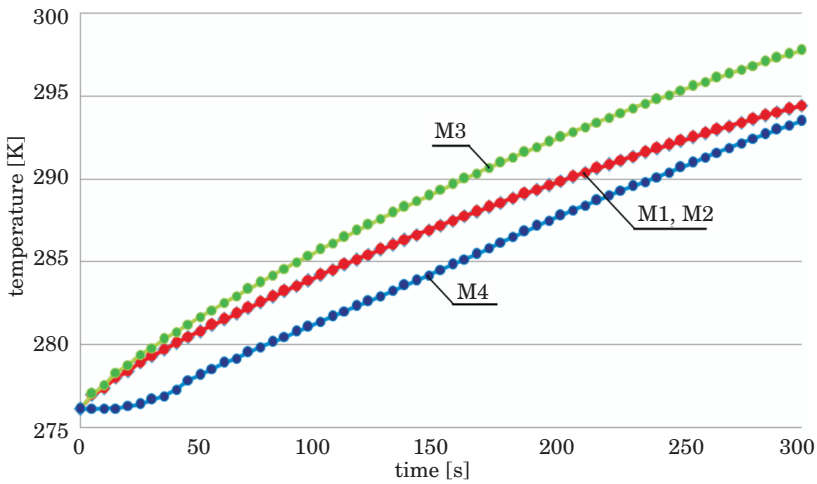


Fig. 3. Changes in average temperature of the liquid

The local oscillations in the temperature accompanied by the local whirls of the liquid gradually vanished after about 100 s of heating. The oscillations could be caused by the numerical method applied during simulation. Figure 7 presents the spatial distributions of temperature and velocity of the liquid simulated using the model M3 after 300 s of heating.



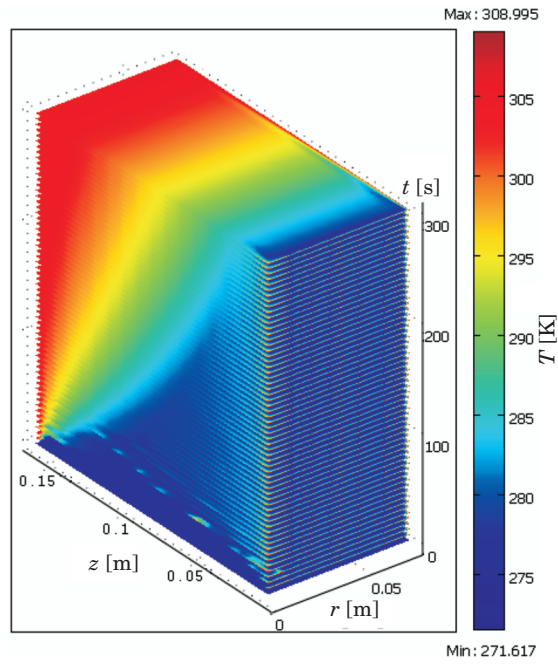


Fig. 4. Changes in temperature of the liquid simulated according to the models M1 and M2

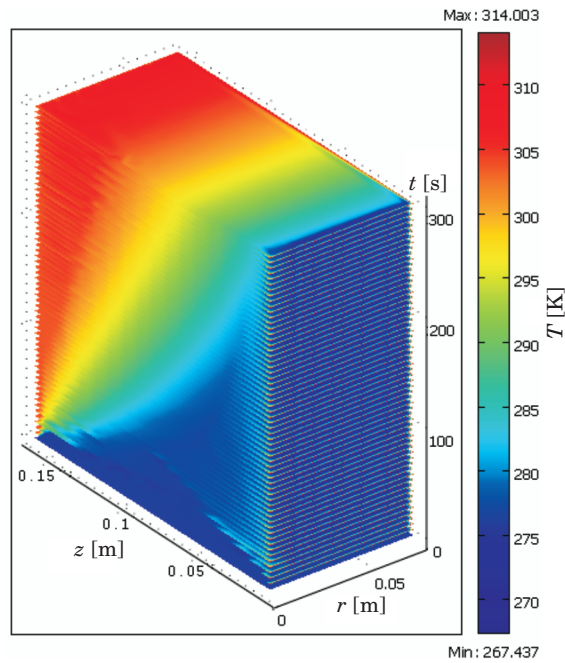


Fig. 5. Changes in temperature of the liquid simulated according to the model M3

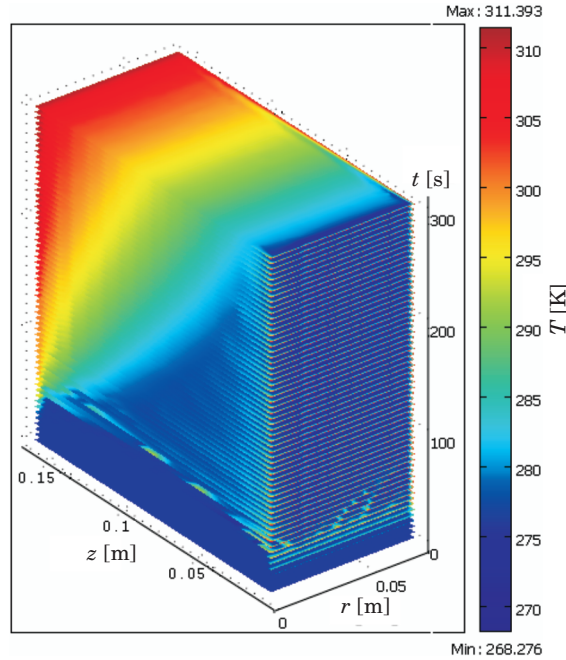


Fig. 6. Changes in temperature of the liquid simulated according to the model M4

It can be concluded from figures 2–6 that simulated courses of the local and average temperature of the liquid depend on the applied model of the physical properties of the liquid and on the type of the boundary conditions used. However, the changes in temperature of the liquid vs. time of heating simulated using models M1 and M2 are similar. This means that using thermal conductivity, thermal capacity and viscosity of liquid described as functions of the temperature do not influence significantly the results of simulation.

It should be emphasized that the method of defining the physical properties of the liquid in COMSOL MULTIPHYSICS software was important not only for the quality of the simulation model but also was important for receiving reliable results. It can be done in COMSOL by two different methods: (1) by defining an algebraic expression, (2) by using built-in data acquisition editor. It was observed that the most effective method of defining the temperature dependant physical properties of the liquid is using built-in data acquisition editor. In this case COMSOL environment uses automatically the implemented approximation procedures. The model built that way is more stable with less oscillations of the results in comparison with the model built using

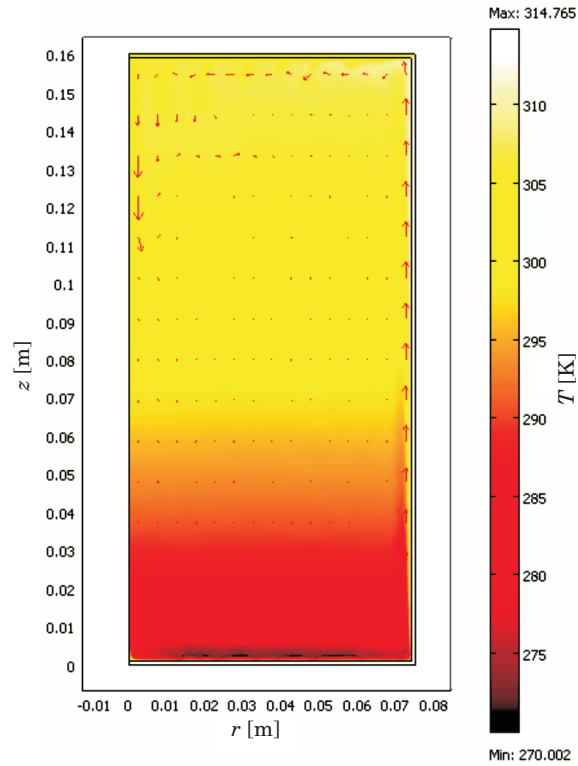


Fig. 7. Simulated spatial distributions of temperature and velocity of the liquid received using the model M3 after 300 s of heating

algebraic functions used for describing physical properties of the liquid. Also time of problem solving is reduced significantly when the built-in data acquisition editor is applied.

## Conclusions

Natural convection heating within a can of liquid food was simulated by solving the governing equations for continuity, momentum and energy conservation for an axial symmetry case using a commercial COMSOL Multiphysics software (Comsol Co.). A physical experiment and computer simulations were studied using four different models. The results allowed to suggest a method of defining the properties the material and boundary conditions applied and to determine the usefulness of COMSOL Multiphysics package in modeling the natural convection heating of liquid food. Using the built-in data acquisition

editor for defining the temperature dependant physical properties of the liquid ensures receiving stable solutions with less oscillations of the results in shorter time in comparison to the model built using algebraic functions used for describing physical properties of the liquid.

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