

## TAKING SUBSOIL SUSCEPTIBILITY INTO ACCOUNT IN DESIGNING COLUMNS IN REINFORCED SINGLE-STOREY STRUCTURES

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**Key words:** subsoil susceptibility, effective length of columns, method based on nominal stiffness, second order effects, reinforced concrete buildings, reinforced concrete structures.

### Abstract

When the Polish norm for designing concrete constructions will be replaced in 2010 by the Eurocode, its limits referring to columns' slenderness will no longer bide. While designing reinforced slender columns in single-storey buildings, it is vital to consider all the factors which may influence the bearing capacity, one of which is the increment of the bending moment caused by the rotation of foundations supported by susceptible soils. The following article presents examples of the second order calculations taking into account the influence of subsoil susceptibility and columns' nominal stiffness, as defined in the Eurocode.

## UWZGLĘDNIANIE PODATNOŚCI PODŁOŻA W PROJEKTOWANIU SŁUPÓW HAL ŻELBETOWYCH

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**Słowa kluczowe:** podatność podłoża, długość obliczeniowa słupa, metoda nominalnej sztywności, efekty drugiego rzędu, hale żelbetowe, konstrukcje żelbetowe.

### Abstrakt

Po wycofaniu (w 2010 r.) polskiej normy projektowania konstrukcji z betonu i zastąpieniu jej przez Eurokod przestaną obowiązywać zawarte w tej normie ograniczenia smukłości słupów.

Projektując smukłe żelbetowe słupy w halach, trzeba rozpatrzyć wszystkie czynniki, które mogą mieć wpływ na nośność – jednym z nich jest przyrost momentu zginającego wywołany obrotem fundamentu na podatnym podłożu. W artykule przedstawiono przykłady obliczeń według teorii II rzędu z uwzględnieniem wpływu podatności podłoża i nominalnych sztywności słupów, zdefiniowanych w Eurokodzie.

## Introduction

While designing columns in reinforced concrete single-storey buildings, it is not usual to take into account the influence of subsoil susceptibility on bending moments, illustrated in Fig. 1b.

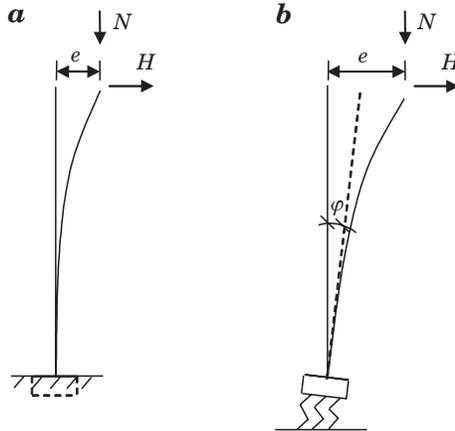


Fig. 1. Eccentricity increase: *a*) only resulting from the deflection of the column, *b*) resulting from the deflection and foundation rotation through angle  $\varphi$

The PN-B-03264 norm, which still bides, recommends that the column's slenderness should not be larger than  $l_0/i = 104$  ( $l_0/h = 30$ ).

This norm is to be withdrawn in 2010 and replaced by the Eurocode, which does not contain any arbitrary limits in case of the slenderness of columns; any slenderness may be used as long as it can be proved by calculations that the bearing capacity is sufficient.

Canceling the limits of slenderness and using better materials will make it possible to design columns that are even more slender than before. According to the new general rules for designing (point 5.8.7 in Eurocode 2 (abbr. EN)), "Where relevant, soil-structure interaction should be taken into account".

As a result of the deflection of a column due to the first and second order effects (Fig. 1a), we obtain the increment of eccentricity of the applied

longitudinal force, as well as the increment of the bending moment of the column. The foundation rotation causes an additional increment of the moment as shown in Fig. 1b.

The moment increments associated with the second order effects may be calculated using two methods. The first one is a simplified method based on the concept of isolated members. To use it, it is necessary to calculate the effective length of the column being designed. EN does not give any defined effective lengths of isolated columns supported by elastic soil but only recommendations (Fig. 5.7 f, g in EN 1992-1-1) which are too general for immediate usage. The effective length of columns directly supported by foundations depends on soil susceptibility and the size of foundations. Formulae for effective lengths of isolated columns were presented in publications by KOBIAK and STACHURSKI (1989) which had been based on the works of the German scientists FISCHER (1965) and KANY (1974).

The second method is the exact method based on the second order analysis of the whole frame structure. To use the exact method does not require calculating effective lengths – the shape of deformed elements and the associated increment of moments are determined in direct calculation. The examples of how this method may be used were presented in publications by KNAUFF and KLEMPKA in 2009.

The next part of the article will present the method of calculating the coefficient of subsoil susceptibility and shaping the foundation rotation supported by elastic soil. This will be followed by calculation results for some examples of frame structures.

### **Subsoil susceptibility coefficient in Winkler model**

Winkler's rotation angle of foundation on soil (Fig. 2a) may be calculated using the following formula

$$\varphi = \frac{M}{C_z I_F} \quad (1)$$

where

$I_F$  – the moment of inertia of the area of the foundation's base,

$C_z$  – subsoil elasticity coefficient.

Coefficient  $C_z$  is not a material constant because it does not depend only on the physical characteristics of the soil but also on the dimensions of foundations. To determine its value we have to take into account uniform elastic

half-space with characteristics defined by modulus  $E_0$  and Poisson's ratio  $\nu$ . The value of the rotation angle of the foundation may be determined from formula (2) which comes from the article by GORBUNOV-POSADOV (1956).

$$\varphi = \frac{4M(1 - \nu_0^2)}{\pi E_0 b' l^2} \quad (2)$$

(1) and (2) show that the soil elasticity coefficient used in Winkler's model has to be expressed by the following dependence:

$$C_z = \frac{\pi E_0 b' l^2}{4I_F(1 - \nu_0^2)}$$

Taking into account that

$$I_F = \frac{b'(2l)^3}{12} = \frac{2}{3} b' l^3$$

We obtain

$$C_z = \frac{3\pi}{8l} \frac{E_0}{(1 - \nu_0^2)} \quad (3)$$

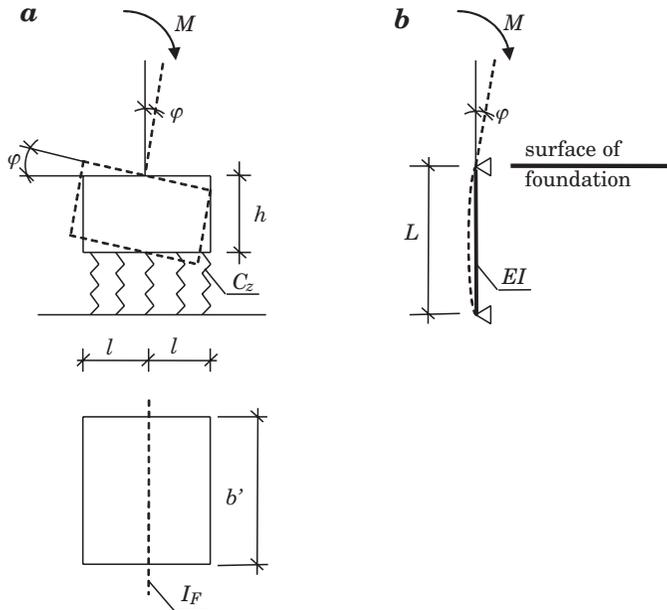


Fig. 2. Foundation supported by elastic soil (a); the way of shaping the influence of the ground in static calculations of foundations (b)

If we want to take the foundation rotation into account in static calculation of frames, we can assume that our model of a column's support will be a hypothetical bar like the one in Fig. 2b (with its length expressed by  $L$  and its stiffness by  $EI$ ). The rotation angle on the bar's support, caused by moment  $M$  can be derived from formula (4):

$$\varphi = \frac{ML}{3EI} \quad (4)$$

It may be seen from (1) and (4) that length  $L$  and stiffness  $EI$  of the hypothetical bar should be selected in such a way that its flexural stiffness  $3EI/L$  satisfies the following dependence

$$\frac{3EI}{L} = I_F C_z \quad (5)$$

### Analysis of calculation results for examples of single-storey frames

Below we present examples of calculations of bending moments in columns of single-storey reinforced concrete buildings. For the calculations we used the exact method based on the second order analysis taking into account the nominal stiffness as described in the article by KNAUFF and KLEMPKA (2009). We analysed the case of columns fully fixed to the foundation base as well as the case of a column fixed to the foundation supported on elastic subsoil. We assumed the same foundation base –  $3.0 \times 2.0$  m – for all the cases. The base is fixed on genesis-C cohesive, hard saturated plastic soil where  $I_L = 0.20$ ,  $E_0 = 20$  MPa,  $\nu = 0.32$ .

The moment of inertia of the area of the foundation's base is

$$I_F = \frac{2}{3} b'l^3 = \frac{2}{3} 2 \cdot 1.5^3 = 4.5 \text{ m}^4$$

And the subsoil elasticity coefficient is

$$C_z = \frac{3\pi}{8l} \frac{E_0}{(1 - \nu_0^2)} = \frac{3\pi}{8 \cdot 1.5} \frac{20}{(1 - 0.32^2)} = 17.50 \text{ MN/m}^3$$

Using formula (5) we calculated the flexural stiffness of the hypothetical bars shaping the support

$$\frac{3EI}{L} = I_F C_z = 4.5 \cdot 17.50 = 78.75 \text{ MNm}$$

**Example 1**

Computational longitudinal forces in three-nave single-storey building (Fig. 3) are  $P_1 = 450 \text{ kN}$  in the edge columns and  $P_2 = 900 \text{ kN}$  in internal columns. The eccentricity resulting from cover load in edge columns is  $0.15 \text{ m}$ .

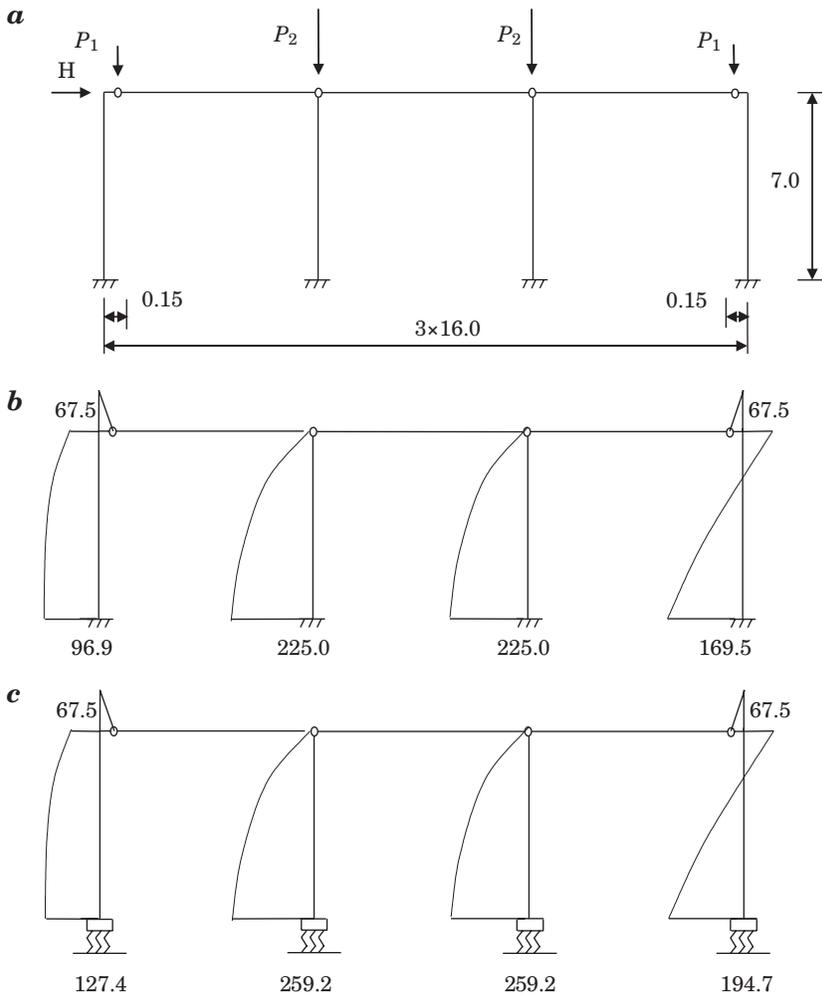


Fig. 3. Example 1. a) the diagram of the single-storey building, b) bending moments for fully fixed columns [kNm], c) moments calculated with subsoil susceptibility [kNm]

The horizontal force caused by wind pressure and suction equals  $H = 36$  kN. We assumed that the stiff roof construction imposes identical horizontal shift of the top ends of all the columns. Columns (concrete C40/50, steel A-III) have identical cross sections  $b = 40$  cm,  $h = 45$  cm. We also assumed that the reinforcement in the edge columns is  $3\phi 20$  ( $A_s = 9.42$  cm<sup>2</sup>), and in the internal columns is  $5\phi 20$  ( $A_s = 15.71$  cm<sup>2</sup>) on each side of the cross section.

Imperfections according to point 5.2 EN equal

$$\alpha_h = \frac{2}{\sqrt{l}} = \frac{2}{\sqrt{7.0}} = 0.756, \alpha_m = \sqrt{0.5 (1 + 1/m)} = \sqrt{0.5 (1 + 1/4)} = 0.790$$

The angle of inclination is

$$\Theta_i = \Theta_0 \alpha_h \alpha_m = \frac{1}{200} 0.756 \cdot 0.790 = 0.00298$$

Horizontal forces caused by imperfections:

In edge columns  $H_1 = \Theta_i P_1 = 0.00298 \cdot 450 = 1.341$  kN,

In internal columns  $H_2 = \Theta_i P_2 = 0.00298 \cdot 900 = 2.682$  kN.

More details referring to the model with fully fixed columns can be found in the article by KNAUFF and KLEMPKA (2009). Bending moments are presented in Figure 3.

### Example 2

We assumed that the horizontal force caused by wind pressure and suction equals  $H = 30$  kN. We also assumed that the stiff roof construction imposes identical horizontal shifts of the top ends of all the columns. Columns like the ones in example 1 have on each side of their cross section reinforcement  $4\phi 16$  ( $A_s = 8.04$  cm<sup>2</sup>) in edge columns and  $7\phi 16$  ( $A_s = 14.07$  cm<sup>2</sup>) in internal columns. The calculations were conducted for two cases of loading with longitudinal forces:

#### Case 1

$P_1 = 200$  kN in edge columns,

$P_2 = 900$  kN in an internal column

Case 2

$P_1 = 450$  kN in edge columns,

$P_2 = 790$  kN in an internal column.

Bending moments for case 1 and 2 are presented in Figure 4 and 5 respectively.

The cases show calculations for the soil of small stiffness. Generally, the influence of subsoil susceptibility on bending moments is not very significant. The results for example 2 presented in Figure 5 are an exception. As a result of taking into account the foundation on elastic soil, the bending moments in the edge columns increased by 17% compared to the calculation results for the columns fully fixed to the foundation.

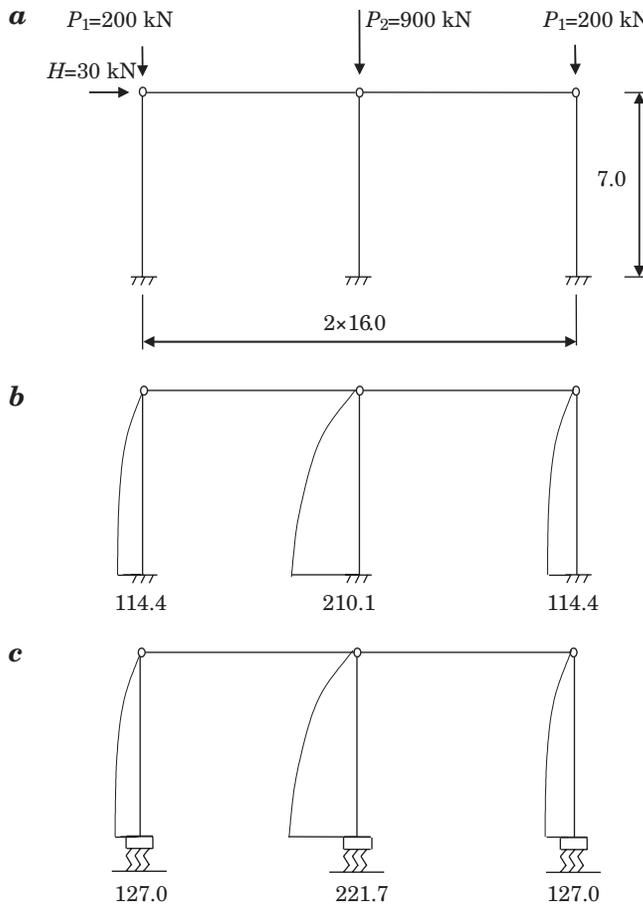


Fig. 4. Example 2- loading for case 1; a) the diagram of the single-storey building, b) bending moments for fully fixed columns [kNm], c) moments calculated with subsoil susceptibility [kNm]

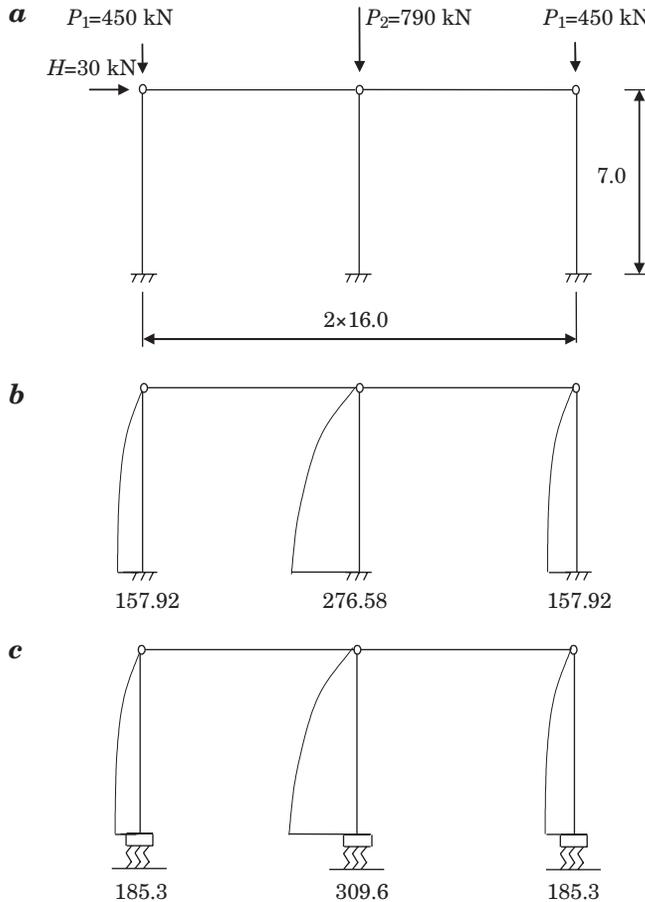


Fig. 5. Example 2 – loading for case 2. a) the diagram of the single-storey building, b) bending moments for fully fixed columns [kNm], c) moments calculated with subsoil susceptibility [kNm]

## Conclusions

The article presents a method for shaping the support of a column fixed to the foundation on elastic soil. This method may be used in standard computer programmes for static calculations. We presented examples of calculations using the exact method based on the second order analysis and taking into account the nominal stiffness for frames with columns supported in the ways described in the article. Foundation rotation leads to the increase in the final values of moments in the columns, which may be of vital importance in case of fixing columns on soils of small stiffness.

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## References

- Eurocode 2, 2008. Design of Concrete Structures. Part 1-1. General Rules and Rules for Buildings. EN 1992-1-1.
- FISCHER K. 1965. *Beispiele zur Bodenmechanik*. Ernst u. Sohn. Berlin.
- GORBUNOW-POSADOW M.T. 1956. *Obliczanie konstrukcji na podłożu sprężystym*. Budownictwo i Architektura, Warszawa.
- KANY M. 1974. *Berechnung von Flächengründungen*. Ernst u. Sohn. Berlin.
- KNAUFF M., KLEMPKA K. 2009. *Effective lengths of reinforced concrete columns in single-storey frame structures in the light of the Eurocode*. Technical Sciences, 12, 71–82.
- KNAUFF M., KLEMPKA K. 2009. *Projektowanie smukłych słupów żelbetowych według Eurokodu 2*. Inżynieria i Budownictwo, 12: 663–665.
- KOBLAK J., STACHURSKI W. 1989. *Konstrukcje żelbetowe*. T. 3. Arkady, Warszawa.
- LEWIŃSKI P. 2007. *Analiza współpracy żelbetowych zbiorników cylindrycznych z podłożem*. Wyd. ITB, Warszawa.
- PN-B-03264: 2002. *Konstrukcje betonowe, żelbetowe i sprężone. Obliczenia statyczne i projektowanie*.