

## QUASIGEOID FOR THE AREA OF POLAND COMPUTED BY LEAST SQUARES COLLOCATION

*Adam Łyszkowicz*

Chair of Land Surveying and Geomatics  
University of Warmia and Mazury in Olsztyn

Key words: gravity field, geoid, collocation.

### Abstract

The quasigeoid models recently computed in Poland e.g. (ŁYSZKOWICZ, KRYŃSKI 2006) and abroad e.g. (AL MARZOOQI et al. 2005) were computed from Stokes'a integral by Fast Fourier Technique (FFT). At present because of significant capability of personal computers and proper strategy of computation more often for geoid/quasigeoid computation the least squares collocation is used.

In the present paper is described the first quasigeoid computation by least collocation for the area of Poland. The quasigeoid model was computed in two version, namely from the gravity data only and from the gravity and vertical deflections data simultaneously. The differences between these two versions are small and do not exceed 1–2 mm.

In order to evaluate the advantages coming from the collocation the third pure gravimetric model using Stokes'a integral was computed and compared with the gravimetric model computed by collocation. The differences between these versions are significant and at the level of 20 cm, beside the collocation model is better.

## WYZNACZENIE PRZEBIEGU QUASI-GEOIDY DLA OBSZARU POLSKI METODĄ KOŁOKACJI NAJMNIEJSZYCH KWADRATÓW

*Adam Łyszkowicz*

Katedra Geodezji Szczegółowej  
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: pole siły ciężkości, geoida, kolokacja.

### Abstract

Wszystkie ostatnio liczone grawimetryczne modele quasi-geoidy w Polsce (np. ŁYSZKOWICZ, KRYŃSKI 2006) i na świecie (np. AL MARZOOQI et al. 2005) były liczone na podstawie całki Stokesa, do której oszacowania wykorzystywano szybką transformatę Fouriera (FFT). Obecnie ze względu na znaczne możliwości komputerów oraz odpowiednio opracowaną strategię obliczeniową coraz częściej do wyznaczenia przebiegu geoidy/quasi-geoidy wykorzystuje się metodę kolokacji.

W pracy przedstawiono pierwsze w Polsce wyznaczenie przebiegu quasi-geoidy metodą kolokacji. Model quasi-geoidy wyznaczono w dwóch wariantach, a mianowicie tylko z danych grawimetrycznych, a następnie łącznie z danych grawimetrycznych i astro-geodezyjnych odchyłeń pionu. Różnice między tymi dwoma wariantami są minimalne i wynoszą 1–2 mm.

W celu oceny korzyści wynikających z metody kolokacji grawimetryczną quasi-geoidę policzono również metodą FFT i porównano z quasi-geoidą obliczoną metodą kolokacji. Z przeprowadzonego porównania wynika, że rozbieżność między modelami jest rzędu 20 cm, z tym że quasi-geoida kolokacyjna jest lepsza.

## Introduction

The potential improvement of efficiency and lowering costs of height determination technology by replacing spirit levelling with GNSS technique is one of major factors generating research interest in developing regional quasigeoid models of high quality. Research on high-resolution precise quasigeoid models is thus of great importance not only for scientific purposes, in geodesy, geodynamics, geophysics, geology but also in surveying practice.

The extensive research on precise quasigeoid modelling in Poland with the use of all available data including gravity data and deflections of the vertical, conducted in 2002–2005 resulted in generating regional quasigeoid models: gravimetric quasigeoid obtained from gravity data with the use of “remove-restore” method, and astro-gravimetric quasigeoid calculated from astro-geodetic and astro-gravimetric deflections of the vertical with the use of astronomic levelling (KRYŃSKI 2007).

The aim of the following research was to investigate the contribution of collocation method and the existing deflections of the vertical in Poland to the improvement of the quality of the regional gravimetric quasigeoid in Poland and to determine a suitable methodology of combined use of gravity data with the deflections of the vertical for quasigeoid modelling.

## Quasigeoid by remove-restore technique

Geoid calculation in a regional scale by the „remove-restore” technique is realized by summation of the three terms according to the formula

$$N = N_{GM} + N_{\Delta g_{res}} + N_H \quad (1)$$

where  $N_{GM}$  is computed from geopotential model,  $N_{\Delta g_{res}}$  is computed from residual Faye’s gravity anomalies and  $N_H$  express the influence of topography. The residual gravity anomalies are compute from

$$\Delta g_{\text{res}} = \Delta g_F - \Delta g_{\text{GM}} \quad (2)$$

where  $\Delta g_F$  is Faye gravity anomaly and  $\Delta g_{\text{GM}}$  is gravity anomaly computed from geopotential model.

Term  $N_{\text{GM}}$  is computed from the general formula e.g. (TORGE 2001)

$$N_{\text{GM}}(r, \phi, \lambda) = N_0 + \frac{\text{GM}}{r\gamma} \sum_{n=2}^{n_{\text{max}}} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin\phi) \quad (3)$$

and the term  $\Delta g_{\text{GM}}$  is computed from

$$\Delta g_{\text{GM}}(r, \phi, \lambda) = \frac{\text{GM}}{r\gamma} \sum_{n=2}^{n_{\text{max}}} (n-1) \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin\phi) \quad (4)$$

where  $C_{nm}$ ,  $S_{nm}$  are fully normalized spherical harmonic coefficients of degree  $n$  and order  $m$ ,  $n_{\text{max}}$  is the maximum degree of geopotential model,  $\text{GM}$  is product of the Newtonian gravitational constant and mass of the geopotential model,  $r$ ,  $\phi$ ,  $\lambda$  are spherical polar coordinates,  $a$  is the equatorial radius of geopotential model and  $P_{nm}$  are the fully normalized associated Legendre's functions.

The term  $N_0$  is the zero term due to the difference in the mass of the Earth used in IERS Convention and GRS80 ellipsoid. It is computed according to the well known formula

$$N_0 = \frac{\text{GM} - \text{GM}_0}{R\gamma} - \frac{W_0 - U_0}{\gamma} \quad (5)$$

where the parameters  $\text{GM}_0$  and  $U_0$  correspond to the normal gravity field on the surface of the normal ellipsoid. For the GRS80 ellipsoid we have  $\text{GM}_0 = 398\,600.5000 \times 10^9 \text{ m}^3 \text{ s}^{-2}$  and  $U_0 = 62\,636\,860.85 \text{ m}^2 \text{ s}^{-2}$ . The Earth's parameter  $\text{GM}$  used in quasigeoid computation from geopotential models and the constant gravity potential  $W_0$  on the quasigeoid according to IERS Conventions have been set to the following values:  $\text{GM} = 398\,600.4415 \times 10^9 \text{ m}^3 \text{ s}^{-2}$ ,  $W_0 = 62\,636\,856.00 \text{ m}^2 \text{ s}^{-2}$ . The mean Earth radius  $R$  and the mean normal gravity  $\gamma$  on the reference ellipsoid are taken equal to  $6\,371\,008.771 \text{ m}$  and  $9.798 \text{ m s}^{-2}$  respectively (GRS80 values). Based on the above conventional choices, the zero degree term from equation (5) yields the value  $N_0 = -0.442 \text{ m}$ , which has been added to the geoid heights obtained from the corresponding spherical harmonic coefficients series expansions of geopotential model.

The numerical computations for the spherical harmonic values of  $N$  from EGM08 model have been performed with the *geocol* software program that was kindly provided by dr Gabriel Strykowski from *Danish National Space Center*. The final GM geoid heights computed from equation (3) refer to the tide free system, with respect to a geometrically fixed reference ellipsoid (GRS80).

Term  $N_{\Delta g_{\text{res}}}$  can be compute from Stokes' integral or by least squares collocation method. When the geoid is computed by least squares collocation method then the term  $N_{\Delta g_{\text{res}}}$  can be computed from the formula e.g. (MORITZ 1989)

$$N_{\Delta g_{\text{res}}} = \mathbf{C}_{Nl} \mathbf{C}_{ll}^{-1} \mathbf{I} \quad (6)$$

where  $\mathbf{I}$  is a vector of observations,  $\mathbf{C}_{ll}$  is autocovariance matrix of observations,  $\mathbf{C}_{Nl}$  is crosscovariance matrix between the  $\mathbf{I}$  and  $N$ .

Once the geoid is computed from gravity anomaly  $\Delta g$  and vertical deflection  $\xi, \eta$ , then vector of observations  $\mathbf{I}$  can be in the form

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}_{\xi\eta} \\ \mathbf{I}_{\Delta g} \end{bmatrix} \quad (7)$$

where vector  $\mathbf{I}_{\xi\eta}$  contain components of vertical deflections  $\xi, \eta$ , and vector  $\mathbf{I}_{\Delta g}$  contains gravity anomalies from the consider region.

In order to determine the term  $N_{\Delta g_{\text{res}}}$  the knowledge of the matrix  $\mathbf{C}_{ll}$  and  $\mathbf{C}_{Nl}$  is necessary, which can be computed from the proper model of covariance function. In our case the logarithmic function was used (FORSBERG 1987). This model is defined by three parameters: variance  $C_0$  of gravity anomalies, and parameters  $D$  and  $T$  that determine the degree of damping high and low frequencies of the gravity signal, respectively.

Suitable choice of the parameters of the covariance function is obtained by its proper fit to the empirical data. Practically, parameters  $C_0, D$  and  $T$  can be determined using gravity anomalies from the area of interest, e.g. with the use of the *gpfitt* program of the *GRAVSOFT* package (TSCHERNING et al. 1992).

In the case when the sphere is approximated by the plane the Stokes' integral in two dimensional coordinate system  $x$  and  $y$  is (VANIČEK, CHRISTOU 1994)

$$N_{\Delta g_{\text{res}}}(x, y) = \frac{1}{\gamma} \Delta g_{\text{res}}(x, y) * l_N(x, y) \quad (8)$$

where

$$l_N(x, x) = \frac{1}{2\pi} (x^2 + y^2)^{-\frac{1}{2}} \quad (9)$$

is planar form of Stokes' kernel function and \* denotes convolution. The equation (8) can be evaluated by direct and inverse Fourier transform. For more details see e.g. (VANIČEK, CHRISTOU 1994).

The displacement of the topographic masses in gravity reductions changes the gravitational potential and thus the geoid. Therefore the computed surface is not the geoid itself but slightly different surface cogeoid. The vertical distance between the geoid and cogeoid can be computed from (e.g. WICHEN-CHAROEN 1982)

$$N_H \approx \frac{\pi G \rho}{\gamma_m} H_P^2 \quad (10)$$

where  $G$  is gravitational constant,  $\rho$  is mass density,  $H_P$  is topographic height of the point  $P$  and  $\gamma_m$  is mean normal acceleration.

The transformation from geoid undulation  $N$  to the height anomaly  $\xi$  representing the quasigeoid height over the ellipsoid is realized with the use the Bouguer anomaly  $\Delta g_B$  using the formula e.g. (HEISKANEN, MORITZ 1967).

$$\xi = N - \frac{\Delta g_B}{\gamma_m} H \quad (11)$$

## Data used in quasigeoid computation

### Gravity data

The determination of gravimetric quasigeoid on the territory of Poland is based on gravity anomalies presented on Fig. 1, which consist of 202 403 point and mean Faye'a anomalies.

Basic data set consist of gravity data from area of Poland and contains 147 530 mean  $1' \times 1'$  Faye'a anomalies, computed from point data, clean from outstanding observation, recomputed to the POGK99 gravity and ETRS80 system and finally corrected due to topography (KRYŃSKI 2007).

Gravity data sets from Czech, Slovakia, Hungary, Romania and Western Ukraine contain mean Faye'a anomalies in blocks  $5' \times 5'$  obtained from the

Bouguer anomaly maps. Remaining data sets are very different and contain point and mean data from maps or from direct measurements.

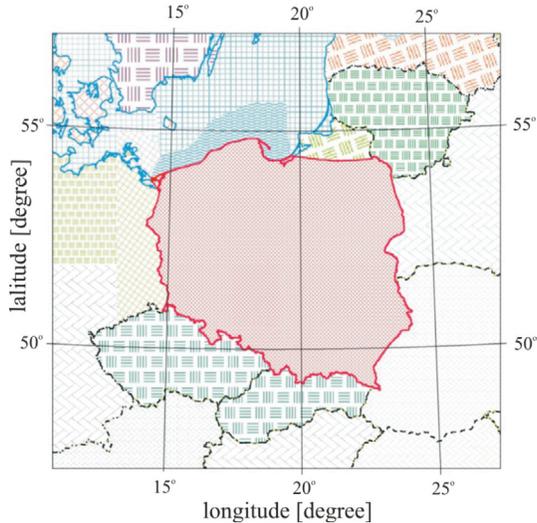


Fig. 1. Distribution of terrestrial and marine gravity data

### Vertical deflections

Archival astrogeodetic data are kept in eleven catalogues calculated in years 1967–1981. They do not contain any materials from the area of Poland to 1939. Unfortunately the archival materials are not source materials and do not contain the results of astronomical observations.

Archival astronomical data kept in repository contain mean value of determined coordinates together with the epoch of observation (time). They possess also information concerning the number of observed star pairs, the method of observation, used instrument and star catalogue used to reduction of observation. Generally, observations were reduced on the bases of the catalog FK3. In ten cases the catalog FK4 was used and three observations were reduced using Chelberger catalogue.

Saved information also contains the values of the reduction to the conventional mean pole and correction to the TU1 time system according to the conventions of International Astronomical Union. These deflections were reduced on geoid (BOKUN 1961b, p. 116).

Various instruments and different methods were used in astronomical observations and coordinate computation. For latitude determination Talcott,

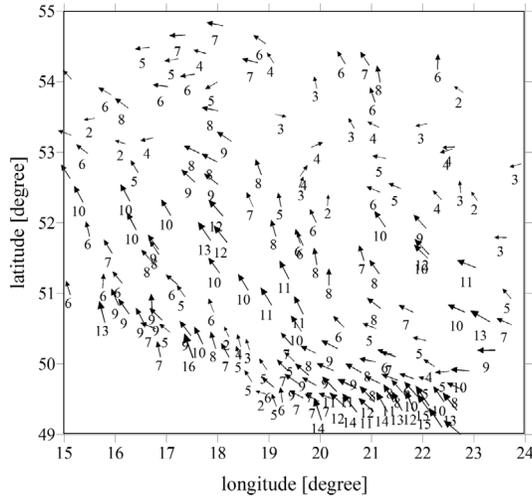


Fig. 2. Distribution of 171 astrogeodetic vertical deflections referred to GRS80 ellipsoid (all values in second of arc)

Sterneck and Piewcow methods were used. In the case of longitude determination Zinger and Mayer methods were applied.

Existing astronomical data contain 171 points (Fig. 2). The standard error of vertical deflection of  $\xi$  and  $\eta$  was estimated on  $\sigma_{\xi} = \pm 0.2''$  and  $\sigma_{\eta} = \pm 0.3''$  (BOKUN 1961) while in (KAMELA 1975, p. 12) accuracy of both components is evaluated as  $\pm 0.45''$ .

### Topographic data

Presently in Poland there are two available numerical terrain models, namely model DTED (*Digital Terrain Elevation Data*) and model SRTM (*Shuttle Radar Topography Mission*).

The model DTED was worked out according to the NATO standards by polish military group. As source material in this study military topographical maps in scale 1:50 000 with the contours interval 10 m in the horizontal system of coordinates called system "1942". Horizontal coordinates of the model were expressed in WGS84 system, however heights became referred the tide gauge in Kronsztad. Resolution of the model in the northern part of Poland is  $1' \times 2'$ , and for the southern part is  $1' \times 1'$ . The accuracy of the vertical component of the model on the area of Poland is estimated on  $\pm 2 - \pm 7$  meters, while horizontal accuracy of the model is estimated on  $\pm 15 - \pm 16$  meters.

## Quasigeoid from gravity data

### Stokes integral

In order to calculate the gravimetric quasigeoid, equation (1), from gravity data using Stokes integral, in the first step the geoid undulation  $N_{GM}$  and gravity anomalies  $\Delta g_{GM}$  from EGM08 model were computed using *geocol* software for the territory  $47^\circ < \varphi < 57^\circ$ ,  $11^\circ < \lambda < 27^\circ$  on a grid  $1.5' \times 3.0'$  (Table 1).

Statistics of computed  $N_{GM}$ ,  $\Delta g_{GM}$  from EGM08 model

Table 1

Specification	Number of points	Mean	Std dev	Min.	Max
$N_{GM}$ [m]	128 721	34.21	6.98	19.34	52.03
$\Delta g_{GM}$ [mGal]	128 721	1.18	19.93	-72.66	212.63

In the next step the residual anomalies from the area of Poland and neighboring countries (202 403 values) were computed using *geoip* software (Fig. 3). Such residual anomalies on the territory of Poland are very small, usually 2 mGal, and they do not exceed value of 4 mGal what means that the EGM08 model well represent Polish gravity field.

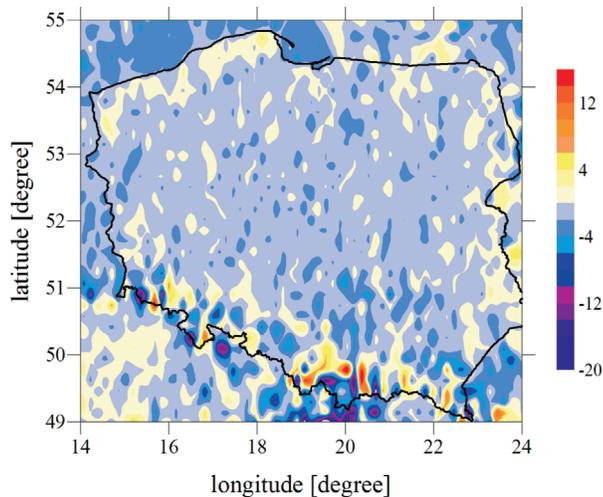


Fig. 3. Residual anomalies (isoline 2 mGal)

The evaluation of Stokes integral by FFT method require the distribution of gravity data on a regular grid. The interpolation of gravity data on a grid  $1.5' \times 3.0'$  were done by least square collocation method using *geogrid* software. First the empirical covariance function of reduced due to EGM08 model mean  $1' \times 1'$  for the area of Poland were computed (Fig. 4). Then the parameters of such empirical covariance function were determined. The computation were dane using *gpfit* from GRAVSOFTE Software.

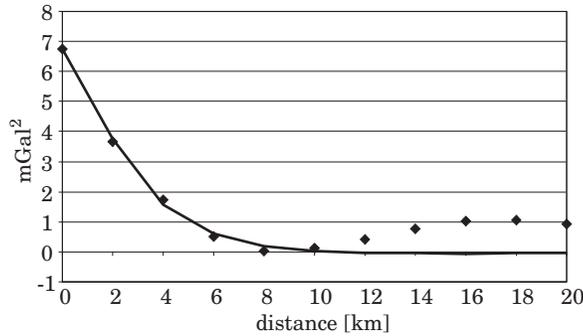


Fig. 4. Empirical covariance function of residual gravity anomalies (blue rhombus) and fitted analytical logarithmic function (red line)

The numerical value of the estimated parameters are the following: root square of the residual anomalies is  $\sqrt{C_0} = \sqrt{6.76} = 2.6$  mGal, the correlation length is  $\xi = 2.55$  km, the parameter  $D$  is 1 km and parameter  $T$  is equal 5 km. These parameter were used in interpolation of a mean Faye anomalies on a grid  $1.5' \times 3.0'$ . It yields the residual gravity data set which was used in further computation.

Next residual geoid ellipsoid undulations  $N_{\Delta g_{\text{res}}}$ , according to the formula (8), were computed. The term  $N_H$  was computed from formula (10) using SRTM3 topographic heights in a grid  $1.5' \times 3.0'$ . In this computation the density  $\rho$  was assumed equal  $2.67$  g cm $^{-3}$ . The result of  $N_H$  computation are shown on Fig. 5. It is seen that cogeoid corrections for the main part of Poland do not exceed  $-3$  mm, in the mountains is not bigger than  $-50$  mm.

After that the geoid model was computed and then transform into the quasigeoid by adding the correction computed from the formula (11). In order to compute quasigeoid corrections the knowledge of Bouguer anomalies is necessary. The Bouguer anomalies were estimated from available Faye'a anomalies by the formula

$$\Delta g_B = \Delta g_F - 0.1119 \times H \quad (12)$$

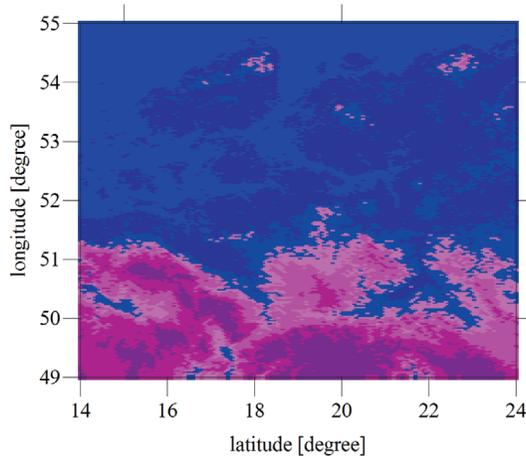


Fig. 5. Cogeoid correction  $N_H$  for the area of Poland. All correction are negative. Blue from 0 to -1 mm, dark blue from -1 to -2 mm, pink from -2 to -3 mm, dark pink from -17 to -620 mm

for the area of  $47^\circ < \varphi < 57^\circ$ ,  $11^\circ < \lambda < 27^\circ$  on a grid  $1.5' \times 3.0'$ , where  $H$  is a height obtained from model SRTM30 or SRTM3. The final result of computation is shown on Fig. 6.

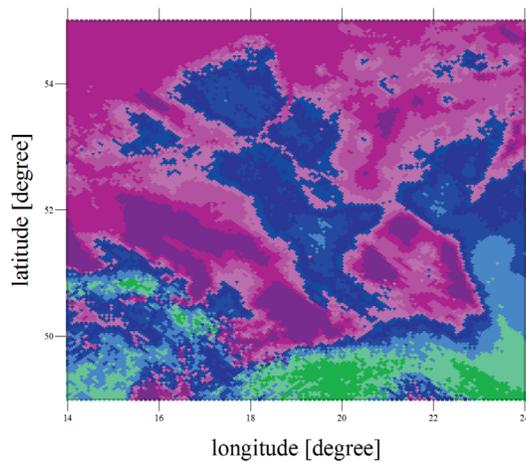


Fig. 6. Quasigeoid correction for the area of Poland. Dark pink from -11 to 0 mm, pink from 0 to 1 mm, light pink from 1 to 2 mm, blue from 3 to 6 mm, green from 17 to 766 mm

From Fig. 6 it is seen that the sign of correction depend on the sign of Bouguer anomaly and on the main part of Poland is few millimeter while in mountains can reach value even plus 80 cm.

The quasigeoid model computed in this chapter from gravity data by the FFT method was called *quasi09a*.

### Collocation approach

Subsequently from the same gravimetric data the next gravimetric model of the quasigeoid was computed by the least square collocation method. Taking into account parameters of the empirical covariance function computed in previous chapter the residual geoid distances of the geoid from ellipsoid were computed by the collocation method, equation (6), using *gpcol* software. They were computed on a grid  $1.5' \times 3.0'$  for the area  $47^\circ - 57^\circ; 11^\circ - 27^\circ$ .

Because of a large number of gravity data, computation of residual geoid heights was not possible in one step. Therefore the consider area was divided into 160 blocks  $1^\circ \times 1^\circ$  each. For each block the gravity data was prepare in three data sets (Fig. 7). First set contained all accessible data for the area  $1.2^\circ \times 1.4^\circ$ . In the second set data were collected from the surrounding area on width  $1^\circ \times 2^\circ$  on a grid  $3.0' \times 6.1'$ . In the third data set the data were collected for the remaining area with a mesh  $9.0' \times 18.0'$  (Fig. 7).

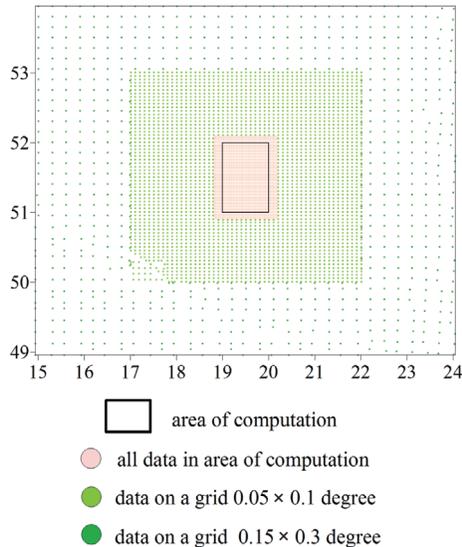


Fig. 7 The way in which gravity data for the block  $51^\circ - 52^\circ; 19^\circ - 20^\circ$  was formed

From the results of computation in 160 blocks the final solution was created by adding all blocks together. The residual geoid ellipsoid separations were calculated for the area  $47^\circ - 57^\circ; 11^\circ - 27^\circ$  with a mesh  $1.5' \times 3.0'$ . Together

with geoid computation first time in Poland the accuracy of computed residual geoid separation were estimated and the results are shown on Fig. 8. From the Fig. 8 appears that the accuracy of the term  $N_{\Delta g_{res}}$  is from  $\pm 3$  – to  $\pm 4$  mm.

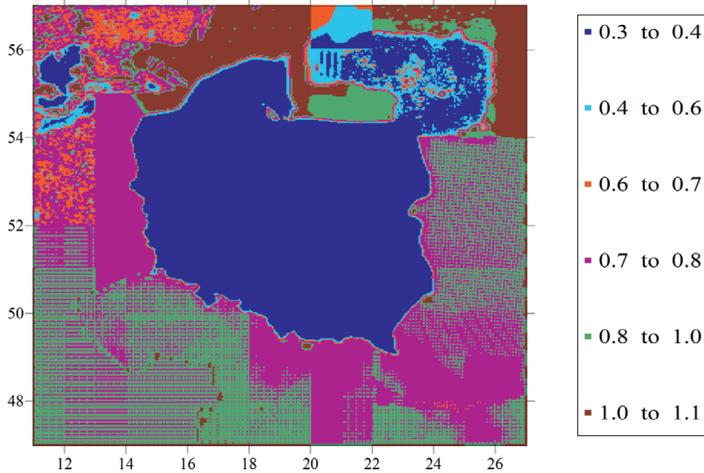


Fig. 8. Error distribution (in cm) of residua geoid ellipsoid separation  $N_{\Delta g_{res}}$  computed by the least squares method from gravity data for the area of Poland

The final geoid solution was obtained applying formula (1) and then quasigeoid solution was obtained through the transformation of the geoid into quasigeoid according to the formula (11). Computed in such a way the gravimetric model of the quasigeoid was called *quasi09c*.

### Quasigeoid from gravity data and vertical deflections

Combined quasigeoid from gravimetric data and astrogeodetic vertical deflection was calculated according the rules given in sec. 2. In our calculation residual gravity anomalies (Fig. 3) and residual vertical deflections  $\xi$  and  $\eta$  referred to EGM08 model were used. Mean value of residual gravimetric anomalies is  $-0.93$  mGal, while mean value for the vertical deflection is  $-0.15'$  for the component  $\xi$  and  $0.13'$  for the component  $\eta$ . It means that the EGM08 model well represents the gravity field of Poland.

From residual data i.e. gravity anomalies and vertical deflections, the term  $N_{\Delta g_{res}}$  by the least squares collocation was computed (eq. 6). These calculation were done for each  $1^\circ \times 1^\circ$  block and final residual solution was obtained by summation all computed blocks. The combined geoid was obtained by the adding indirect effect (eq. 11) and quasigeoid was obtained by transforming

geoid into quasigeoid accordingly with (eq. 11). Such computed quasigeoid model was called *quasi09b*.

### Accuracy assessment and quasigeoid comparison

In the present chapter three computed models, namely:

- model *quasi09c* computed from gravimetric data by least squares collocation,
  - model *quasi09a* computed from gravimetric data by FFT method,
  - model *quasi09b* computed from gravimetric and vertical deflection data by the least squares collocation method,
- were evaluated on the satellite GPS/levelling networks.

The quasigeoid evaluation is based on the comparison the quasigeoid distances  $\zeta^{\text{gps/niw}}$  computed from satellite GPS observations and levelling of the POLREF network with distances  $\zeta^{\text{grav}}$  from appropriate gravimetric models.

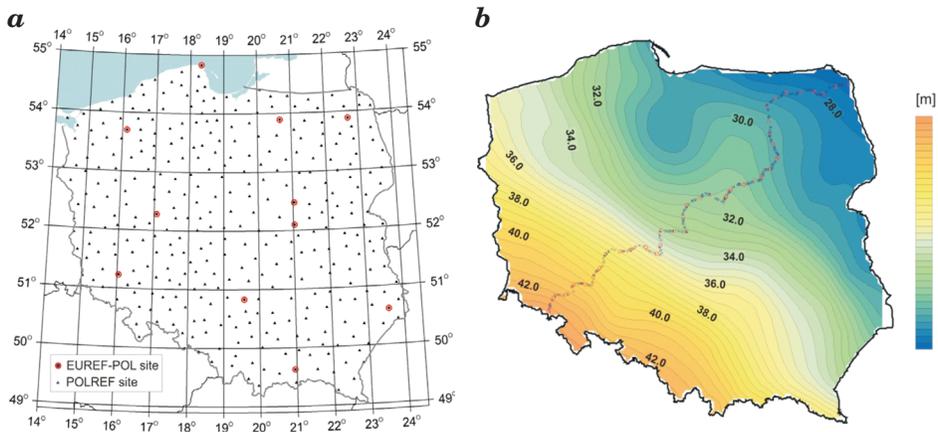


Fig. 9. EUREF-POL92 i POLREF points (a) and precise traverse (b)

For verification of quasigeoid models as well as for estimation of their accuracy and evaluation of interpolation algorithms used for application of GPS/levelling quasigeoid, a GPS/levelling control traverse has been established across Poland (KRYNSKI 2006). The traverse of 868 km surveyed in 2003–2004 consists of 190 stations (1/4.6 km) of precisely determined ellipsoidal and normal heights.

Observation strategy developed and processing methodology applied ensure accuracy of quasigeoid heights at traverse points at a centimeter level. The 49 stations of the traverse considered as the 1<sup>st</sup> order control were

surveyed in one or two 24 h sessions. The remaining 141 stations were surveyed in 4h sessions. The coordinates of 1<sup>st</sup> order control were determined using the EPN strategy with the Bernese v.4.2. Accuracy of the coordinates determined in such a way is at the level of single millimeters. The coordinates of 141 points were calculated using the Pinnacle program with the 1<sup>st</sup> order control as reference (CISAK, FIGURSKI 2005).

In order to estimate the absolute accuracy of the gravimetric model the differences

$$\varepsilon_i = \zeta_i^{\text{gps/niv}} - \zeta_i^{\text{graw}} \quad (13)$$

were computed in each point of satellite GPS network and the mean value was a valuated from

$$\hat{x} = \frac{1}{n} \sum_{i=1}^n \Delta_i \quad (14)$$

The empirical standard deviation is computed from

$$\hat{\sigma} = \pm \sqrt{\frac{\sum \varepsilon_i^2}{n-1}} \quad (15)$$

which gives the numerical description of the accuracy of the tested quasigeoid model.

Below are given the results of the accuracy evaluation of the models: *quasi09a*, *quasi09b* and *quasi09c* at the POLREF network (Table 2) and precise traverse points (Table 3).

From the results given in Table 2 it is seen that the model *quasi09c* computed by collocation method is visible (8%) more exact then the model *quasi09a* computed from the same data but by FFT method. From the Fig. 10 results that the differences between these two models are on the level of two decimeters.

Table 2  
Accuracy evaluation of models *quasi09a*, *quasi09c* i *quasi09b* (in m) at 360 points of POLREF network (bias in not removed)

Specification	<i>quasi09a</i>	<i>quasi09c</i>	<i>quasi09b</i>
Mean	0.374	0.181	0.180
Std dev	0.035	0.032	0.032
Min.	0.241	0.103	0.103
Max	0.482	0.309	0.308

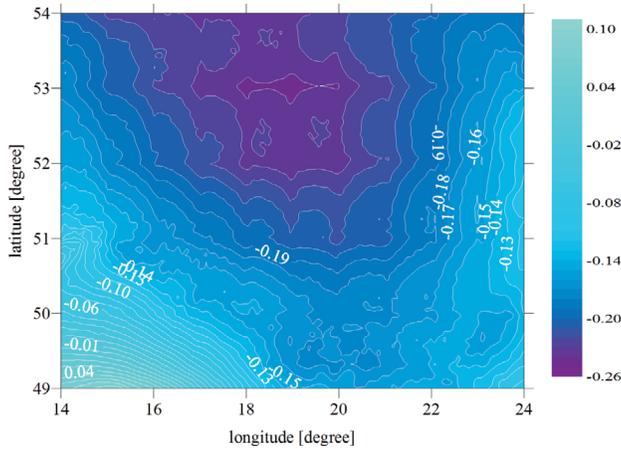


Fig. 10. Differences between the *quasi09a* and *quasi09c* models at POLREF sites (isolines 1 cm)

Considerably better accuracy of computed models was obtained when they were tested at precise traverse points (Table 4). From the quoted table results that the *quasi09c* model computed by least squares collocation method is a significantly (33%) better than model *quasi09a* computed from the same data but by FFT method. From the Fig. 10 results that the differences between these two model are 1–2 dm on the area of Poland and bias about -20 cm is visible (see Fig. 11).

Table 3  
Accuracy evaluation of models *quasi09a*, *quasi09c* and *quasi09b* (in m) at the 190 points of precise traverse (bias in not removed)

Specification	<i>quasi09a</i>	<i>quasi09c</i>	<i>quasi09b</i>
Mean	0.348	0.141	0.140
Std dev	0.027	0.018	0.018
Min.	0.279	0.093	0.091
Max	0.401	0.187	0.187

Influence of additional data i.e. astro-geodetic vertical deflection on the accuracy of gravimetric quasigeoid computed by least squares method do not improve in visible way such combined solution.

Impact of vertical deflection data on the pure gravimetric solution results from -2 mm to +1 mm (see Fig. 12 and Fig. 13)

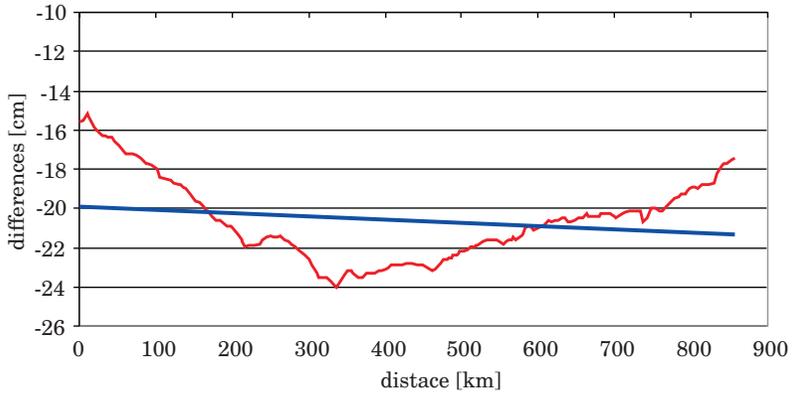


Fig. 11. Differences between the *quasi09a* and *quasi09c* models at the 190 points of precise traverse – red line, and bias – blue line

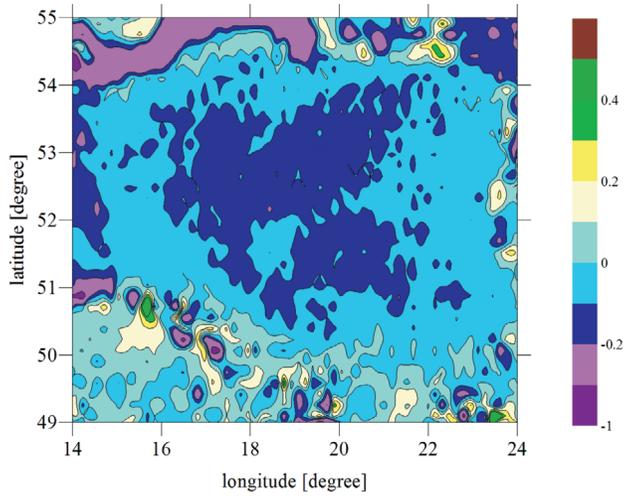


Fig. 12. Differences between the *quasi09b* and *quasi09c* models at the 360 POLREF sites (isolines 1 cm)

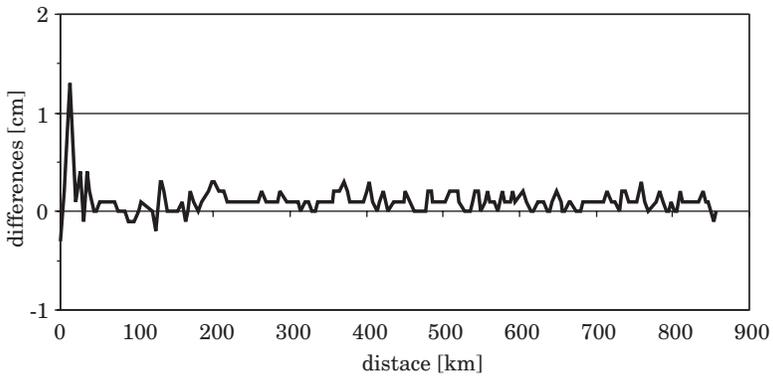


Fig. 13. Differences between the *quasi09b* and *quasi09c* models at the 190 points of precise traverse

## Summary and conclusions

The gravimetric models of quasigeoid were computed using “remove-restore” technique. In this quasigeoid modelling EGM08 potential model was used since it is five time better than the last EGM96 model and can be characterized (in absolute sense) by the empirical standard deviation from  $\pm 2$  to  $\pm 3$  cm (ŁYSZKOWICZ 2009).

Gravity data used in geoid modelling comprise gravity anomalies from Czech Republic, Slovakia, Hungary, Romania, West Ukraine, Byelorussia, Lithuania, Latvia, Denmark, Germany and the main amount of gravity data was from Poland. Additionally in quasigeoid modelling the topographical data were used, namely DTED and SRTM model.

Gravimetric quasigeoid from gravity data was computed by the FFT method (*quasi09a*) and by the least squares collocation method (*quasi09c*).

Simple accuracy evaluation of *quasi09a* model at the 360 points of POLREF network gives value  $\pm 3.5$  cm, while the identical evaluation at the 190 points of precise traverse gives value  $\pm 2.7$  cm.

In order to assess the quality of the least squares collocation the quasigeoid from the same data again was computed. This model was computed using *gpcol* software, which main part is logarithmic model of covariance function proposed by R. Forsberg (FORSBERG 1987).

Logarithmic function was fitted to the mean residual gravity anomalies from the territory of Poland and then following parameters were obtained:  $\sqrt{C_0}$  mGal,  $D = 6$  km,  $T = 30$  km. These parameters were used in *quasi09c* computation. Because of large number of the gravity data residual geoid distances were computed for the 160 block with size  $1^\circ \times 1^\circ$ . One estimates that the accuracy of *quasi09c* model evaluated at 360 points of POLREF network is  $\pm 3.2$  cm and evaluated at 190 points of precise traverse is  $\pm 1.8$  cm.

The third quasigeoid model *quasi09b* was computed from gravity and vertical deflection data and EGM08 geopotential model. Evaluated accuracy of this model at the POLREF network is  $\pm 3.2$  cm, while evaluated at the points of precise traverse is  $\pm 1.8$  cm.

Finally we can state that quasigeoid computed by least square collocation method gives better results then by FFT method.

In the last years a considerably progress in precision of geoid calculation from decimeters to centimeters was achieved. In case of decimeters accuracy the result of calculation by FFT or collocation method was identical. At present when the residual geoid ellipsoid separations on the area of Poland are calculated with accuracy 3 – mm (see Fig. 8) appears that local solution by collocation is better than regional solution by FFT.

Inclusion additional data i.e. components of astro-geodetic deflections of the vertical in our case gives difference of few millimeters (Fig. 12) and tests conducted at the POLREF network and precise traverse do not show, in terms of standard deviation, which solution is better.

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