TECHNICAL SCIENCES
Abbrev.: Techn. Sc., No 13, Y 2010

DOI 10.2478/v10022-010-0022-7

# AN ATTEMPT TO IMPROVE THE AXISYMMETRIC MODEL OF A PNEUMATIC TIRE

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Key words: tire, cord-rubber composite, deformation modeling, finite element method.

Abstract

The existing method of a tire cord-rubber ply modeling involved the determination of effective material constants relative to ply thickness. This paper investigates the use of the same method in reference to plies with the thickness of a cord. Cord-rubber plies with orthotropic properties were separated by layers of isotropic rubber to maximize the composite susceptibility to deformation. When the improved model was applied in the simulations of tire pumping to nominal pressure, an increase was observed in the displacements of the tire characteristic points, maximum equivalent stress in rubber layers located between cord-rubber plies and the shear strain of rubber.

## PRÓBA DOSKONALENIA OBROTOWO-SYMETRYCZNEGO MODELU OPONY PNEUMATYCZNEJ

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Słowa kluczowe: opona, kompozyt kodowo-gumowy, modelowanie deformacji, metoda elementów skończonych.

Abstrakt

Dotychczasowy, powszechnie stosowany sposób modelowania warstwy kord-guma w oponie polegał na wyznaczaniu stałych efektywnych materiału w odniesieniu do grubości warstwy. Zbadano możliwość zastosowania tego samego sposobu, ale w odniesieniu do warstwy o grubości nici kordu. Warstwy kordu o właściwościach ortotropowych rozdzielono warstwami gumy izotropowej, co wpłynęło na zwiększenie podatności kompozytu na odkształcenia. Wykorzystując udoskonalony model do symulacji pompowania opony testowej do ciśnienia o wartości nominalnej, zaobserwowano wzrost wartości przemieszczeń jej punktów charakterystycznych, maksymalnych naprężeń zredukowanych w gumie między warstwami oraz odkształceń postaciowych w gumie.

## Introduction

In axisymmetric computational models of pneumatic tires based on the finite element method, the rubberized cord ply is usually discretized in such a way that one of the finite element dimensions is equal to the thickness of that ply (Kaga, Okamoto, Tozawa 1977, Tseng et al. 1991, Pelle 1994, Pelc 1995, Pelc 2000). In this case, the mechanical properties of a layer comprising cords, the rubber between cords and two rubber layers covering the cord-rubber layer are averaged in the area of the finite element and are presented in the form of effective constants of the composite ply (Jones 1975). This material is characterized by greater rigidity than the material of the rubber layers (referred to as separators) isolating cord-rubber plies with the thickness of a cord (i.e. steel or textile yarn). The finite elements of adjacent plies share nodes along their boundary, therefore, the relative displacement of those plies is blocked despite the existence of compliant rubber separators. The method for the cord-rubber composite material modeling with rubber separators has been indicated by Walter (1978) and used by Cembrola and Dudek (1985) in modeling of deformations of a plane composite specimen. This study investigates the effect of separators in modeling of deformation of a truck tire subjected to internal pressure.

## Constants for cord-rubber plies and for rubber materials

Effective (averaged) constants of the cord-rubber composite ply may be calculated based on the equations proposed by Halpin and Tsai (1969) which take on the following form on the assumption that cords have a much higher shear modulus than rubber:

$$\begin{aligned} E_1 &= E_c v_c + E_r(1 - v_c) \\ E_2 &= \frac{E_r( + 2v_c)}{1 - v_c} \\ \nu_{12} &= \nu_c v_c + \nu_r (1 - v_c) \\ G_{12} &= G_r \frac{1 + v_c}{1 - v_c} \end{aligned} \tag{1}$$

where: letters $E/G/\nu$ – indicate Young modulus, Kirchhoff modulus and the Poisson ratio, indices $c/r$ denote cord and rubber. Axis 1 is parallel to the cord, and axis 2 is the perpendicular axis in the layer plane. At constant density of cord $e_p$ with cross-section area $A_c$, the values of the above constants are determined by the ply thickness $t$ and the ply cord volume fraction:

$$v_c = \frac{e_p A_c}{t} \tag{2}$$

When rubber layers covering the cord on both sides are included in the cord-rubber composite ply, the value of parameter $v_c$ decreases in comparison with that corresponding to the cord-rubber ply with the thickness of a cord.

The tire purely rubber elements (tread, sidewall, etc.) are considered to be incompressible, and their properties are described with the use of a one-parameter, neo-Hookean model.

## Characteristic of an axisymmetric tire model

The model has the features of an axisymmetric system if only internal pressure is applied to the tire. In this case, only the tire cross-section is discretized, and the problem has class 2.5D. The analyzed tire cross-section and the standard mesh of elements with one finite element at the thickness of the rubber/cord-rubber/rubber composite ply are presented in Figure 1a and Figure 1b, respectively. The mesh is also used to generate 3D tire models.

To eliminate the effect of the discretization degree on the results, a refined mesh of finite elements was generated and used in both the standard model of cord-rubber composite plies (Fig. 2a) as well as in the improved model, i.e. a model accounting for rubber separators between reinforcement layers, when cord-rubber plies have the thickness of a cord (Fig. 2b). It has been assumed that the nodes of tire beads that come into contact with the rim are fully fixed, therefore, the tire contact with the rim has been disregarded as insignificant for the needs of this analysis. The nodes located in the tire plane of symmetry (*yz*) may be displaced only in the radial direction. Internal pressure was applied to the tire (Fig. 1b).

The problem was solved using the MARC program with user subroutines supporting the description of the composite specific properties. The composite geometric parameters in the tire are determined based on the properties of the non-vulcanized, raw layer on the tire building drum, the geometric parameters of the drum and the vulcanization mold. In the FEM model, the element overlaying technique was used to improve the model numeric properties (PELC 2002). Cord-rubber composite plies and bead wire were modeled with four-noded axisymmetric solid elements, while five-noded incompressible Hermann elements were applied to model rubber layers and purely rubber elements. The number and type of elements are presented in Table 1.

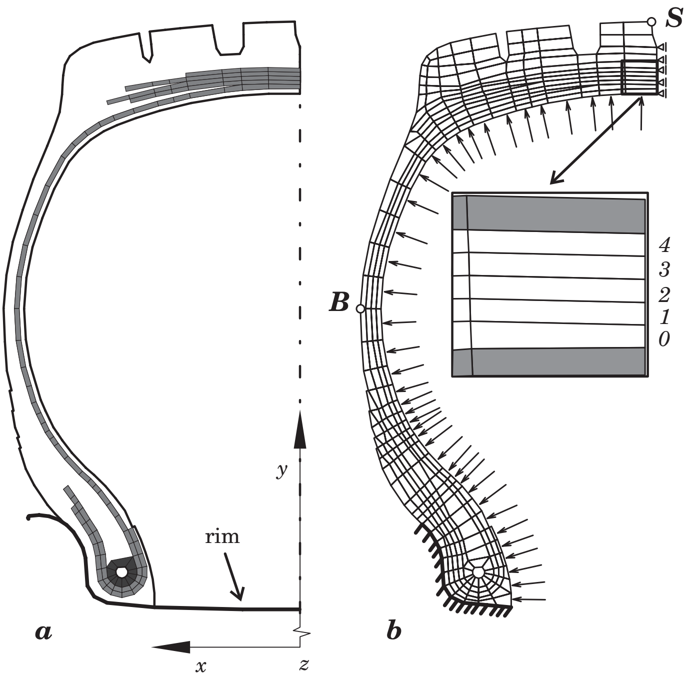


Fig. 1. Axisymmetric model of a truck tire: *a* – cross-section with an indication of plies, *b* – typical finite elements mesh: *0* – body ply; *1, 2, 3* and *4* – successive belt plies

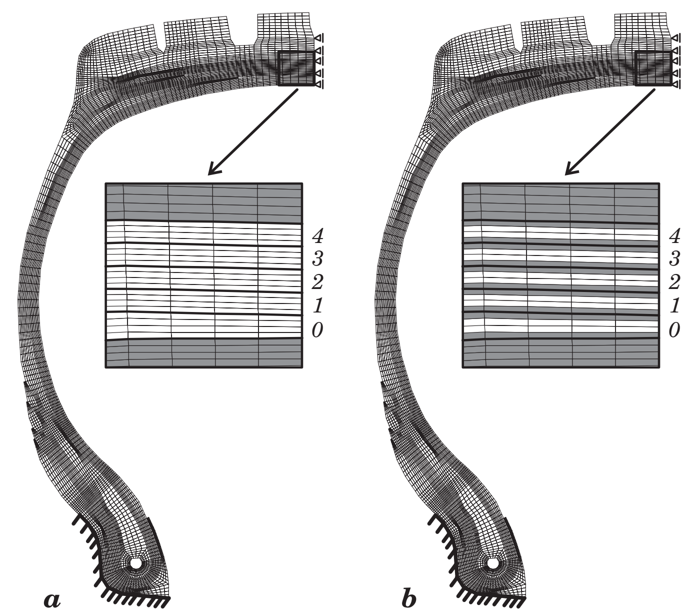


Fig. 2. Axisymmetric model of a truck tire: *a* – standard model, *b* – improved model: *0* – body ply; *1*, *2*, *3* and *4* – successive belt plies

Table 1

Number of elements in every tire zone

| Zone | Number of elements | | |
|---|---|---|---|
| | typical model (Fig. 1b) | standard model (Fig. 2a) | improved model (Fig. 2b) |
| Cord-rubber plies | 96 | 1536 | 768 |
| Rubber under plies | 96 | 1536 | 768 |
| Bead wire (steel) | 11 | 176 | 176 |
| Pure rubber | 227 | 3632 | 3632 |
| Rubber between plies | 0 | 0 | 768 |
| Total | 430 | 6880 | 6112 |

## Calculation results

Due to the model non-linear character, the Newton-Raphson method was selected to calculate the solution. The tire was subjected to incremental load by increasing internal pressure to nominal pressure of 800 kPa.

Displacement components of the tire two characteristic points, point $B$ and point $S$ (Fig. 1b), are presented in Table 2. In the improved model, maximum axial and radial displacement of the two points increased by 8% and 3%, respectively, in comparison with the standard model.

Table 2

Displacement components of the tire characteristic points

| Tire model | Point located in tire profile | $B$ | | $S$ | |
|---|---|---|---|---|---|
| | Displacement [mm] | $x$ | $y$ | $x$ | $y$ |
| Standard | | 2.11 | 1.05 | 0 | 4.32 |
| Improved | | 2.28 | 1.19 | 0 | 4.44 |
| Displacement difference [%] | | 8.3 | 13.3 | 0 | 2.8 |

Percent changes in the maximum values of Cauchy stress in rubber located between reinforcement layers are given in Table 3.

Table 3

Changes in the maximum values of Cauchy stress in rubber

| Ply | Change in Cauchy stress in rubber [%] ← between plies → | Ply |
|---|---|---|
| Body | 10 | belt No. 1 |
| Belt No. 1 | 11 | belt No. 2 |
| Belt No. 2 | -5 | belt No. 3 |
| Belt No. 3 | -8 | belt No. 4 |

The greatest changes in the maximum equivalent stress values in rubber reached 10% and were observed between the body ply and the first belt ply, and between the first and the second belt.

The presented isoline maps of shear strain $E_{xy}$ indicate that the separation of cord-rubber plies with isotropic rubber layers produces higher shear strain values in rubber in comparison with the model where rubber separators were not applied (Fig. 3). In the improved model, the maximum strain values were 18% higher than in the standard model.

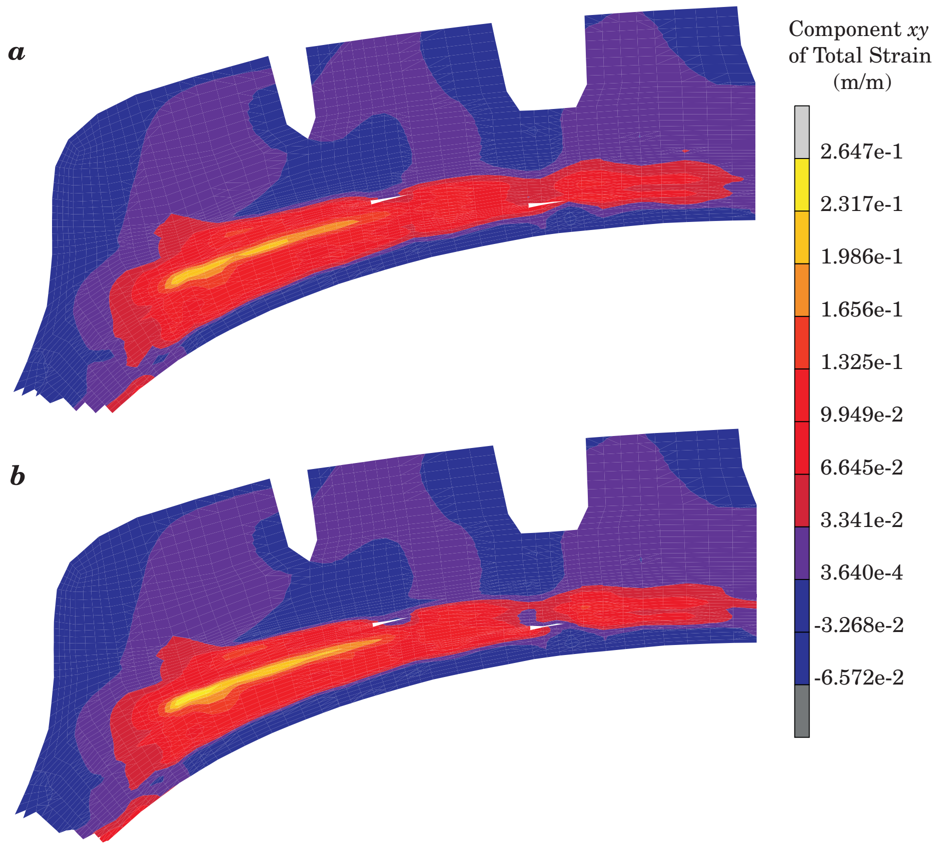


Fig. 3. Isoline maps of shear strain $E_{xy}$ in rubber in the tread zone of an inflated tire: *a* – standard model, *b* – improved model

## Conclusions

This study analyzed the effect of improved modeling of pneumatic tire plies in the axisymmetric model of a test tire subjected to nominal pressure of 800 kPa. In the improved modeling technique, cord-rubber plies were separated with rubber layers. The axial displacement of the characteristic point on the sidewall increased by 8%, and the radial displacement of the point situated at the apex of the tire increased by 3% in comparison with the results yielded by the standard model. The discussed technique for modeling of layered materials significantly enhances the system flexibility. Cord-rubber plies are more readily deformed – they are separated by layers of isotropic rubber, and finite elements representing layers of the cord-rubber composite do not have shared nodes.

The presented approach to the cord-rubber plies modeling may seem natural, but to date, it has not found any applications in deformation modeling of tires. The discussed model requires a drastic increase in the number of the finite elements in the mesh (14-fold in the analyzed tire) to preserve the right proportions between the length of the sidewalls. In view of the computing power of contemporary computers, attempts should be made to use the improved method of layered composite material modeling, which gives a more reliable computational model of a tire. Unfortunately the suggested approach requires the generation of two finite elements meshes for tire cross-section: one (refined) for axisymmetric analyses and the other, with a reduced number of elements, for 3D tire analyses.

Accepted for print 11.05.2010

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