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JET PUMPS – NUMERICAL MODELING POSSIBILITIES UPON THE BIFURCATION PHENOMENA

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Key words: flow bifurcation, jet pumps, CFD.

Abstract

This paper presents the results of numerical simulations for modeling the air-air jet pump in 2D canal. The conducted simulation studies indicate the possibility of presence of strong bifurcations in jet pumps with the degree of intensity depending on geometric indicators and thermodynamic parameters of the system. Appearance of bifurcation in the mixing chamber causes periodic appearance of free spaces between the bifurcating liquid jet flowing out of the working nozzle and the walls of the chamber in which compression of the medium sucked and transport of it to the diffuser occurs. The paper presents and discuss the results of earlier works on bifurcation phenomenon in the systems of "nozzle – mixing chamber" type and qualitative results of computer simulation of sample ejector operation. The paper also contains the analysis of the influence of various factors on the results of computations. The goal of the work was confirmation of the possibility of numerical modelling jet pumps based on the bifurcation phenomena.

MOŻLIWOŚCI NUMERYCZNEGO MODELOWANIA STRUMIENIC Z WYKORZYSTANIEM ZJAWISKA BIFURKACJI

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Słowa kluczowe: przepływy bifurkacyjne, strumienice, CFD.

Abstrakt

W srtykule przedstawiono wyniki symulacji numerycznych dotyczących modelowania strumienicy powietrzno-powietrznej w przestrzeni 2D. Wyniki badań symulacyjnych wskazują na możliwość występowania w strumienicach silnych bifurkacji, o stopniu intensywności zależnym od wyróżników geometrycznych oraz parametrów termodynamicznych układu. Wystąpienie bifurkacji w komorze mieszania powoduje okresowe pojawianie się przestrzeni – między wypływającym z dyszy roboczej bifurkującym dżetem płynu a ściankami komory, w których następnie dochodzi do sprężenia zasysanego czynnika i transportowania go do dyfuzora. Przedstawiono również wyniki wcześniejszych prac dotyczących zjawiska bifurkacji w układach typu „dysza – komora mieszania" oraz jakościowe wyniki symulacji komputerowych pracy przykładowej strumienicy. Przeanalizowano wpływ różnych czynników na wyniki obliczeń. Celem badań było sprawdzenie możliwości modelowania numerycznego strumienic bazującego na zjawisku bifurkacji.

Introduction

Currently three basic mixing mechanisms in jet pumps are identified:

1. impact mixing – this mixing mechanism assumes presence, along a certain section of the mixing chamber, of the parallel flow of the working medium (in the form of central, compact stream) and the suction medium (in the form of a cylinder between the working stream and the walls), which next, in a rapid way, transforms into the vertical flow. After the mixing zone the liquid is the major fraction while the gas is present in it in the form of bubbles. The impingement model was developed for liquid-gas jet pumps in the algebraic form (WHITTE 1969).

2. friction mixing – this mixing mechanism assumes that as a consequence of friction (increased by turbulent movement) direct transfer of kinetic energy from the feeding stream to the suction stream occurs. That means balancing of velocities of both phases although there is no mixing of them. That model was developed for any type of jet pump in the form of differential equations (SPALDING 1992).

3. pulsation mixing – while the two preceding mechanisms were related to the stationary phenomenon of mixing, the pulsation mixing mechanism is based on dynamic phenomena occurring locally in the area of the catching nozzle and the mixing chamber. That mechanism contains certain elements identical with the mechanism of impingement mixing (no influence of viscosity) or friction mechanism (non-stationary nature of multiscale turbulence) (CUNNINGHAM 1974, FLÜGEL 1951). In the pulsation model, side surfaces of bifurcating mainstream substitute for the blades of the traditional compressor (Fig. 1).

This paper mainly refers to the third model. Further in the paper simulation results are presented in case of which the suction effect on passive inlets was achieved resulting from flow bifurcation phenomenon only. In those simulations where bifurcations did not appear there was also no suction of passive medium. The presented analysis represents a continuation of earlier studies on appearance of bifurcations in systems of "nozzle – mixing chamber" type (BADUR, SOBIESKI 2001, BADUR, SOBIESKI 2005) and it is not based on

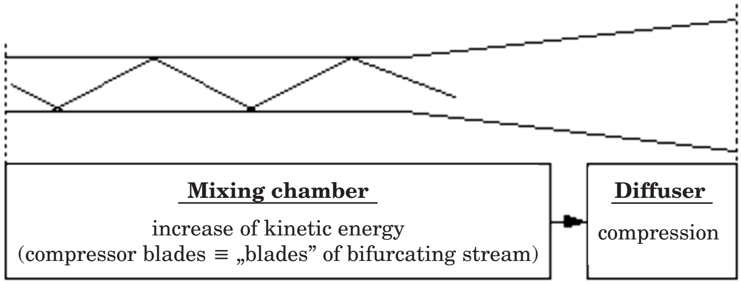


Fig. 1. Nature of pulsation model functioning

a specific, actual jet pump. The general nature of the paper is a consequence of the fact that so far a correct jet pump simulation model has not been developed and the literature does not offer appropriate publications.

Introduction to bifurcation theory

In mathematical terms bifurcation is a phenomenon of the abrupt change of mathematic model characteristics resulting from a small change in its parameters (e.g. initial conditions of the process or the boundary conditions). That notion is particularly important and frequently met during solving differential equations and studies on fractals (and chaos theory). In the model with parameter λ, λ_0 is the bifurcation point if, in every surrounding of it, two points exist for which the characteristics of the model are not identical. In case of differential equations bifurcation occurs when the solution of the nonlinear differential equation:

$$\frac{dx}{dt} = f(x, \lambda)$$

changes qualitatively its character with the change of the active parameter λ (Fig. 2).

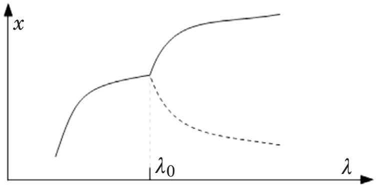


Fig. 2. Visualization of the nature of bifurcation
Source: MELLIS (2006), TRYBULEC (2006), WILSON (2006).

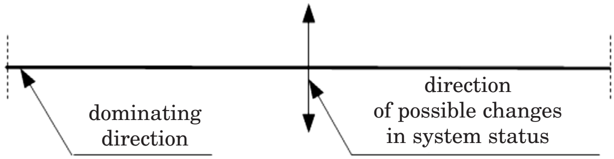


Fig. 3. Conditions for appearance of bifurcation

In case of passing the critical value the behavior of the system can be of two types (BADUR, SOBIESKI 2001, KURNIK 1997):
– the system will change its status into another one of stationary character – this type of bifurcation is referred to as stationary or divergent bifurcations,
– the system will change its status periodically – this type of bifurcation is referred to as non-stationary or oscillatory bifurcations.

The critical value is usually a specific parameter characterizing a given system. In case of flow system that role can be played by, e.g. Mach number or Reynolds number. Examples of the bifurcation phenomena in a solid-body and in a fluid flow are presented on Fig. 4.

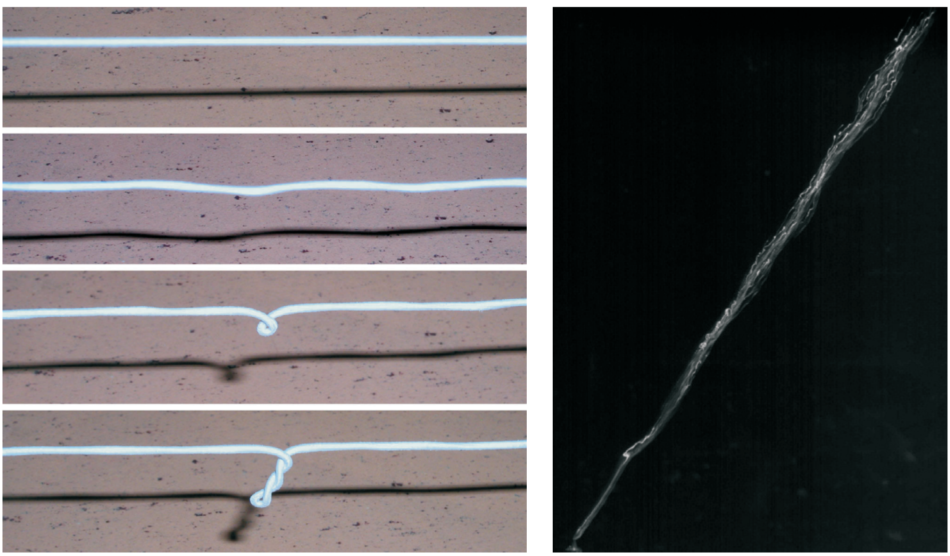


Fig. 4. Examples of bifurcation in a solid (left) [Lega] and liquid (right)
Source: MANASSEH et al. (2006).

Types of bifurcation in flow systems

The issue of bifurcation in mechanics of liquids is a frequent subject of scientific studies and analyses. Authors of papers indicate the possibilities of appearance of different bifurcation types in flow systems characterized by a specific type of geometry. The examples are:

a) flow takes place in a channel with a sudden widening – Hopf bifurcations possible (BADUR et al. 1999, BATAGLIA et al. 1997, ROSENFELD 1995, SOBIESKI 2009a, SOBIESKI 2009b),

b) flow takes place in a channel with a perpendicular branch – Hopf bifurcations possible (KHODADADI 1991, SCHINAS and MATHIOULAKIS 2000),

c) flow takes place as a result of convection – Rayleigh-Benard bifurcations possible (HARAMINA 2005, MANNEVILLE, ZHANG 2001),

d) flow takes place between rotating cylinders – Taylor vortices possible (BAIER 1999, GREBE 2004, YOUD).

The goal of the studies described in the paper was to find the basic relations between the geometry of a "nozzle – mixing chamber" system and its parameters and the intensity of bifurcations appearing in the flow. The system in which Hopf bifurcations occur was assumed as the optimal.

Conditions for development of Hopf bifurcations

Hopf bifurcations are a type of oscillatory bifurcations in which changes of system state occur in a periodic pattern. Such bifurcations can occur, e.g. during outflow of the liquid jet from a nozzle to a larger space that is limited by walls. Such a situation occurs, inter alia in jet pumps in the area of outflow of liquid from the working nozzle into the mixing chamber. In literature papers can be found concerning analysis of bifurcations in the flow systems of that type. It is found that the major parameters influencing the behavior of the main liquid stream are the geometrical conditions (BATAGLIA et al. 1997, DYBAN et al. 1971, SOBIESKI 2009a, SOBIESKI 2009b).The ratio between channel or mixing chamber width D and the width of the working nozzle outlet d is the basic parameter characterizing the system in this case (the symbols D and d are from (BATAGLIA et al. 1997). With the increase of that parameter consecutive recirculation zones larger in size and with the progressively larger flow oscillations amplitude appear (Fig. 5) (BATAGLIA et al. 1997). The second parameter influencing the behavior of the system in considered case is the flow velocity (Fig. 6). Depending on it, symmetrical or asymetrical (bifurcation) forms can appear (BATAGLIA et al. 1997). The importance of other parameters, such as e.g. viscosity, seems to be lower (DYBAN et al. 1971).

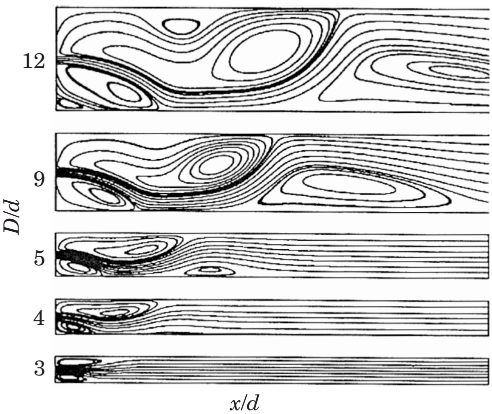


Fig. 5. Current contour lines for D/d = 3,4,5,9,12, c = 15 m/s, x/d = 50 i Re = 60
Source: BATAGLIA et al. (1997).

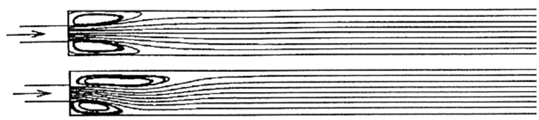


Fig. 6. Current contour lines for Re = 53 (upper) and Re = 67 (lower)
Source: BATAGLIA et al. (1997).

Types of bifurcations in nozzle – mixing chamber systems

Earlier completed own simulation studies based on laboratory experiments of other authors (DYBAN et al. 1971 resulted in identification of several flow structure types (BADUR and SOBIESKI 2001, BADUR and SOBIESKI 2005, ŚWIĄTECKI 2004):

– TYPE A – oscillations of the stream along the entire length of the channel, by the closing wall separation into two parts and symmetrical or alternating return towards side channels (Fig. 7-A),

– TYPE B – after reaching the closing wall, asymmetrical, cyclic return of the flow towards the side channels (Fig. 7-B),

– TYPE C – asymmetric return in the central part of the channel with characteristic, periodically changing reflections of the stream (regular or not) (Fig. 7-C),

– TYPE D – symmetrical return of the stream at the specific distance from the main nozzle outlet (ca. 1 – 1.5 width of the nozzle). In the other part of the channel no clear flow structures (Fig. 7-D),

– TYPE E – symmetrical return of the stream at the wall closing the channel (Fig. 7-E).

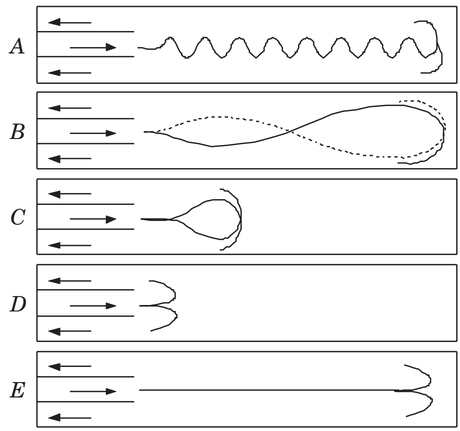


Fig. 7. Bifurcation types present in a flat, dead end channel

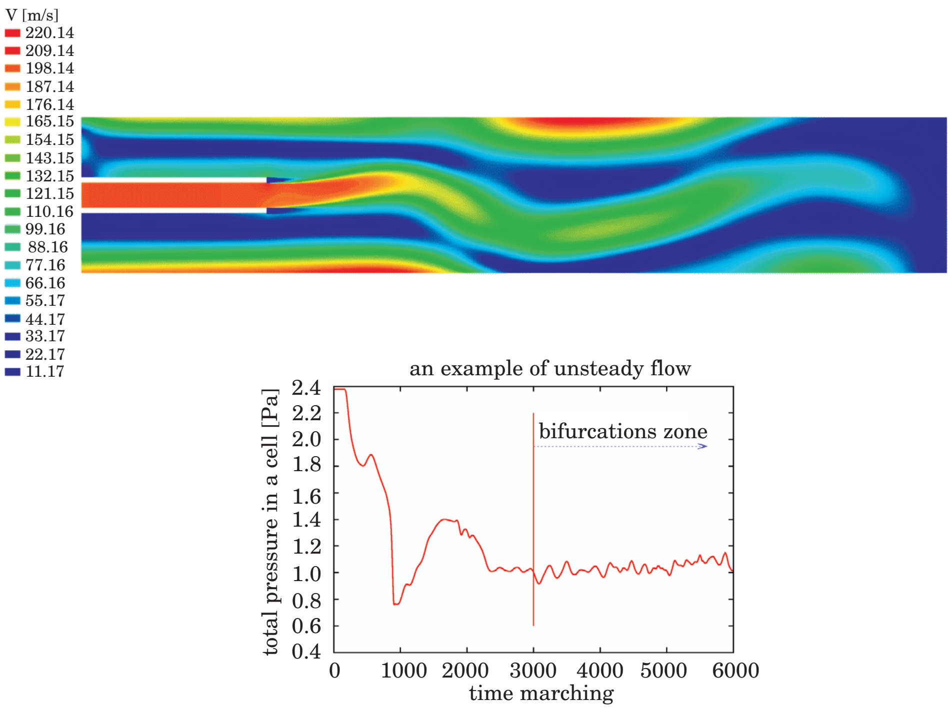


Fig. 8. Total velocity in the channel for b/B = 0.16, H/B = 4.4, p_{in} = 120 000 [Pa] (upper) and record of total pressure in a selected cell (lower)

Source: ŚWIĄTECKI (2004)

The results of simulations obtained in the study (BADUR, SOBIESKI 2001, BADUR, SOBIESKI 2005) were consistent with the description of the laboratory experiment and allowed revealing of a new type of flow (type E) not mentioned by the authors of the paper (Dyban 1971). All numerical simulation series were performed in the Multi Flower 2D version 3.0 software package and some of them were then processed in FLUENT 6.1 software. During the studies the technique of thermodynamic parameters tracing in selected cells of the computation net was applied, which facilitated classification of the type of bifurcation present. Figure 8 presents an example of such analysis with Hopf bifurcations evidently present. The symbols in the description: H – channel length [m], B – channel width [m], b – working nozzle width [m], p_{in} – inlet pressure [Pa].

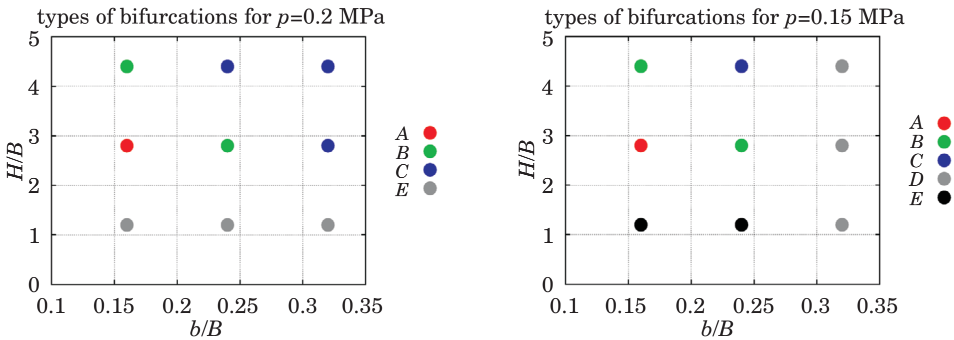


Fig. 9. Areas in which individual types of bifurcation were present at different values of pressure in the working nozzle

The simulations carried out allowed development of generalized characteristics providing graphic presentations of areas of individual types of bifurcation appearance depending on two geometric parameters, H/B and b/B (Fig. 9). The generalized characteristics were developed for a narrow range of pressures in the working nozzle (linked to the parameters of the laboratory experiment), for 0.12, 0.15 and 0.2 MPa. The developed characteristics also allowed determining the behavior of the system with elongation of the channel (Fig. 10).

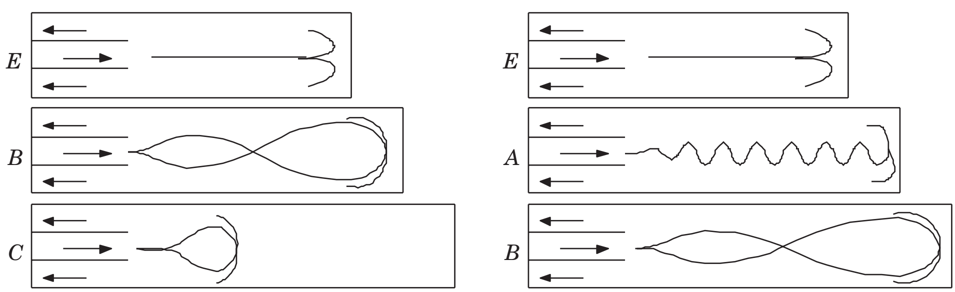


Fig. 10. Transitions of bifurcation types with increasing the parameter H/B for: $p = 0.15$ MPa and $b/B = 0.24$ (left), $p = 0.20$ MPa and $b/B = 0.16$ (right)

Possibilities of bifurcations appearance in jet pumps

The simulation analysis example presented in the preceding point covered the channel, the end of which was closed with a wall that made further flow impossible. In case of liquid-gas jet pumps the role of the wall limiting the flow is assumed by the impact mixing zone characteristic for those devices.

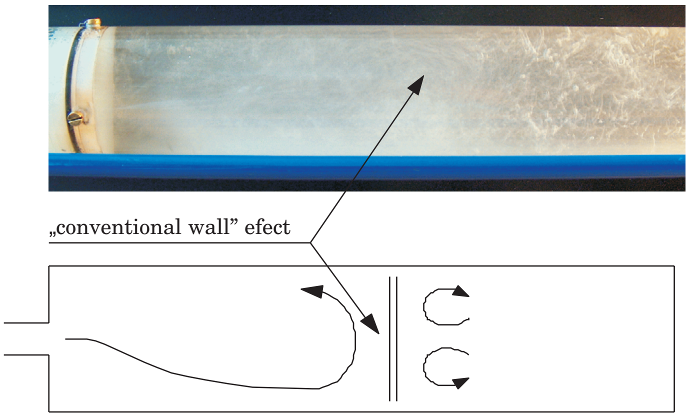


Fig. 11. Example of cavitation flow with stream bifurcation

The effect of the "conventional wall" is also encountered in other types of flows. Figure 11 presents a photograph from own studies on visualization of cavitation phenomenon (SOBIESKI 2004, SOBIESKI 2005), in which zones of different velocities are clearly visible.

Analyzing the results of numerical simulations the following conclusions can be reached:

– in jet pumps possessing no mixing zone or the mixing zone is positioned relatively far from working nozzle outlet, the oscillation bifurcations, including Hopf bifurcations, can occur (in Fig. 7 cases A, B or intermediate),

– in jet pumps possessing mixing zone (in particular when it is situated close to the working nozzle outlet) the effect of "conventional wall" and oscillation or divergent bifurcation can occur (in Fig. 7 cases B, C or intermediate).

The presented analogy leads to the question, if Hopf bifurcations can appear in real jet pumps and, if yes, then how these bifurcations influence operation of the jet pump. An attempt at solving that issue will be presented further in the paper.

Numerical modeling of air-air jet pumps

Aiming at verification of the possibility of bifurcation occurrence in flow devices a series of numerical simulations was completed. The following assumptions were made for the study:

– simulation will be done in 2D space for the example of air-air jet pump,

– jet pump geometry will be selected on the basis of Fig. 9, for the zone in which the Hopf bifurcations occurrence probability is the highest (that case seems to be the most interesting one),

– the influence of viscosity is dismissed from the computations.

The computer model parameters were as follows:

– geometric indicator b/B: 0.16, 0.20, 0.24 (b = 4 mm),

– mixing chamber length H: 400, 600, 800, 1000 mm,

– operational medium: air, water steam, 293 K (ideal gas model),

– medium sucked in: air, 293 K (ideal gas model),

– total pressure p_{in} at working inlet: 150, 200, 250 kPa (Mach Number in all cases under 0.3),

– static pressure of the medium sucked in: 101.3 kPa,

– static pressure p_{out} on the outlet: 105, 110, 115, 120 kPa,

– grid type: structured, 52 600 cells,

– numerical scheme: TVD (Total Variation Diminishing),

– time scheme: implicit,

– CFL number: 10,

– minimum number of iterations: 10 000.

Sample results of computer simulation are presented in Fig. 12. It was found out that in the area of the assumed geometric discriminants and pressures in the working nozzle appearance of bifurcations, including Hopf bifurcation is very typical. In such cases the main stream, after leaving the working nozzle, was subject to periodic deviations from jet pump axis in part or along the entire length of the mixing chamber. Those deviations caused appearance of characteristic, alternating zones (air-chambers) forming at both walls of the mixing chamber. The consequence of development of those "chambers" as well as oblique sections of the main stream was that the medium was sucked into the side channels of the jet pump. It should be highlighted that the suction appeared only in case of appearance of bifurcation. In other cases the backward flow occurred on the external sides of the main stream (the pressure on the outlet was slightly higher than on the passive inlet).

During the simulation tests of the influence of individual parameters of the jet pump on the effect of passive stream suction were conducted. The results of studies (Figures 13, 14 and 15) indicate that the geometric discriminant

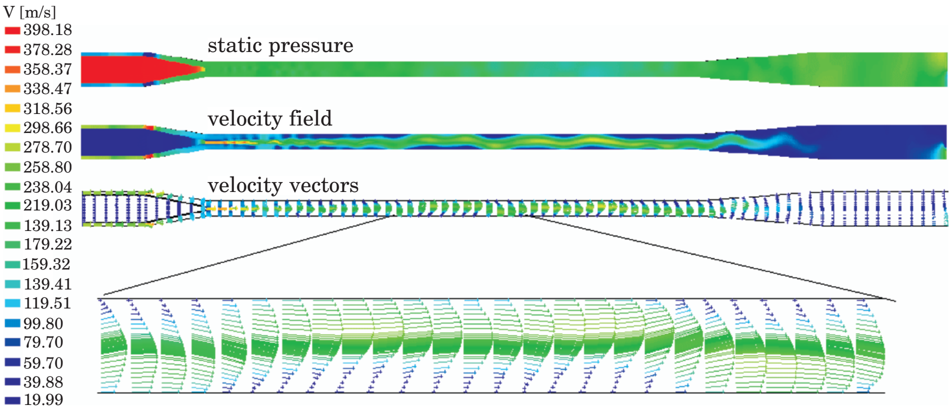


Fig. 12. Example of working stream bifurcation for b/B = 0.16, H = 800 [mm], p_{in} = 150 [kPa], p_{out} = 110 [kPa]
Source: CUDAKIEWICZ (2005).

b/B and inlet pressure p_{in} in the working nozzle had a large influence on jet pump operation. In case of small discriminants and low pressures regular bifurcations (including Hopf bifurcations) are achieved. With the increase of the difference between working nozzle width and channel width b/B and the inlet pressure p_{in}, bifurcations became increasingly irregular and the suction speed decreased almost to zero. The highest suction speed was achieved for the jet pump with the following parameters: p_{in} = 150 [kPa], b/B = 0.16, p_{out} = 110 [kPa].

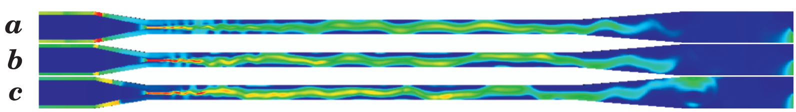


Fig. 13. Distribution of total velocity for: *a*) p_{in} = 150 [kPa], b/B = 0.16, p_{out} = 110 [kPa]; *b*) p_{in} = 200 [kPa], b/B = 0.16, p_{out} = 110 [kPa]; *c*) p_{in} = 250 [kPa], b/B = 0.16, p_{out} = 110 [kPa]
Source: CUDAKIEWICZ (2005).

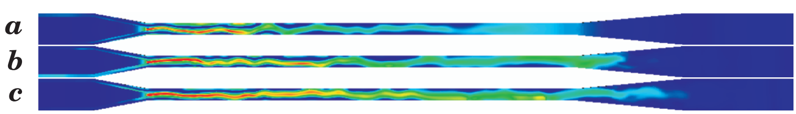


Fig. 14. Distribution of total velocity for: *a*) p_{in} = 150 [kPa], b/B = 0.20, p_{out} = 110 [kPa]; *b*) p_{in} = 200 [kPa], b/B = 0.20, p_{out} = 110 [kPa]; *c*) p_{in} = 250 [kPa], b/B = 0.20, p_{out} = 110 [kPa]
Source: CUDAKIEWICZ (2005).

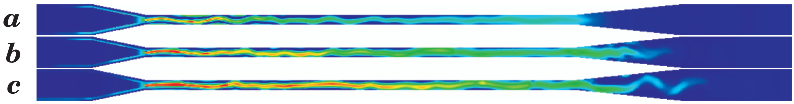


Fig. 15. Distribution of total velocity for: *a)* p_{in} = 150 [kPa], b/B = 0.24, p_{out} = 110 [kPa]; *b)* p_{in} = 200 [kPa], b/B = 0.24, p_{out} = 110 [kPa]; *c)* p_{in} = 250 [kPa], b/B = 0.24, p_{out} = 110 [kPa]
Source: CUDAKIEWICZ (2005).

The influence of mixing chamber length on jet pump operation was another aspect considered during studies. For that purpose analysis was carried out on jet pumps with four lengths of the mixing chamber: H = 400, 600, 800, 1000 [mm], with the feeding nozzle inlet pressure p_{in} = 150 [kPa], jet pump outlet pressure p_{out} = 110 [kPa] and geometric discriminant b/B = 0.16.

The studies showed that in case of mixing chamber short length (H = 400 [mm]) the stream bifurcation occurs still in the diffuser, where it should be extinguished (Fig. 16a). With the change of the mixing chamber length to 600 [mm], bifurcation becomes regular, but it is still not extinguished before the diffusers (Fig. 16b). Further elongation of the mixing chamber to 800 [mm] causes regular bifurcations along the entire length of the mixing chamber and their extinction in the diffuser area (Fig.16c). Another change of chamber length to 1000 [mm] causes that bifurcation is irregular and ends in the end part of the mixing chamber (Fig. 16d). The higher suction velocity was obtained in the jet pump with the mixing chamber of H = 800 [mm].

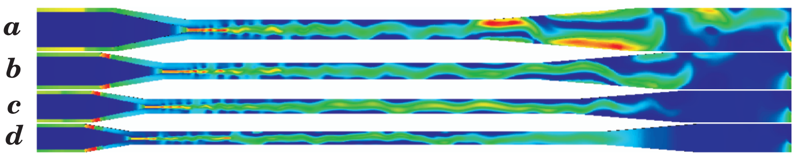


Fig. 16. Distribution of total velocity for: *a)* H = 400 [mm], *b)* H = 600 [mm], *c)* H = 800 [mm], *d)* H = 1000 [mm] (the figures are not in the same scale)
Source: CUDAKIEWICZ (2005).

Next the influence of static pressure on the diffuser outlet on jet pump operation was investigated. For that purpose cases of four pressure values: p_{out} = 105, 110, 115 and 120 [kPa] were analyzed. The total pressure on feed nozzle inlet was constant at 150 [kPa]. For that series of tests the jet pump with geometric discriminator b/B = 0.16 and mixing chamber length of H = 800 [mm] was selected.

The completed series of numerical simulations showed that the most developed bifurcations (occurring along the entire mixing chamber length) appeared at the pressure of around 110 kPa (Fig. 17b). Lower values of the outlet pressure cause a significant shortening of the bifurcations area and deterioration of the effect of suction on passive inlets (Fig. 17a). Increasing the outlet pressure on the other hand caused appearance of increased pressure resistance and damping of bifurcation. In case of higher pressures the clearly visible backward flows in the area of passive nozzle also appeared (Fig. 17c and d).

a
b
c
d

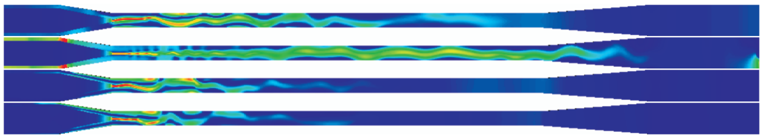
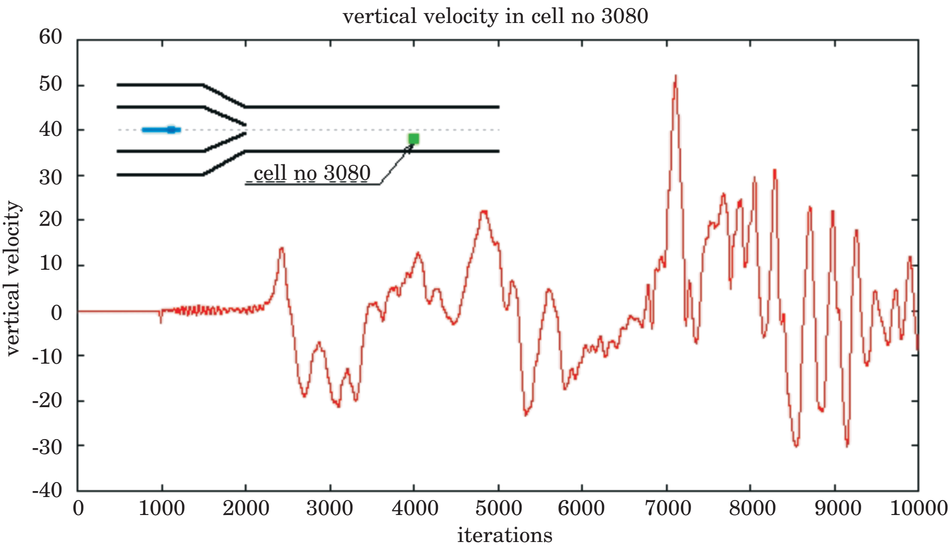


Fig. 17. Distribution of total velocity for: *a)* p_{out} = 105 [kPa], *b)* p_{out} = 110 [kPa], *c)* p_{out} = 115 [kPa], *d)* p_{out} = 120 [kPa]

Source: CUDAKIEWICZ (2005).



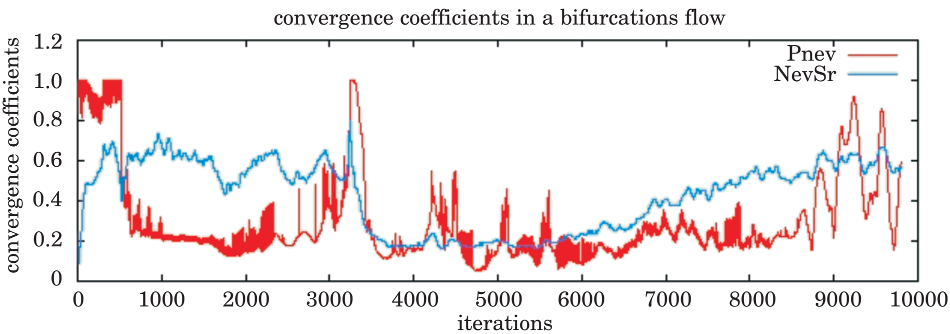


Fig. 18. Record of velocity x component in a selected computation grid cell and the computations convergence process (lower)

The studies completed so far indicate that the highest velocities of the sucked medium could be obtained in a jet pump possessing the following parameters: p_{in} = 150 [kPa], b/B = 0.16, H = 800 [mm] and p_{out} = 110 [kPa]. Changes of static pressure and velocities in directions x and y over time in one of jet pump cells were analyzed to further analyze that case and determine the type of bifurcation. The cell was selected in such a way that it was not positioned on the mixing chamber axis and at the same time it was not too close to chamber wall. The record of jet pump parameters was made as of the beginning of the iteration process. During the first part (up to ca. 8000 iterations) the process of convergence of computations is observed (Fig. 18). The results in that range are not "physically" reasonable and only after stabilization of the processes of numerical computations they become consistent with physics and reasonable for analysis. Figure 18 allows to observe the changes in the position of the main stream. That phenomenon can also be followed to a certain extent by observing the progress of computations convergence process.

Generally the studies carried out showed that the more regular the bifurcation the more efficient the process of medium suction is.

Summary

Results of numerical studies allow assuming that the pulsation model can be applicable to any type of jet pump on condition of appropriate configuration of the geometry and thermodynamic parameters. The following conclusions can be formulated on the basis of the studies completed:

1. numerical modeling allows qualitative determination of the influence of various factors, mainly thermodynamic parameters and geometric discriminants on operation of the studied steam-air jet pump;

2. assuming rightness of bifurcational jet pump operation model, it is possible to determine the range of parameters at which the jet pump operation is the most effective (the velocity in the inlet section of the suction medium was assumed to be the measure of effectiveness: the higher it is the more favorable the jet pump operation conditions are);

3. jet pumps based on bifurcational model should possess the following characteristics:

a. the jet pump should have the appropriate relation of the feeding nozzle width and the mixing chamber width; in the studies the best suction was obtained for b/B = 0.16, which resulted directly from conditions of the bifurcation phenomenon,

b. the inlet pressure should be selected in such a way that bifurcation should occur and that it should have a possibly regular character; within the tested range the pressures of 150 000 [Pa] were the most favorable,

c. the outlet pressure should not be too high because of the resistance of flow; in the studies the best suction occurred at around 110 000 [Pa],

d. the chamber length should be selected in such a way that bifurcation should end at the diffuser inlet, the effect of the "blades" of the working stream is experienced the best at that case and no additional motion resistance (excessively long chamber) or irregularity in mixing of streams and pulsation operation of the jet pump (to short chamber) occur in that case;

4. simulations concerning the influence of outlet pressure on jet pump operation showed the necessity of presence of certain resistance facilitating "breaking" the stability of stream and development of bifurcation phenomenon;

5. presentation of the results of studies on bifurcation in the form of collective results proved to be very useful as it facilitated selection of jet pump geometry and preparation of the computer model;

6. the study does not exhaust the subject of jet pumps modeling; in future work the number of cases investigated should be increased and they should be verified using other numerical codes or, at best, experimentally.

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