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EXAMPLES OF USING THE FINITE VOLUME METHOD FOR MODELING FLUID-SOLID SYSTEMS

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Key words: CFD, Porous Media Model, Eulerian Multiphase Model, fluid-solid systems.

Abstract

The article presents two mathematical models designed for the numerical modeling of fluid flow through stationary and non-stationary porous beds, so called Porous Media Model and Eulerian Multiphase Model. After a brief description, both models are illustrated with several examples of their use. All examples were made in the package ANSYS Fluent. All cases come from studies on the actual objects, usually various laboratory stands. This article aims to show the differences and determinants of both approaches. Motivation stemmed from own experience of the author, indicating that in some cases the choosing of appropriate model is not obvious.

PRZYKŁADY ZASTOSOWANIA METODY OBJĘTOŚCI SKOŃCZONYCH DO MODELOWANIA OŚRODKÓW TYPU FLUID-SOLID

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Słowa kluczowe: CFD, Porous Media Model, Wielofazowy Model Eulera, ośrodki fluid-solid.

Abstrakt

W artykule zaprezentowano dwa modele matematyczne – Porous Media Model oraz Eulerian Multiphase Model, służące do numerycznego modelowania przepływów płynów przez stacjonarne i niestacjonarne złoża porowate. Oba modele, po krótkim ich przedstawieniu, zilustrowano kilkoma przykładami praktycznymi. Wszystkie przedstawione w pracy wyniki symulacji numerycznych bazują na rzeczywistych obiektach, zazwyczaj na konkretnych stanowiskach laboratoryjnych i zostały wykonane w pakiecie obliczeniowym ANSYS Fluent. Celem artykułu było przedstawienie podstawowych różnic dotyczących zastosowań obu modeli oraz uwarunkowań ich stosowania. Motywacją do napisania artykułu były doświadczenia autora, wskazujące, że wybór odpowiedniego modelu może w pewnych przypadkach nie być oczywisty.

Introduction

In industry there exist many systems in which a fluid flows through a bed consisting of particles with similar characteristics (density, shape, surface properties and other). From the standpoint of analysis of such systems it is very important, whether under the influence of forces arising from the motion of fluid, the particles of solid body are on the movement or not. If not, there is a typical case of fluid flow through a porous bed (Fig. 1a). If yes, there is a case with non-stationary porous bed (Fig. 1b). Both types of flows usually require use of different mathematical model. The situation is not so simple, because the boundary between the two systems can be very thin. One type may change into the second type after even relatively minor change in parameters, e.g. in the so called filtration velocity. In next sections are described two models dedicated for stationary and non-stationary porous beds, respectively.

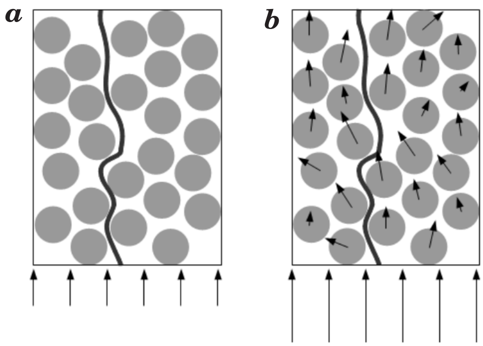

Fig. 1. Stationary (*a*) und non-stationary porous bed (*b*)

Porous Media Model

Porous Media Model (PMM), dedicated for stationary porous beds, is an extension of a basic system of mass (1) and momentum balance (2) equations intended for one phase flow (Fluent User Guide 2006, Fluent Tutorial Guide 2006:

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho\vec{v}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla \cdot (\rho\vec{v}\otimes\vec{v}) = -\nabla p + \nabla \cdot \overset{\leftrightarrow}{\tau} + \vec{S_f} \tag{2}$$

where: ρ – density [kg m^{-3}], $\vec{v}$ – velocity [m s^{-1}], p – static pressure [Pa], τ – total stress tensor [Pa], $\vec{S_f}$ – source of forces [N m^{-3}]. In general case the energy equation can be added to the equations (1) and (2).

In PMM the medium resistance – resulting from the presence of solid fraction – is treated as the additional source in momentum balance equation (2). To calculate a value of this resistance the very popular Forchheimer law (1901) can be used (HELLSTRÖM, LUNDSTRÖM 2006, EWING et al. 1999, ANDRADE et al. 1999) or other similar law. The Forchheimer law, taking into account losses due to viscosity and inertia, can be written in 3D domain as follow

$$S_{mf,i} = -\sum_{j=1}^{3} A_{ij} \cdot \mu \cdot v_j - \sum_{j=1}^{3} B_{ij} \cdot \frac{\rho \cdot |v| \cdot v_j}{2} \tag{3}$$

Symbol $S_{mf,i}$ denotes source of mass forces for the i^{th} space dimension (x, y and z), μ – dynamic viscosity coefficient [kg (m s^{-1})], v_j – *the* j^{th} component of velocity [m s^{-1}], $|v|$ – absolute value of velocity [m s^{-1}]. A_{ij} and B_{ij} are constant diagonal matrices, with diagonal terms equal to the inverse of permeability coefficient κ [m^2] and doubled value of Forchheimer coefficient β [1/m] respectively. Off-diagonal terms in both matrices are null. If the matrix B_{ij} will be equal to zero, the formula (3) will be simplified to the Darcy law (1856) (SAWICKI et al. 2004, NAŁĘCZ 1991) which is very known too. In general case, both A_{ij} and B_{ij} can be functions of coordinates, time, temperature and other factors. This means in practice that the flow resistance can change over time and space.

In PMM only the accurate value of resistance is important. It does not matter which medium generates these resistances and how it happens. In the literature many mathematical formulas describing the permeability coefficient κ and Forchheimer coefficient β can be found (SAWICKI et al. 2004, NAŁĘCZ 1991, MITOSEK 2007, PAZDRO, BOHDAN 1990, BELYADI A. 2006, BELYADI F. 2006, LORD et al. 2006, AMAO 2007). All of them are usually functions of particle diameter, porosity, tortuosity or other geometrical parameters. These models also contain many of constants. The application of these formulas in practice is severely limited because they are often correct only for very specific cases. The best way to obtain a correct value of model parameters (that mean κ and β) for a specific medium is to perform an own experiment. If appropriate values are already known, numerical simulations for this medium can be freely created.

Figure 2 shows an example of using the PMM. It is a water flow through a pipe filled by glass beads. The average diameter of particle was 1.95 [mm]. Experimental data were derived from own measurements performed on an existing laboratory stand, located in the Department of Mechanics and Machine Design at the University of Warmia and Mazury in Olsztyn. The aim of this study was to select of the best methods of determining the permeability coefficient and the Forchheimer coefficient. It turned out that in this case the best is so called Forchheimer Plot Method (SOBIESKI 2009d). The selection was made using a method described in the work of SOBIESKI (2010). The sensitivity of numerical model to changes in the selected parameters was also examined. It is worth adding that the use of the PMM is quite simple if only high quality experimental data are available. In the described case differences between measurements and calculations were always less than 3% (SOBIESKI and TRYKOZKO submitted). Similar accuracy on this laboratory stand was obtained for a few other porous media.

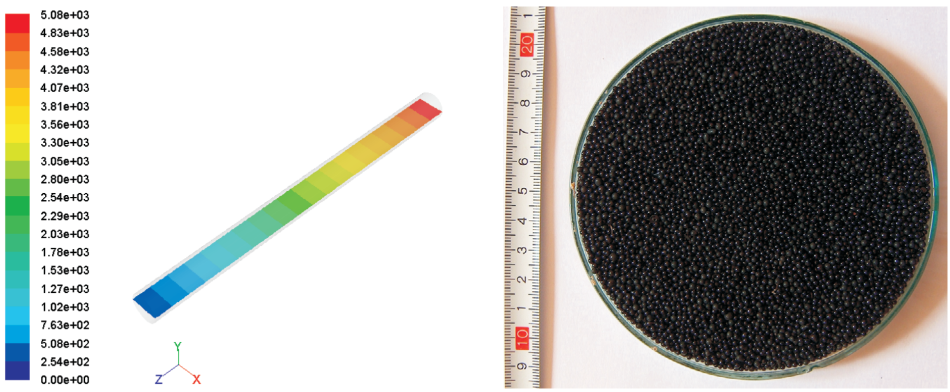

Fig. 2. Static pressure distribution in a porous column
Source: own calculations.

The second example is shown in Figure 3. There is a ground heat exchanger described in the work (NALEPA et al. 2008). This time the working medium was the air. The calculation domain consists of three parts: inlet zone (on the left), porous zone filled with gravel (the large section in the center) and outlet zone (on the right). The flow resistance was included only in the porous zone. The result presented here refers to an existing test stand, but the accurate experimental data are not yet available. After obtaining appropriate experimental data the numerical model can be further developed. The laboratory stand is located in the Department of Electrical and Power Engineering at the University of Warmia and Mazury in Olsztyn.

Another example with air flow through a porous bed is shown in Figure. 4. Presented results refer to the laboratory stand which was designed and constructed specifically for the validation of the numerical model. The device is located in the Departament of Biosystems Engineering at the University

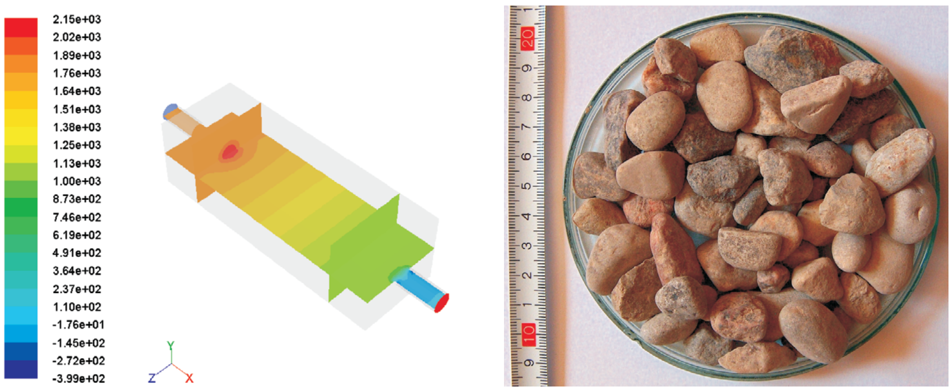

Fig. 3. Static pressure distribution inside a ground heat exchanger
Source: own calculations.

of Manitoba in Winnipeg. In this case soybean seed was the porous medium. The average diameter of grain was equal to 6.2 [mm]. The aim of the project was to study the impact of the spatial structure on the flow resistance in three directions. In this case all needed mathematical models were made in ANSYS Fluent, too, but by using so called User Defined Function and User Defined Memory. The Forchheimer law, Ergun Equation and Carman-Kozeny equation were used. In addition the porosity was defined as a function, not as a constant value. A new method for calculation of the tortuosity, needed for the Kozeny-Carman equation, was developed (SOBIESKI, LIU, ZHANG, submitted). In this example the errors are in most cases less than 1%.

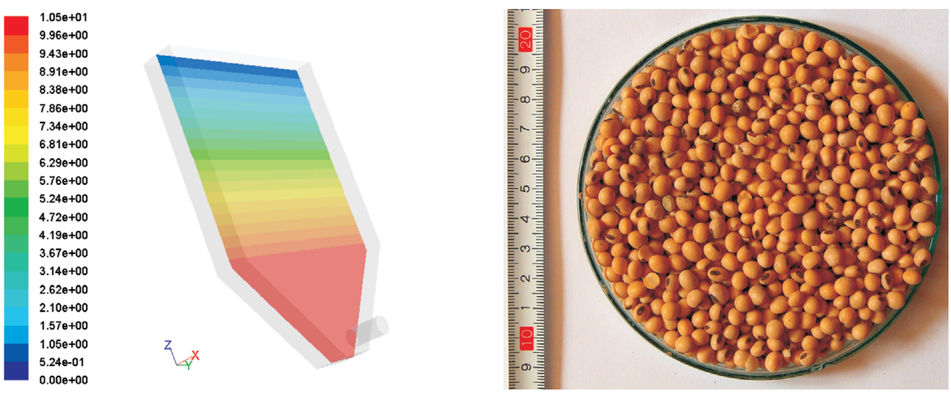

Fig. 4. Static pressure distribution inside a grain tank
Source: own calculations.

Eulerian Multiphase Model

Eulerian Multiphase Model (EMM), dedicated for flows with particle motion, is intended for description of mixtures consisting of any number of phases: gases, liquids and particles of solids. In a general case a separate system of mass, momentum and energy equations is solved for each of the phases. Coupling of phases occurs through pressure and the so-called interphase mass, momentum and energy exchange coefficients. Those coefficients are a characteristic feature of the model and play the key role in it. The description of interactions between individual phases depends mainly on whether just liquid or simultaneously liquid and solid phases are present in the flow. In this model, the Eulerian treatment is used for each phase. In literature other names of this model: Two-Fluid Model or Multi-Fluid model are also found. The equations of mass (4) and momentum (5) for q^{th} phase in the flow are as follow (Fluent User Guide 2006, Fluent Tutorial Guide 2006):

$$\frac{\partial}{\partial t}(\varepsilon_q \rho_q) + \nabla \cdot (\varepsilon_q \rho_q \vec{v}_q) = S_{m,q} \tag{4}$$

$$\frac{\partial}{\partial t}(\varepsilon_q \rho_q \vec{v}_q) + \nabla \cdot (\varepsilon_q \rho_q \vec{v}_q \otimes \vec{v}_q) = -\varepsilon_q \nabla p - \nabla p_q + \nabla \cdot \overleftrightarrow{\tau}_q + \vec{R}_q + \vec{S}_{f,q} \tag{5}$$

where: ε_q – volume fraction of component q[-], ρ_q – density of phase q [kg m^{-3}], $\vec{v}_q$ – velocity of phase q [m s^{-1}], $S_{m,q}$ – source of mass of phase q [kg m^{-3} s^{-1}], p – static pressure of the mixture [Pa], p_q – granular pressure of component q [Pa], $\vec{R}_q$ – source of momentum for phase q exchanged between phases during movement [N m^{-3}], $\vec{S}_{f,q}$ – source of mass forces influencing phase q [N m^{-3}].

Modeling the flow dynamics requires precise definition of mass and momentum exchange between flow components

$$S_{m,q} = \sum_{p=1}^{n}(\dot{m}_{pq} - \dot{m}_{qp}) \tag{6}$$

$$\vec{R}_q = \sum_{p=1}^{n}(\vec{R}_{pq}\ \dot{m}_{pq}\vec{v}_{pq} - \dot{m}_{qp}\vec{v}_{qp}) \tag{7}$$

where: $\dot{m}_{pq}$ – mass transfer from phase p to phase q [kg m^{-3} s^{-1}], $\dot{m}_{qp}$ – mass transfer from phase q to phase p [kg m^{-3} s^{-1}], $\vec{R}_{pq}$ – force of interaction between phases p and q [N m^{-3}], $\vec{v}_{pq}$ – interphase velocity [m s^{-1}]. The value of interphase velocity depends on the direction of mass transfer:

$$\text{if } \dot{m}_{pq} > 0 \text{ then } \vec{v}_{pq} = \vec{v}_p \tag{8}$$

$$\text{if } \dot{m}_{pq} < 0 \text{ then } \vec{v}_{pq} = \vec{v}_q \tag{9}$$

The interphase momentum exchange coefficient β_{pq} [kg (m^{-3} s^{-1})] plays the key role in the model. It describes the forces of interaction between phases in the following way

$$\sum_{p=1}^{n} \vec{R}_{pq} = \sum_{p=1}^{n} \beta_{pq}(\vec{v}_p - \vec{v}_q) \tag{10}$$

where the additional dependences $\beta_{pq} = \beta_{qp}$ and $\vec{R}_{qq}$ must be satisfied. The interphase momentum exchange coefficient β_{pq} may be calculated from many formulas.

Fig. 5. Solid phase distribution in a spouted bed grain dryer
Source: own calculations.

Figure 5 shows an example of modeling a spouted bed grain dryer. The interphase momentum exchange coefficient was described by the Gidaspow model, with a few own extensions. The model was made on the basis of previous experiments performed at the Department of Agricultural Process Engineering, University of Warmia and Mazury in Olsztyn. During the study a very good agreement between experiment and the numerical model was obtained (SOBIESKI 2008). In the first stage the sensitivity of EMM to the change in value of experimental data was investigated. In the next stages dozens of "closures" for momentum exchange (SOBIESKI 2009b), switch function and sphericity coefficient in the Gidaspow model (SOBIESKI 2009a) and the influence of changing the drag coefficient were tested.

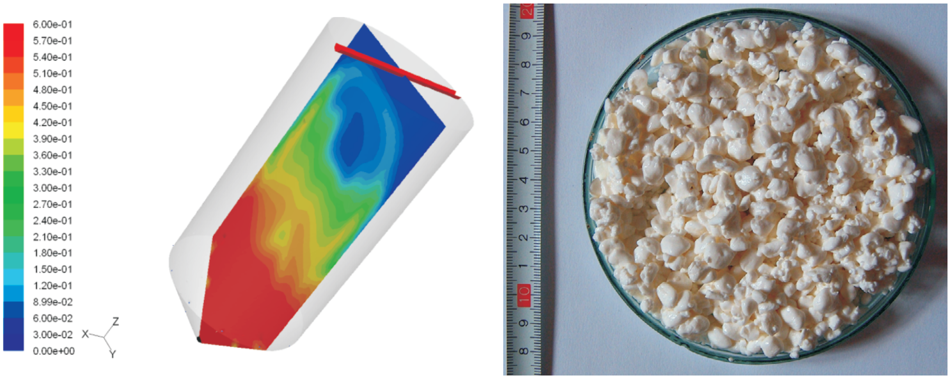

Fig. 6. Solid phase distribution in a washer cooler
Source: own calculations.

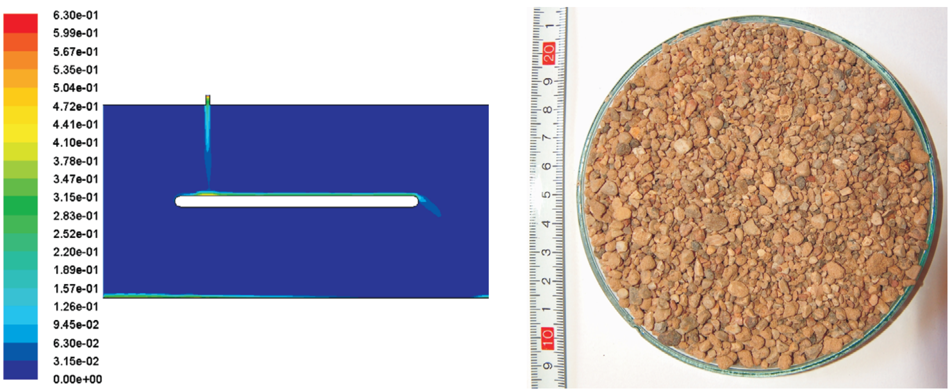

Fig. 7. Solid phase distribution in a conveyor belt
Source: own calculations.

Another example relates to an actual device used in the production process of cottage cheese (Fig. 6) (SOBIESKI 2009c). The first phase was water and the second cottage cheese mentioned above. The average diameter of particle was equal to 9 [mm]. The goal of the research was the qualitative identification of flow structures in the reservoir of the washer cooler. Such data were not previously known. In this case the validation of numerical model was not possible due to costs of an appropriate experiment. Nevertheless, the simulation results remain consistent with the available knowledge about the work of this type of equipment. A characteristic feature of this example is a different medium. Grains of cottage cheese are indeed relatively soft and delicate. The research was conducted jointly with the TEWES-BIS company from Barczewo.

Figure 7 shows third example of using the EMM. In this case the possibilities to model transport of granular media with using a conveyor belt were investigated. In the first stage an appropriate numerical model was developed. For the second stage a simple experimental stand was designed and built (KOWALEWSKI 2009). The first phase was air and the second phase was gravel with fraction 2–8 [mm]. So far, a qualitative compliance has been obtained. In the future, further studies in this direction are planned.

Conclusions

Following conclusions related to Porous Media Model and Eulerian Multiphase Model can be formulated:

– In PMM the shape and location of the porous zone must be defined rigid on the building stage of the numerical model. Subsequent developments in this area require rebuilding the model. In EMM those elements are only the initial conditions and may change in calculation time.

– The local values of flow resistance in EMM are different and depend on current distributions of the parameters: porosity (or volume fraction), filtration velocity and other. In the PMM the resistance value is constant in space (for one direction), unless calculations were made with using distribution functions (i.e. for porosity).

– In both models mass transfer (i.e. dissolution, drying), heat transfer (i.e. heating or cooling) and other phenomena can be added, but mathematical approach for the description is different. In the PMM all “closures” must be defined “globally” for whole porous zone. In the EMM all phenomena must be defined “locally”, on the level of interaction between particles and surroundings.

– If there is a confidence that the modeled system is stationary, it is recommended to use the PMM. The EMM is more complicated than the PMM. The EMM is also more difficult to use.

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References

AMAO A. M. 2007. *Mathematical Model For Darcy Forchheimer Flow With Applications To Well Performance Analysis*. MSc Thesis. Department of Petroleum Engineering. Texas Tech University.

ANDRADE J.S., COSTA U.M.S., ALMEIDA M.P., MAKSE H.A., STANLEY H.E. 1999. *Inertial Effects on Fluid Flow through Disordered Porous Media*. Physical Review Letters, 82(26) 28 June 1999.

BELYADI A. 2006. *Analysis Of Single-Point Test To Determine Skin Factor*. PhD Thesis. Department of Petroleum and Natural Gas Engineering, Morgantown, West Virginia.

BELYADI F. 2006. *Determining Low Permeability Formation Properties from Absolute Open Flow Potential*. PhD Thesis. Department of Petroleum and Natural Gas Engineering, Morgantown, West Virginia.

EWING R., LAZAROV R., LYONS S.L., PAPAVASSILIOU D.V., PASCIAK J., QIN G.X. 1999. *Numerical Well Model For Non-Darcy Flow*. Computational Geosciences, 3(3–4): 185–204.

Fluent 6.3 Tutorial Guide. Fluent Inc., September 2006.

Fluent 6.3 User's Guide. Fluent Inc., September 2006.

HELLSTRÖM J.G.I., LUNDSTRÖM T.S. 2006. *Flow through Porous Media at Moderate Reynolds Number*. International Scientific Colloquium, Modelling for Material Processing, Riga, June 8–9.

KOWALEWSKI K. 2009. *A stand for tests of dynamics of loose material in the model belt conveyor*. MSc Thesis, University Of Warmia And Mazury In Olsztyn, The Faculty of Technical Sciences.

LORD D.L., RUDEEN D.K., SCHATZ J.F., GILKEY A.P., HANSEN C. W. 2006. *DRSPALL: Spallings Model for the Waste Isolation Pilot Plant 2004 Recertification*. SANDIA REPORT SAND2004-0730, Sandia National Laboratories, Albuquerque, New Mexico 87185 and Livermore, California 94550. February 2006.

MITOSEK M. 2007. *Fluid Dynamics in Engineering and Environmental Protection*. Publisher Warsaw University of Technology, Warszaw, Poland.

NALEPA K., NEUGEBAUER M., SOŁOWIEJ P. 2008. *The concept and construction of a ground heat exchanger as part of the ventilation system of a residential building*. Agricultural Engineering. 2(100): 203–208.

NAŁĘCZ T. 1991. *Laboratory of Fluid Mechanics – exercise*. ART Publishing House, Olsztyn.

PAZDRO Z., BOHDAN K. 1990. General Hydrogeology. Geological Publisher, Warsaw, Poland.

SAWICKI J., SZPAKOWSKI W., WEINEROWSKA K., WOŁOSZYN E., ZIMA P. 2004. *Laboratory of Fluid Mechanics and Hydraulics*. Technical University of Gdańsk Publisher, Gdańsk, Poland.

SOBIESKI W. 2008. *Numerical analysis of sensitivity of Eulerian Multiphase Model for a spouted bed grain dryer*. Drying Technology, 26(12): 1438–1456.

SOBIESKI W. 2009a. *Switch Function and Sphericity Coefficient in the Gidaspow Drag Model for Modeling Solid-Fluid Systems*. Drying Technology, 27(2): 267–280.

SOBIESKI W. 2009b. *Momentum Exchange in Solid-Fluid System Modeling with the Eulerian Multiphase Model*. Drying Technology, 27(5): 653–671.

SOBIESKI W. 2009c. *Opportunities and strategies for numerical modeling of flow dynamics and thermal phenomena in the washing-cooling column*. Research report 1/2009 for TEWES-BIS Company, Barczewo, Poland.

SOBIESKI W. 2009d. *Application of simulation methods to validate the results of the experiment*. Chapter in book "Knowledge Engineering and Expert Systems", pp. 113–124. EXIT Publisher, Warsaw 2009.

SOBIESKI W. 2010. *Use of Numerical Models in Validating Experimental Results*. Journal of Applied Computer Science, 18(1): 49–60.

SOBIESKI W., LIU C., ZHANG Q. (submitted). *Calculating tortuosity in a porous bed consisting of spherical particles with known sizes and distribution in space*. Transport in Porous Media.

SOBIESKI W., TRYKOZKO A. (submitted). *Sensitivity analysis of Forchheimer law*. Transport in Porous Media.