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BATHYMETRY CHANGES AND SAND SORTING DURING SEDIMENTATION OF WATERWAYS. PART 1 – CONSERVATION OF SEDIMENT MASS

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K e y w o r d s: sediment transport, conservation of sediment mass, bathymetry changes, sediment velocity, silting up.

A b s t r a c t

In this paper, an equation has been derived from the principle of mass conservation which enables us to produce mathematical description of changes in the seabed bathymetry in time and space. A detailed analysis of this equation has been made in the context of dependence between transport intensity and thickness of densely packed sand grains in sediment being in motion. For the condition when sediment transport is in hydrodynamic equilibrium, i.e. when the flux of sediments falling on the bed is offset by the flux of sediments lifted from the bed, it has been suggested to describe this dependence in the form of a linear function, a proposal which later was verified experimentally. In the mathematical description of changes in the seabed bathymetry, a clear distinction has been made between sediment transport in the positive, onshore direction and transport in the negative, offshore direction, associated, respectively, with sediment transport during the wave crest and wave trough phases.

ZMIANY BATYMETRII I SEGREGACJA OSADÓW W PROCESIE ZAPIASZCZANIA TORÓW WODNYCH. CZĘŚĆ 1 – ZASADA ZACHOWANIA MASY

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S ł o w a k l u c z o w e: transport osadów, zasada zachowania masy, zmiany batymetrii, prędkość rumowiska, zapiaszczanie.

Abstrakt

Na podstawie zasady zachowania masy wyprowadzono równanie służące do matematycznego opisu zmian batymetrii dna w czasie i przestrzeni. Szczegółowo przeanalizowano równanie ze względu na zależność między natężeniem transportu a miąższością gęsto upakowanych ziaren będącego w ruchu rumowiska. Dla warunku, kiedy transport osadu znajduje się w równowadze hydrodynamicznej, tj. kiedy strumień osadów opadających na dno jest równoważony strumieniem osadów podrywanych z dna, zaproponowano opis tej zależności w postaci funkcji liniowej, którą następnie potwierdzono wynikami z eksperymentu laboratoryjnego. Przedyskutowano także efekty związane z nieliniową postacią tej zależności. W opisie matematycznym zmian batymetrii dna wyraźnie podzielono transport osadów na transport w kierunku dodatnim – dobrzegowym, i ujemnym – odbrzegowym, związany z transportem rumowiska odpowiednio w fazie grzbietu i doliny fali.

Introduction

It is extremely important to predict accurately bathymetric changes in the seabed near an approach canal to a harbour so as to be able to maintain its navigable depth. Changes in the bathymetry of the seabed in time and space are usually described using equations derived from the principle of mass conservation.

The review written by NICHOLSON et al. (1997) states that morphodynamic models use classical shock capturing schemes for bed level simulations. JOHNSON and ZYSERMAN (2002) applied a modified second-order accurate Lax-Wendroff scheme. This scheme, however, is burdened with a numerical dispersion error, which is reflected by additional oscillations in the results of numerical calculations. As JOHNSON and ZYSERMAN showed (2002), spatial oscillations generated by numerical schemes are caused by the dependence of the celerity of the bed level oscillations on the bed level, which are a result of the non-linear relationship between the sediment transport rate and the bed level.

Some of the numerical schemes applied to simulation of changes in the bed level have been discussed by LONG et al. (2008), who analyzed the accuracy and stability of these schemes. Their discussion seems to suggests that the best scheme for simulation of sediment transport is a fifths-order Euler-Weno scheme, which relies on the upwinding concept, also implemented in this paper. The Euler-Weno scheme is shown to have significant advantages over schemes with artificial viscosity and filtering process. It is highly recommended especially for phase-revolving sediment transport models, when the sediment transport rate is postulated to be split into parts associated with the bedform propagation in the positive and negative x-directions.

This paper suggests that there is a linear relationship between the sediment transport rate and the thickness of the layer of densely packed, moving sand grains, which consequently enables us to apply the first-order upwind scheme for solving an equation which describes changes in the seabed bathymetry in time and space. Moreover, additional effects connected with the non-linearity of this dependence are discussed. Noteworthy is the suggestion that there is a linear relationship between sediment transport and the bed level. The latter relationship continues to be viewed as a non-linear one.

Part 2 of the article describes a three-layer model of transport of sediments with sand grains of various size, derived by KACZMAREK (1999) from the principle of the conservation of water and sediment flow in the nearbed layer. It has been demonstrated that this model is applicable (alongside the equation derived from the mass conservation principle, described in this part) to mathematical description of changes in bathymetry and changes in the distribution of grain size composition of sediments which constitute the seabed near an approach waterway to a harbour under given wave and current conditions.

Conservation of sediment mass

Using Euler coordinates, a continuity equation derived from the mass conservation principle is determined for control volume V limited by closed control space A. The amount of mass contained in the thus determined, immobile space can change in time due to a sediment flow through the control space (cf., for example, MITOSEK, 2001, PUZYREWSKI, SAWICKI, 2000). A change in the mass enclosed in the V space, dependent on the change in the density ρ_r of sediment (treated as fluid) in time dt, equals:

$$\iiint_V \frac{\partial \rho_r}{\partial t}\, dt dV \tag{1}$$

where

$$\rho_r = \rho_s C \tag{2}$$

$$\rho_s = \frac{m_s}{V_s} \tag{3}$$

$$C = \frac{V_s}{V_p + V_s} \tag{4}$$

$$\rho_r = \rho_s C = \frac{m_s}{V_s}\frac{V_s}{V_p + V_s} = \frac{m_s}{V_p + V_s} \tag{5}$$

In dependences (1)÷(5), the following denotations were introduced:
ρ_s – ground skeletal density [kg/m^3],
C – volume concentration [m^3/m^3],
m_s – ground skeletal mass [kg],
V_s – ground skeletal volume [m^3],
V_p – volume of pores [m^3].

The mass of sediment which flowed though surface A in a given time, equal to the difference in the mass flowing into the control volume and flowing out of that volume, is:

$$-\iint \rho_r \boldsymbol{u}\mathbf{d}\boldsymbol{A}\mathrm{d}t = -\iint \rho_r u_n \mathrm{d}A\mathrm{d}t \tag{6}$$

where
d*A* is a vector of the value equal to the dA field, normal to this surface and oriented outside the area.

Should the sign "minus" be omitted, the integral would signify the loss of sediment from control volume V. The scalar product of the velocity vector and surface vector is equal to the product of the component velocity u_n normal to this surface and the field of an area dA.

Having compared the expressions (1) and (6) and after simplification by dt, the general, integral form of the continuity equation is obtained:

$$\iiint_V \frac{\partial \rho_r}{\partial t}\, dV + \iint_A \rho_r u_n \mathrm{d}A = 0 \tag{7}$$

From dependence (2) and assuming that ρ_s=const., equation (7) can be transformed to:

$$\iiint_V \frac{\partial C}{\partial t}\, dV + \iint_A C u_n \mathrm{d}A = 0 \tag{8}$$

When including Gauss-Ostrogradsky theorem, the following form of the continuity equation is obtained:

$$\iiint_V \frac{\partial C}{\partial t}\, dV + \iiint_A div(C\boldsymbol{u}) dV = 0 \tag{9}$$

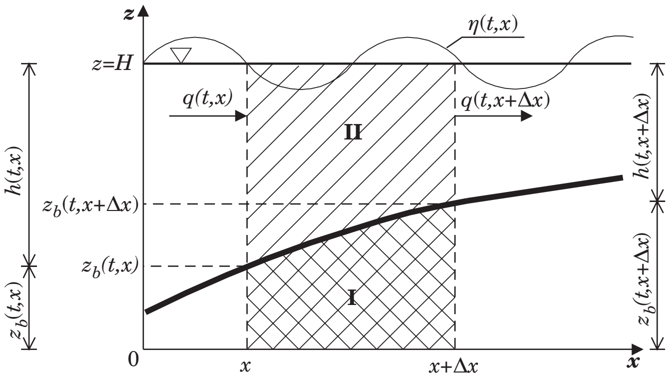

Fig. 1. Diagram of the analyzed area

Next, assuming the control volume as shown in Figure 1, i.e. analyzing the plane bed concept, and assuming the constant level of filling up H=const., and bearing in mind that $z_b + h = H$, one can obtain:

$$\int_0^{\Delta x} dx \left[\int_0^H \frac{\partial C}{\partial t}\, dz + \int_0^H \frac{\partial}{\partial x}\,(Cu)dz \right] = 0 \tag{10}$$

which can be rewritten as:

$$\int_0^H \frac{\partial C}{\partial t}\, dz + \int_0^H \frac{\partial}{\partial x}\,(Cu)dz = 0 \tag{11}$$

In the motion of water caused by surface waves $\eta(t,x)$, instantaneous values of the velocity and concentration, in Euler's set, can be written in the following form:

$$u = \langle u \rangle + \tilde{u} \tag{12}$$

$$C = \langle C \rangle + \tilde{C} \tag{13}$$

where

$\langle ... \rangle = \dfrac{1}{T}\displaystyle\int_0^T (...)\, \mathrm{d}t$ is the averaged value in a wave period (T) whereas $\tilde{u}$ and $\tilde{C}$ stand for the oscillation component of, respectively, velocity and concentration.

By substituting dependences (12) and (13) to equation (11), and then averaging this equation over a wave period, the following is obtained:

$$\int_0^{\eta} \frac{\partial \langle C \rangle}{\partial t}\, dz + \int_0^{\eta} \frac{\partial}{\partial x} \left(\langle u \rangle \langle C \rangle + \langle \tilde{u}\tilde{C} \rangle \right) dz = 0 \tag{14}$$

Taking into consideration that:

$$\frac{\partial}{\partial x} \langle \tilde{u}\tilde{C} \rangle = U_s \frac{\partial \langle C \rangle}{\partial x} \tag{15}$$

where
U_s is known under the name of Stokes drift (cf. FREDSØE and DEIGAARD 1992) and

$$U_L = \langle u \rangle + U_s \tag{16}$$

where
U_L is the Lagrangian-type averaged velocity,
equation (14) can be rewritten to:

$$\int_0^{\eta} \frac{\partial \langle C \rangle}{\partial t}\, dz + \int_0^{\eta} \frac{\partial}{\partial x} \left(U_L \langle C \rangle + \langle \tilde{u}\tilde{C} \rangle \right) dz = 0 \tag{17}$$

on the assumption that when there are no vertical currents, by virtue of the mass conservation equation for water $\frac{\partial}{\partial x} U_L = 0$.

Assuming that the vast majority of sediment is transported near the bed as bedload and sheet flow, respectively, and assuming that for $z=\eta$ concentration $\langle C \rangle$ is negligibly small, equation (17), according to Leinbniz integration law, can be written as:

$$\frac{\partial}{\partial t} \int_0^{\eta} \langle C \rangle\, dz + \frac{\partial}{\partial x} \int_0^{\eta} \left(U_L \langle C \rangle \right) dz = 0 \tag{18}$$

Solving equation (18) for the control volume as in Figure 1, we obtain:

$$\frac{\partial}{\partial t}\int_0^{z_r}\langle C^I\rangle\, dz + \frac{\partial}{\partial t}\int_{z_r}^{\eta}\langle C^{II}\rangle\, dz + \frac{\partial}{\partial x}\int_0^{z_r}(U_L^I\langle C^I\rangle)\, dz + \frac{\partial}{\partial x}\int_{z_r}^{\eta}(U_L^{II}\langle C^{II}\rangle)\, dz = 0 \tag{19}$$

where:

z_r – reference level and in Fig. 1 it was assumed to be at the bed level $z_b = z_r$, although obviously another choice is possible, too,

U_L^I, C^I – sediment velocity and concentration in the sub-layer I (Fig. 1), respectively,

U_L^{II}, C^{II} – sediment velocity and concentration in the sub-layer II (Fig. 1), respectively.

From equation (19) it can be concluded that the total sediment transport concentration per volume can be described with the following relationship:

$$\int_0^{z_r} C^I U_L^I dz + \int_{z_r}^{\eta} C^{II} U_L^{II}\, dz = q(t, x) \tag{20}$$

which means that equation (19) can be rewritten to the form:

$$\frac{\partial}{\partial t}\int_0^{z_r} C^I\, dz + \frac{\partial}{\partial x}\int_{z_r}^{\eta} C^{II} dz + \frac{\partial q}{\partial x} = 0 \tag{21}$$

Should we assume that the sediment concentration $C^I = (1 - n_p)$ in the layer $0 \le z \le z_r$ is constant, then for $z_r = z_b$ from equation (21) we obtain:

$$(1 - n_p)\frac{\partial z_b}{\partial t} + \frac{\partial}{\partial t}\int_{z_b}^{\eta} C^{II}\, dz + \frac{\partial q}{\partial x} = 0 \tag{22}$$

where

$$n_p = \frac{V_p}{V_p + V_s} \tag{23}$$

denotes the porosity of sediment.

Equation (22) can be named the equation of sediment transport. It can be a useful mathematical tool for describing bathymetric changes of the seabed in time and space. As shown above, the transport equation was derived from the mass conservation principle. The expression $(1 - n_p)\, z_b + \int_{z_b}^{\eta} C^{II} dz$ corresponds quantitatively to the total volume of sediment per level surface unit of the sediment lying on the bed (first component of the expression) or suspended (second component). If the volume of the suspended sediment can be omitted, then equation (22) can be written as:

$$\frac{\partial z_b}{\partial t} + \frac{1}{(1 - n_p)} \frac{\partial q}{\partial x} = 0 \tag{24}$$

Sediment transport in equilibrium with load hydrodynamics

As mentioned before, in our discussion on the sediment transport equation, the bed level is usually taken as a reference level, which means that $z_r = z_b$. Of course, it is possible to take another level for reference. In this paper, for example, it is proposed to assume that $z_r = z_m$, where z_m stands for the thickness of densely packed sand grains in sediment which is moving.

Because it is assumed that intensive transport of sediment takes place during a storm, it can be expected that under such hydrodynamic conditions the bed remains flat (devoid of any bed forms) and locally horizontal. In this case, the value z_b can be replaced by z_m, which denotes the thickness of the layer of densely packed sand grains of moving sediment. The propagation velocity of this layer should not depend on the value of z_m because concentration $(1 - n_p)$ in the layer of densely packed sand grain in the moving sediment is constant. As demonstrated by KACZMAREK and OSTROWSKI (2002), in the layer of sediment where the velocity of the transported sediment is a function of the depth, the concentration also depends on the depth coordinate z.

Let z_m^+ and z_m^- stand, respectively, for the thickness of the cells $z_m^+ \times dx^+ \times 1$ and $z_m^- \times dx^- \times 1$, which are eroded from the profile transverse to the shore (direction x) in time dt due to the transport of sediment q_x^+ and q_x^- directed, respectively, on- and offshore:

$$z_m^+ = \frac{1}{(1 - n_p)} \frac{q_x^+ \cdot dt}{dx^+} \tag{25}$$

$$z_m^- = \frac{1}{(1 - n_p)} \frac{q_x^- \cdot dt}{dx^-} \tag{26}$$

where

$$z_m = z_m{}^{+} + z_m{}^{-} \tag{27}$$

and

$$z_b(\mathrm{x,t} + dt) = z_b(\mathrm{x,t}) + \frac{\partial z_m}{\partial t}\, dt \tag{28}$$

The equilibrium conditions appear in the whole area of cells except near their edges. It can be assumed that the total amount of the sediment in motion, dragged or suspended (mainly in the contact layer near the bed) is picked up directly from the bed and equals the amount of the sediment moving in the bed as densely packed grains. It also means that sediment transport promptly adjust itself to the flow conditions and, therefore, the bed "responds" immediately to the given hydrodynamic conditions.

The following assumptions are derived:

$$1.\ \text{for } 0 \le z \le z_m \begin{cases} (1 - n_p) = \langle C^I \rangle \\ U_L^I = U_{L_1} \end{cases} \tag{29}$$

2. for $z_m < z \le \eta$ we can write:

$$\int_{z_m}^{\eta} U_L \langle C \rangle dz = \overline{U}_L \int_{z_m}^{\eta} \langle C \rangle dz = U_{L_1} \int_{z_m}^{\eta} \kappa \langle C \rangle dz = U_{L_1} \int_{z_m}^{\eta} \langle C_1 \rangle dz = \int_{z_m}^{\eta} U_L^{II} \langle C^{II} \rangle dz \tag{30}$$

where

$$\frac{\overline{U}_L}{U_{L_1}} = \kappa \text{ and } \langle C_1 \rangle = \kappa \langle C \rangle \tag{31}$$

From the relationship (30), it can be concluded that:

$$U_L^{II} = U_{L_1} \text{ and } \langle C^{II} \rangle = \langle C_1 \rangle \tag{32}$$

At this point, it is worth noticing that progressive speed U_{L_1} is identifiable with the speed of propagation of the mass centre of a bed form z_m thick, which is moving but not changing its shape under the effect of the surface transport of sediment at speed $\overline{U}_L$. This progressive motion of a bed form of the thickness z_m can be compared to the movement of a sand dune, which is set in motion as a result of the surface motion of sand grains caused by a wind. The nature of this motion implies mixing of the grains involved in the motion and the ones which are at rest in the bed layer of the thickness of $2z_m$ (KACZMAREK et al. 2004). It is so because as a flow of sediments fills up half the control volume of the length $dx/2$ during time dt, during the same time, a flow of sediment empties the other half of the control volume (likewise, of the length $dx/2$). If the bed level is to remain unchanged, for instance, than the flow entering the control volume and the flow leaving it should be equal. Thus, by virtue of equations (25) and (26), these flows, depending on the direction, are equal, respectively, $q_x^{+/-}\, dt = (1 - n_p)\, 2z_m^{+/-}\, dx^{+/-}/2$.

As the sediment transport is in equilibrium with load hydrodynamics, it is therefore postulated that:

$$(1 - n_p)\, z_m = \int_{z_m}^{\eta} \langle C_1 \rangle\, dz \tag{33}$$

Considering relationships (29) and (32), equation (19) can be presented as follows:

$$(1 - n_p)\, \frac{\partial z_m}{\partial t} + \frac{\partial}{\partial t} \int_{z_m}^{\eta} \langle C_1 \rangle\, dz + (1 - n_p)\, U_{L_1} \frac{\partial z_m}{\partial x} + U_{L_1} \frac{\partial}{\partial x} \int_{z_m}^{\eta} \langle C_1 \rangle\, dz = 0 \tag{34}$$

Next, by including relationship (33), we obtain:

$$2(1 - n_p)\, \frac{\partial z_m}{\partial t} + 2(1 - n_p)\, U_{L_1} \frac{\partial z_m}{\partial x} = 0 \tag{35}$$

which consequently leads to the following equations:

$$\frac{\partial z_m}{\partial t} + U_{L_1} \frac{\partial z_m}{\partial x} = 0 \tag{36}$$

and

$$\frac{\partial}{\partial t} \int_{z_m}^{\eta} \langle C_1 \rangle\, dz + U_{L_1} \frac{\partial}{\partial x} \int_{z_m}^{\eta} \langle \mathrm{C}_1 \rangle\, dz = 0 \tag{37}$$

By including the decomposition of the velocity U_{L_1} into $U_{L_1}^{+}$ and $U_{L_1}^{-}$ and the thickness z_m into z_m^{+} and z_m^{-}, the following can be obtained:

$$q_x^{+} = (1 - n_p)\, U_{L_1}^{+}\, z_m^{+} = U_{L_1}^{+} \int_{z_m^{+}}^{\eta} \langle C_1 \rangle^{+}\, dz \tag{38}$$

$$q_x^{-} = (1 - n_p)\, U_{L_1}^{-}\, z_m^{-} = U_{L_1}^{+} \int_{z_m^{-}}^{\eta} \langle C_1 \rangle^{+}\, dz \tag{39}$$

where

$\langle ... \rangle^{+} = \dfrac{1}{T} \displaystyle\int_{0}^{T_c} (...)\, \mathrm{d}t,$

$\langle ... \rangle^{-} = \dfrac{1}{T} \displaystyle\int_{T_c}^{T} (...)\, \mathrm{d}t,$

T – wave period,
T_c – wave crest duration period.

By substituting relations (38) and (39) into equation (36), we obtain the following equations:

$$\frac{\partial z_m^{+}}{\partial t} + U_{L_1}^{+} \frac{\partial z_m^{+}}{\partial x^{+}} = \frac{\partial z_m^{+}}{\partial t} + \frac{1}{(1 - n_p)} \frac{\partial q_x^{+}}{\partial x^{+}} = 0 \tag{40}$$

$$\frac{\partial z_m^{-}}{\partial t} + U_{L_1}^{-} \frac{\partial z_m^{-}}{\partial x^{-}} = \frac{\partial z_m^{-}}{\partial t} + \frac{1}{(1 - n_p)} \frac{\partial q_x^{-}}{\partial x^{-}} = 0 \tag{41}$$

Next, taking advantage of relations (27) and (28), we obtain the final equation in the form:

$$\frac{\partial z_b}{\partial t} + \frac{1}{(1 - n_p)} \left(\frac{\partial q_x^{+}}{\partial x^{+}} + \frac{\partial q_x^{-}}{\partial x^{-}} \right) = 0 \tag{42}$$

If the wave is propagating over an inclined bed, we should include the fact that:

$$\frac{\partial q_x^+}{\partial x^+} \neq \frac{\partial q_x^-}{\partial x^-} \tag{43}$$

Relation (43) indicates that in order to achieve the right solution of the sediment transport equation (43) in the case of linear relations (25) and (26), it is necessary to divide the volume of transport into two part: q_x^+ and q_x^-. In fact, the values q_x^+ and q_x^- should be interpreted as the volume of transported sediment in hydrodynamic equilibrium, averaged for a wave period T, in the wave crest and trough, respectively, and q_x should be taken as a resultant sediment transport averaged for a wave period T (net transport rate).

In a 2-D case, equation (42) can be presented in a generalized form:

$$\frac{\partial z_b}{\partial t} + \frac{1}{(1-n_p)}\left(\frac{\partial q_x^+}{\partial x^+} + \frac{\partial q_x^-}{\partial x^-} + \frac{\partial q_y}{\partial y}\right) = 0 \tag{44}$$

where,

q_y stands for the volume of longshore transport.

The volume of the longshore transport is divided into two parts because this transport is not connected with unidirectional flow, which would depend on the angle between the wave propagation direction and the direction of the profile transverse to the shore. It is assumed that positive values are taken to express the volume of transport if the sediment is transported according to the positive direction of y axis.

It can be noticed that the equilibrium between the sediment transport in the layer of densely packed sand grains in the moving sediment and the transport in the layer above means that bed disturbances (z_m^+ and z_m^- respectively) propagate without changing the shape. Obviously, this does not mean that – according to relations (27) and (28) – the bed level z_b disturbances move likewise without changing the shape. In other words, because the sediment transport rate is a non-linear function z_b, this equation of mass conservation is also physically a non-linear equation of conservation with respect to the bed level.

Comparison with experimental data

Our considerations about sediment transport under equilibrium conditions, i.e. about the linearity of equations (25) and (26), seem to be confirmed by the results of an experiment conducted at the Institute of Hydroengineering of the Polish Academy of Sciences in Gdańsk in 1996 (cf. KACZMAREK 1999). In a wave flume continuously filled up, regular (tests 1, 2, 3, 4, 11 and 12) and irregular (tests 5, 6, 7, 8, 9, 10) waves were generated. The bed consisted of natural sand of the representative diameter d_{50}=0.22 mm. One of the objectives of the experiment was to determine the amount of sediment transported mainly in the bedload layer onshore and offshore, by measuring the amounts of sediment flowing into a sand trap, which was composed of two cells. Because the generated wave conditions corresponded to the ripple regime, the measurements were not taken until the time necessary for the establishment of the equilibrium conditions had elapsed. The experiment was run several times for each set of hydrodynamic parameters. In total, 141 such measurements were made for 12 tests.

Our comparison of the results of the measurements of sediment transport rates ($q_x^+ + |q_x^-|$) with the results of the measurements of the thickness ($z_m^+ + z_m^-$) of densely packed grains in moving sediment are shown in Figure 2, where the average values of the experimental measurements are presented. The mathematic tool used while modelling the sediment transport rates was a three-layer model of sediment transport, which has been developed for several years now at the Polish Academy of Sciences Institute of Hydroengineering in Gdańsk (cf. KACZMAREK 1999). A detailed description of this model is contained in Part 2 of this article. In our calculations, it was assumed that the grain-size distribution of the sediment corresponded to the measured grain-size distribution curve.

The dependence of the calculated values of ($q_x^+ + |q_x^-|$) on the averaged measured values of the thicknesses ($z_m^+ + z_m^-$) shows a distinctly linear

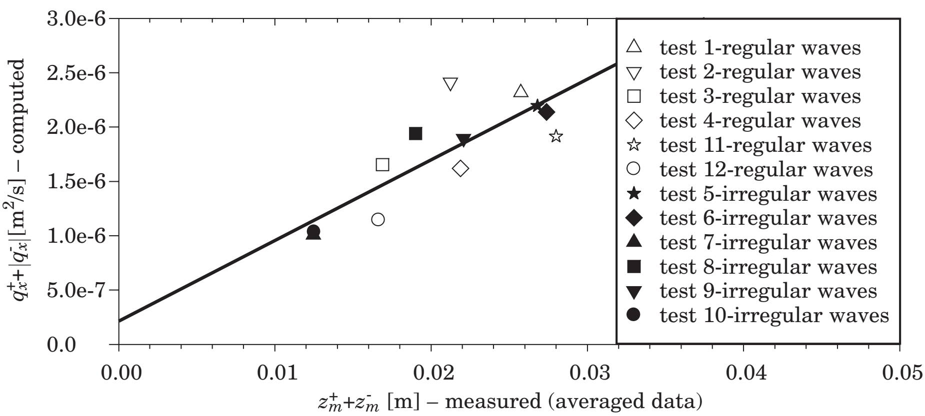

Fig. 2. Comparison of the results of modelling sediment transport rates with averaged measurements of thickness obtained from the experiment carried out at the Polish Academy of Sciences Institute of Hydroengineering in 1996

character. Thus, by virtue of equations (38) and (39), it should be noted that the expressions in the form of $(1 - n_p)\ U^-_{L_1}$ and $(1 - n_p)\ |U^-_{L_1}|$, respectively, represent the coefficient of proportionality of a straight line. In turn, the straight line crossing with the axis of ordinates (y-axis) (Fig. 2) seems to indicate the fact that the linear relation holds true only for such intensity of sediment transport which enables us to determine, for a given time dt, at a length of a sand trap $dx=L_{ST}$, the minimum thickness $(z^+_{m0} + z^-_{m0})$. In other words, for the rate of transport lower than the transport resulting from the crossing of the straight line with the axis of ordinates, the sand trap will not be filled up.

By virtue of equations (25) and (26), it can also be written:

$$B_{ST}\ (q^+_x + |q^-_x|)\ dt = (1 - n_p)(z^+_m + z^-_m)\ L_{ST}B_{ST} \tag{45}$$

where
B_{ST} is the width of a sand trap.

Using definition (31) and dependences (38) and (39), equation (45), once shortened by B_{ST}, can be transformed as follows:

$$\begin{aligned}(q^+_x + |q^-_x|)\ dt &= (1 - n_p)\ \kappa\Big(\int_{z^+_m}^{\eta}\langle C^+\rangle dz + \int_{z^-_m}^{\eta}\langle C^-\rangle dz\Big)L_{ST} \\ &= (1 - n_p)\ (\kappa - 1)(z^+_{m0} + z^-_{m0})L_{ST} + (1 - n_p)(z^+_{m0} + z^-_{m0})\ L_{ST}\end{aligned} \tag{46}$$

and then:

$$(q^+_x + |q^-_x|) = (1 - n_p)\ (\kappa - 1)(z^+_{m0} + z^-_{m0})\ \frac{L_{ST}}{dt}\ (q^+_{x0} + |q^-_{x0}|) \tag{47}$$

For comparison of the results of the calculations with the measurements, it is convenient to present equation (47) in the form:

$$(q^+_x + |q^-_x|)_{\mathrm{comp}} - (q^+_{x0} + |q^-_{x0}|) = (1 - n_p)\ (z^+_m + z^-_m)_{\mathrm{meas.}}\ \frac{L_{ST}}{dt} \tag{48}$$

Non-linear effects

If the rate of sediment transport is to be described with non-linear relations with respect to z_m, the sediment transport is not in equilibrium with load hydrodynamics. This corresponds to a situation where a total amount of the sediment transported in suspension consists of part $\langle C_1\rangle$, which is in equilibrium with load hydrodynamics and part $\langle C_2 \rangle$, which is transported in the form of inert mass. With respect to the bed accumulation, the exchange between the sediment transported in suspension $\langle C_2 \rangle$ and in the bed leads to the deposition process, whereas when the bed is eroded, the sediment is picked up by a current. Thus, we can write that:

$$\begin{cases} \langle C^I \rangle = (1 - n_p) \text{ and } \langle C^{II} \rangle = \langle C_1 \rangle + \langle C_2 \rangle \\ U_L^I = U_{L_1} \quad \text{ and } U_L^{II} = U_{L_1} + \mathrm{U}_{\mathrm{L}_2} \end{cases} \tag{49}$$

Considering dependences (49) and (37), equation (19) can also be written:

$$(1 - n_p) \frac{\partial z_m}{\partial t} + \frac{\partial}{\partial t} \int_{z_m}^{\eta} \langle C_2 \rangle \, dz + (1 - n_p) \, U_{L_1} \frac{\partial z_m}{\partial x} + U_{L_1} \frac{\partial}{\partial x} \int_{z_m}^{\eta} \langle C_2 \rangle \, dz + \\ + \frac{\partial}{\partial x} \int_{z_m}^{\eta} U_{L_2} \langle C_1 \rangle \, dz + \frac{\partial}{\partial x} \int_{z_m}^{\eta} U_{L_2} \langle C_2 \rangle \, dz = 0 \tag{50}$$

where

$$q = (1 - n_p) \, U_{L_1} z_m + U_{L_1} \int_{z_m}^{\eta} \langle C_2 \rangle \, dz + \int_{z_m}^{\eta} U_{L_2} \langle C_1 \rangle \, dz + \int_{z_m}^{\eta} U_{L_2} \langle C_2 \rangle \, dz \tag{51}$$

Let ∇_D denote the rate of sediment deposition, and ∇_E stand for the rate of sediment entrainment into the flow, then:

$$\frac{\partial}{\partial t} \int_{z_m}^{\eta} \langle C_2 \rangle \, dz = \nabla_E - \nabla_D \tag{52}$$

However, if in our considerations the suspended load contribution $\langle C_2 \rangle$ to the total sediment volume stored in the bed can be neglected (cf. LONG et al. 2008), then – taking advantage of dependence (28), equation (50) can be simplified to the form:

$$(1 - n_p) \frac{\partial z_b}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{53}$$

In equation (53), the sediment transport rate q consists of two parts (in respect of the thickness z_m):
– linear

$$q_x^L = q_x = (1 - n_p)\, U_{L_1}\, z_m \tag{54}$$

and
– non-linear

$$q^{nL} = U_{L_1} \int_{z_m}^{\eta} \langle C_2 \rangle\, dz + \int_{z_m}^{\eta} U_{L_2} \langle C_1 \rangle\, dz + \int_{z_m}^{\eta} U_{L_2} \langle C_2 \rangle\, dz \tag{55}$$

Because equation (53) contains a non-linear fragment described by dependence (55), the first order upward scheme cannot be applied in order to obtain the right solution. LONG et al. (2008) suggested using a fifths-order Euler-Weno scheme to solve this equation, simultaneously applying the decomposition of the sediment transport rate q into two parts q^+ and q^-.

In a contrary situation, when sediment is transported only as suspended, inert matter, possibly exchangeable with the sediment lying in the bed, thus causing either deposition of sediment or the picking up o sediment by a current, we can write:

$$\begin{cases} \langle C^I \rangle = (1 - n_p) \text{ and } \langle C^{II} \rangle = \langle C_2 \rangle \\ U_L^I = 0 \quad \text{ and } U_L^{II} = U_{L_2} \end{cases} \tag{56}$$

Considering relations (52) and (56) as well as (28), equation (50) can be written as:

$$(1 - n_p) \frac{\partial z_b}{\partial t} + \frac{\partial q}{\partial x} + \nabla_{\mathrm{E}} - \nabla_{\mathrm{D}} = 0 \tag{57}$$

where

$$q = q^{nL} = \int_{z_m}^{\eta} U_{L_2} \langle C_2 \rangle\, dz \tag{58}$$

The above shows the non-linear part (in respect of z_m) of the sediment transport rate. In a non-linear case, it is not necessary to divide transport q into two parts q^+ and q^-. Furthermore, it should be mentioned that the component describing "storage" of sediment in the bed ($\nabla_E - \nabla_D$) can play a more important role when local, spatially more rapid changes in the intensity of transport q occur, for example in the vicinity of constructions (WHITEHOUSE 1998).

Summary

The purpose of this paper has been to analyze the equation which describes changes in the bathymetry of the bed in time and space, derived from the principle of mass conservation. The analysis of the equation was carried out, *inter alies*, for the hydrodynamic equilibrium conditions, when a flow of sediments falling on the bed is balanced by a flow of sediments picked up from the bed, for which a linear dependence was derived between the sediment transport rate and the thickness of the layer of densely packed sand grains in the sediment in motion. This dependence enables us to apply the first-order upwind scheme to calculations of changes in the bed's bathymetry in time and space. However, in order to correctly solve the equation, it is necessary to divide the sediment transport into the positive direction – onshore – and negative one – offshore. The onshore transport takes place during the wave crest duration while the offshore transport appears during the wave trough time.

The linear form of the dependence between the sediment transport rate and thickness of the layer of densely packed grains, suggested in this paper, has been experimentally confirmed by the results of the experiment conducted in a laboratory of the Polish Academy of Sciences Institute of Hydroengineering in 1996.

This article constitutes a theoretical basis for mathematical description of changes in bathymetry in time and space, which will take into consideration changes in the grain size distribution in sediments which make up the bed. Applicability of the model has been demonstrated in Part 2, where a laboratory experiment run under wave and current conditions was used as a case for modelling changes in bathymetry, changes in the grain size distribution of sediments in the bed as well as vertical profiles of concentrations of suspended sediments near a canal which serves as a model of an approach fairway to a harbour.

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