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# BATHYMETRY CHANGES AND SAND SORTING DURING SEDIMENTATION OF WATERWAYS. PART 2 – MODELLING VERSUS LABORATORY DATA

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Key words: graded sediment, sediment transport, bed grain size distribution changes, bathymetry changes, sediment velocity, sorting, sedimentation.

Abstract

A theoretical, three-layer model of non-uniform sediment transport has been applied in our analysis of the sedimentation of navigable waterways tested under laboratory conditions. This model distinguishes three layers of moving sediment and, additionally, includes an assumption that sediment is not sorted out in the bedload layer but vertical grading occurs only when grains are lifted up in the contact layer. Above that layer, in the outer flow region, the grain size distribution of sediment remains unchanged. Moreover, it has been assumed that during the wave crest phase, sediment is transported in the bedload and contact layers as well as in the outer flow layer under the influence of a resultant current directed, as described in the present article, in the same direction as the propagating surface wave. During the wave trough phase, sediment transport occurs in the bedload and contact layers. The mathematical description of changes in the bed level (bathymetry), contained in this paper, divides sediment transport into the one in the positive direction, during the wave crest phase, in the wave propagation direction, and the one in the negative direction, during the wave trough phase, reverse to the wave propagation direction. The results obtained from modelling changes in bathymetry compared with the results from a laboratory experiment conducted in a wave flume under the wave and current conditions yielded satisfactory coincidence. The effect of including changes in the grain size distribution into the calculations proved to be highly significant. The calculations demonstrated that the applied model could be a useful tool for both predicting changes in the bathymetry in approach waterways to harbours and determining grain size distribution of sediment which fills up a waterway or the rate of sedimentation of a waterway.

**ZMIANY BATYMETRII I SEGREGACJA OSADÓW W PROCESIE ZAPIASZCZANIA TORÓW WODNYCH. CZĘŚĆ 2 – PORÓWNANIE WYNIKÓW MODELOWANIA Z WYNIKAMI POMIARÓW LABORATORYJNYCH**

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Key words: sortowaie osadów, transport osadów, zmiany rozkładów uziarnienia, zmiany batymetrii, prędkość rumowiska, sortowanie, zapiaszczanie.

Abstrakt

Do analizy zapiaszczania torów wodnych w warunkach laboratoryjnych wykorzystano trójwarstwowy model teoretyczny transportu osadów niejednorodnych granulometrycznie. Model ten wyszczególnia trzy warstwy ruchu rumowiska, przy czym założono, że w warstwie wleczenia osady nie są sortowane, a pionowe sortowanie odbywa się tylko w procesie podrywania ziaren w warstwie kontaktowej. Ponad tą warstwą – w zewnętrznym obszarze, rozkład granulometryczny osadu pozostaje niezmienny. Ponadto założono, że w fazie grzbietu fali osady są transportowane w warstwach wleczenia i kontaktowej oraz zewnętrznej – pod wpływem wypadkowego prądu, skierowanego w tę samą stronę, co propagująca fala powierzchniowa. W fazie doliny fali transport rumowiska odbywa się tylko w warstwach wleczenia i kontaktowej. W opisie matematycznym zmian poziomu dna (batymetrii) podzielono transport osadów na transport w kierunku dodatnim, odbywający się w czasie trwania grzbietu fali, w kierunku propagacji fali i kierunku ujemnym, odbywający się w kierunku przeciwnym w czasie trwania doliny fali. Wyniki modelowania zmian batymetrii w porównaniu z wynikami pochodzącymi z eksperymentu laboratoryjnego przeprowadzonego w kanale falowym w warunkach falowo-prądowych dały zadowalającą zgodność. Wpływ uwzględniania w obliczeniach zmian rozkładów uziarnienia osadów okazał się niezwykle istotny. Przeprowadzone obliczenia pokazały, że zastosowany model obliczeniowy może być użytecznym narzędziem służącym do prognozowania zarówno zmian batymetrii w obrębie torów podejściowych do portów, jak i w określaniu rozkładów granulometrycznych osadów wypełniających tor wodny, a także tempa zapiaszczania torów wodnych.

## Introduction

For maintaining the right navigable depth of fairways and planning dredging works, it is crucial to be able to predict accurately amounts of transported bed load and the rate at which approach waterways to harbours are silted up. It is equally important to understand well the mechanisms which cause motion of sediments of different grain size structure, and to be able to predict transported load of particular fractions of sediment as well as to know the actual grain size distribution of sediment in a given fairway. Such knowledge may prove valuable when planning and conducting silting-up works needed for artificial nourishment of shores, when trying to determine the effectiveness of seashore protection schemes, etc. In many cases, silt material is dredged from a waterway and then used to strengthen the shore near a harbour. Then, it is very useful to know the grain size structure of the sand because if too fine silt is used for reinforcing the banks, a situation may arise when artificial seashore nourishment will turn into a useless and unprofitable operation because the sand previously dug out from the seabed and laid on the shores will be washed away very quickly.

This article is a continuation of Part 1, where – based on the mass conservation principle – elementary derivation was presented of an equation applied to describing changes in the seabed bathymetry in time and space. A detailed analysis of this equation was performed in respect of the dependence between the transport rate and thickness of densely packed grains in moving sediment. Based on the theoretical considerations, verified by the results obtained in a laboratory experiment, it was suggested to describe this dependence in the form of a linear function for the so-called hydrodynamic equilibrium condition.

In the paper, we discuss the applicability of the three-layer model of non-uniform sediment transport, as developed by KACZMAREK (1999) according to the principle of conservation of water and sediment momentum in the nearbed layer, to solving specific engineering problems – with sedimentation of a waterway as a case example – and comparing the results to another laboratory experiment carried out in a flume canal at the Delft Hydraulics (VAN RIJN 1986).

## Sand transport

### Momentum equation in the bedload layer

In the discussed model of the transport of sediment with particles different in size, it has been proposed to use a mathematical model to describe interactions between water and sediment as well as mutual interactions between sediment grains in the whole interaction area, from the immobile bed to the formation of "a carpet" of sediments, up to suspended sediments. According to the description presented by KACZMAREK (1999) for uniform sediment under pure oscillatory flow conditions and by KACZMAREK and OSTROWSKI (2002) for flows under the wave and current conditions and by KACZMAREK et al. (2004) for graded sediments, the area in which sediment moves can be divided into three layers: 1) bedload layer, 2) contact load layer and 3) outer flow region.

Employing Sayed and SAVAGE'S (1983) dependences for viscous stresses and for the stresses due to Coloumb friction, for conditions when the transport of sediments is in hydrodynamic equilibrium, i.e. the flow of sediments falling on the bed is offset by the flow of sediments picked up from the bed, a set of equations has been proposed that describes the celerity and concentration of sediments in the bedload layer, which can be presented in the form:

$$\alpha^0 \left(\frac{c_b - c_0}{c_m - c_b}\right) \sin\phi \sin 2\psi + \mu_1 \left(\frac{\partial u_b}{\partial z'}\right)^2 = \rho u_f^2 \tag{1}$$

$$\begin{aligned} &\alpha^0 \left(\frac{c_b - c_0}{c_m - c_b}\right)(1 - \sin\phi \sin 2\psi) + (\mu_0 + \mu_2)\left(\frac{\partial u_b}{\partial z'}\right)^2 \\ &= \left.\left(\frac{\mu_0 + \mu_2}{\mu_1}\right)\right|_{c=c_0} \rho u_f^2 + (\rho_s - \rho) g \int_0^{z'} c_b \, dz' \end{aligned} \tag{2}$$

The factors $\mu_0$, $\mu_1$ and $\mu_2$ are functions of the concentration of sediment $c$, determined by the formulas:

$$\frac{\mu_1}{\rho_s d^2} = \frac{0.03}{(c_m - c_b)^{1.5}} \tag{3}$$

$$\frac{\mu_0 + \mu_2}{\rho_s d^2} = \frac{0.02}{(c_m - c_b)^{1.75}} \tag{4}$$

where:

$c_m (= 0.53)$ – maximum concentration of sediment at rest when grains closely adhere to one another,

$c_b$ – sediment concentration,

$u_b$ – velocity of sediment in the bedload layer,

$c_0$ – concentration of sediment at the theoretical bed level ($c_0 = 0.32$),

$\alpha^0 / \rho_s g d = 1.0$,

$\rho_s$ – density of skeletal ground,

$\rho$ – density of water,

$d$ – diameter of sediment grains,

$\phi$ – quasi-static inner friction angle,

$\psi = \dfrac{\pi}{4} - \dfrac{\phi}{2}$.

Due to strong interactions between particles of each fraction in the bedload layer, it was assumed that all fractions moved at the same speed $u_{br}(z',t)$ and had identical concentration $c_{br}(z',t)$ at a given level $z'$. From this assumption, it has been concluded that sediment in the bedload layer is not being sorted out, which then enables us to describe the transport of sediment using a representative particle diameter $d_r = d_{50}$. Thus, having the representative diameter $d_r$, we can determine the effective coarseness $k_e = k_{er}$ from the formula proposed by KACZMAREK (1999):

$$k_{er} = 47.03\ d_r\ \theta_{2.5}^{-0.658} \tag{5}$$

in which, as NIELSEN (1992) demonstrated

$$\theta_{2.5} = 0.5\ f_{2.5}\psi_1 = 0.5 f_{2.5}\ \frac{(\alpha_{1m}\varpi)}{(s-1)g d_r} \tag{6}$$

and

$$f_{2.5} = \exp\left[5.213\left(\frac{2.5\,d_r}{\alpha_{1m}}\right)^{0.194} - 5.977\right] \tag{7}$$

where

$g$ – acceleration of gravity,

$\alpha_{1m} = U_{1m}/\varpi$ – horizontal amplitude of the nearbed wave motion,

$U_{1m}$ – maximum orbital velocity near the bed,

$s = \rho_s/\rho$ – relative density,

$\varpi = 2\pi/T$ – orbital frequency,

$T$ – wave period.

Next, from FREDSØE'S (1984) integral model, instantaneous friction velocities $u_f(t)$ on the bed surface are determined. From equations (1) – (4) we obtain instantaneous values of the velocity $u_{br}\ (z',t)$ and concentration $c_{br}(z',t)$ of the sediments in the bedload layer of the thickness $\delta_{br}$. The $z'$ axis is orientated positively downwards from the theoretical bed level. The first reference point, where the sediment velocity $u_{br}\ (z',t)$ is equal to zero, is denoted as the coordinate $\delta_{br}$, which describes the thickness of the bedload layer. Noteworthy is the fact that the model adopted to describe changes in the bedload layer removed one of the major simplifications until now broadly applied to the modelling of ungraded sediment transport, namely the presented model assumes that interactions between fractions are so strong that consequently finer fractions are slowed down by coarser ones and eventually all fractions move at the same speed. Thus, simple summation of transport rates of all the fractions in sediment, treated here as uniform deposit, is not applicable to transport processes in this layer, which means that:

$$n_i q_{bi} \neq n_i q_{br} \tag{8}$$

where
$n_i$ – percentage of a given sediment fraction in the whole mixture.

The above conclusion coincides with many laboratory observations (cf. HASSAN et al. 2001, and KACZMAREK et al. 2004).

We should, however, remember that a current causing erosion (due to shearing) of the bed in a cell does not cause changes in the distribution of grain sizes.

Thus, the bedload transport rate in the shoreward $x$-direction $q_{bx}^{+}$ is equal to:

$$q_{bx}^{+} = \frac{1}{T} \int_0^{T_c} \left( \int_0^{\delta_{br}^{+}} u_{br}^{+}(z',t)\ c_{br}^{+}(z',t)\ \mathrm{d}z' \right) \mathrm{d}t \tag{9}$$

whereas the bedload transport rate in seaward $x$-direction $q_{bx}^{-}$ is:

$$q_{bx}^{-} = \frac{1}{T} \int_0^{T} \left( \int_0^{\delta_{br}^{-}} u_{br}^{-}(z',t)\ c_{br}^{-}(z',t)\ \mathrm{d}z' \right) \mathrm{d}t \tag{10}$$

where
$T_c$ and $T$ are wave crest and wave periods, respectively.

## Momentum equation in the contact layer

In the contact layer, turbulent pulsations and chaotic collisions between sand grains cause big differences in the transport of particular fractions of the sediment. In the contact layer, two sub-layers were distinguished, characterized by different mechanisms of current exchange. Near the bed – in the sub-layer where decompositions of the velocity of the i-*th* fraction of sediments quite evidently reveal slip velocity $u_{br}$ – in this sub-layer, there are very strong interactions between particular fractions, caused by chaotic, mutual collisions between sediment grains. Although further away from the bed, interactions between the fractions become weaker, the concentration of the i-*th* fraction is big enough to muffle turbulences, which depend on the grain diameter $d_i$. It can, therefore, be expected that each i-*th* fraction moves at its own speed $u_{ci}\,(z,t)$ and is characterized by its own concentration $c_{ci}(z,t)$. In this model, these instantaneous distributions of speed and concentration for the *i*-th fraction are determined from the equations suggested by KACZMAREK (1999), when the variable of the skin friction velocity $u_{fi}'(t)$ in the wave period is known:

$$\left[\frac{3}{2}\left(\alpha\,\frac{d_i}{w_{si}}\,\frac{\mathrm{d}u_{ci}}{\mathrm{dz}}\,\frac{2}{3}\,\frac{s\,+\,c_M}{c_D}\,+\,\beta\right)^2 d_i^2\,c_{ci}^2\,(s\,+\,c_M)\,+\,l^2\right]\left(\frac{\mathrm{d}u_{ci}}{\mathrm{dz}}\right)^2 = u_{fi}^{\prime 2}(t) \tag{11}$$

$$\left[3\left(\alpha\,\frac{d_i}{w_{si}}\,\frac{\mathrm{d}u_{ci}}{\mathrm{dz}}\,\frac{2}{3}\,\frac{s\,+\,c_M}{c_D}\,+\,\beta\right)^2 d_i^2\,\frac{\mathrm{d}u_{ci}}{\mathrm{dz}}\,c_{ci}\,+\,l^2\,\frac{\mathrm{d}u_{ci}}{\mathrm{dz}}\right]\frac{\mathrm{d}u_{ci}}{\mathrm{dz}} = -w_{si}\,c_{ci} \tag{12}$$

where,
$c_D$ – resistance factor,
$c_M$ – added mass factor,
$l$ – mixing length, expressed as $l = \kappa z'' = 0.4z'$,
$\kappa$ – Karman constant,
with the $z$ axis directed upward.

DEIGAARD (1993) assumed that $(s + c_M) = 3.0$, while $c_D = 1.0$. Values of the proportionality factors $\alpha_s$ and $\beta_s$ are unknown and must be determined by scaling the computational model. The value $u_{fi}'(t)$ is determined from FREDSØE'S model (1984), including the assumption formulated by NIELSEN (1992) that the effective skin roughness $k'_{ei}$ is defined by the relation $k'_{ei} = 2.5d_i/30$. The marginal conditions for all the fractions are identical, i.e.

$$u_{ci}\;(z\,=\,2.5d_i/30,\,t)\,=\,u_{br}\;(z'\,=\,0,\,t) \tag{13}$$

$$c_{ci}\;(z\,=\,2.5d_i/30,\,t)\,=\,c_{br}\;(z'\,=\,0,\,t)\,=\,0.32 \tag{14}$$

Including the effective skin roughness, determined for the representative diameter $k_{er}^{'} = 2.5d_r = 2.5d_{50}$, and using FREDSØE'S integral model (1984), the upper boundary of the contact layer – shared by all the fractions – is determinable ($\delta_{cr}$).

As has been demonstrated by Kaczmarek (1999), this thickness can be determined by the dependence $\delta_{cr} = \delta_1^{'}/2$, in which $\delta_1^{'}$ corresponds to the thickness of the near-wall layer – found from FREDSØE'S (1984) solution for $k_{er}^{'} = 2.5d_r = 2.5d_{50}$ – at the moment of the maximum (during wave period $T$) orbital velocity near the bed.

It is worth noticing that the velocities and concentrations of coarser fractions within the contact layer as computed in the proposed model, are larger than the values these fractions would have attained had the bed been uniform and composed of just one corresponding fraction. This accelerated speed in a mixture is a result of mutual interactions between fractions, where coarser fractions are accelerated by finer ones. The above effect has been demonstrated by several researchers, for example de MEIJER et al. (2002). The laboratory observations reported by these authors prove that coarser fractions of sediment are transported more intensively in an ungraded mixture than in uniform sediment.

As mentioned earlier, instantaneous values of the speed and concentration of sediment for the $i$-th fraction in the presented model are derived from equations (11) and (12). In turn, values of the sediment mixture concentration averaged for the wave period $T$ can be obtained as follows:

$$C_c(z) = \sum_{i=1}^{n} n_i \left( \frac{1}{T} \int_0^T c_i(z,\ t)\ dt \right) \tag{15}$$

whereas the percentage content $k_i$ of the i-*th* fraction can be computed from:

$$k_i\ (z) = \frac{\langle c_{ci}\ (z,\ t)\rangle\ n_i}{C_c(z)} \tag{16}$$

where:

$$\langle c_{ci}(z,\ t)\rangle = \frac{1}{T} \int_0^T c_{ci}\ (z,\ t)\ \mathrm{d}t \tag{17}$$

By knowing the percentage content of $k_i$ of the $i$-th fraction on each level $(z)$ within the contact layer at each step in the computations, it is easy to calculate the grain-size distribution as well as the representative value $d_{50}(z)$ in the suspended sediment found in this layer.

Recapitulating, it should be emphasized that in the presented model of sediment transport both in the contact and bedload layers, simple summation of transport rates of particular fractions, treated as uniform sediment, is not applicable.

Thus, also in this case, the contribution of the $i$-th fraction to the transport of the whole mixture $n_iq_{ci}$ does not equal to the “independent” transport $n_iq_i$, which means that $n_iq_i \neq n_iq_{ci}$.

Finally, the contact load transport in the shoreward $x$ direction $q_{cx}^{+}$ for the mixture can be written as:

$$q_{bx}^{+} = \sum_{i=1}^{N} n_i \left( \frac{1}{T} \int_{0}^{T_c} \left( \int_{0}^{\delta_{cr}} u_{ci}^{+}(z,t)\ c_{ci}^{+}(z,t)\ \mathrm{d}z \right) \mathrm{d}t \right) \tag{18}$$

while, the contact load transport rate in the seaward $x$-direction $q_{cx}^{-}$ as:

$$q_{bx}^{-} = \sum_{i=1}^{N} n_i \left( \frac{1}{T} \int_{T_c}^{T} \left( \int_{0}^{\delta_{cr}} u_{ci}^{-}(z,t)\ c_{ci}^{-}(z,t)\ \mathrm{d}z \right) \mathrm{d}t \right) \tag{19}$$

## Outer flow region

In the outer layer, we encounter certain difficulties while striving to describe correctly the time-variable profile of concentrations of suspended sediment. Higher above the bed, researchers indicate that there is no agreement in the phase between theoretical models and experimental results (DAVIES et al. 1997). Thus, in order to obtain a time-averaged value of the concentration $c_0(z)$ of a mixture of suspended sediment, the dependence in the following form is applied (cf. RIBBERINK and AL-SALEM 1994):

$$\langle C_0(z) \rangle = \langle \mathrm{C}_0(z = \delta_{cr}) \rangle \left( \frac{\delta_{cr}}{z} \right)^{\alpha_1} \tag{20}$$

where, the concentration on the reference level is determined as time-averages of the results obtained for the contact layer:

$$\langle c_0(z = \delta_{cr}) \rangle = \sum_{i=1}^{N} \langle c_{ci}(z = \delta_{cr},\ t) \rangle n_i \tag{21}$$

The exponent of the $\alpha_1$ power has an unknown value, which is determinable from experimental data. As shown by BIEGOWSKI (2006), by selecting $\alpha_1 = 0.6$, the best coincidence is attained between results of calculations and data from field trials and laboratory experiments conducted in a large-scale flume canal. We should add that for $\alpha_1$ = const. it is impossible to speak about any vertical sorting of sediments in a mixture, which means that the grain size distribution in the outer layer remains unchanged. Further, the sediment transport rates in the outer flow zone are determined using the following simplified formulas:

$$q_{ox}^{-} = \int_{\delta_{cr}}^{h} U_{ox}^{-}(z) \langle C_0(z) \rangle \, \mathrm{dz} \tag{22}$$

$$q_{oy} = \int_{\delta_{cr}}^{h} U_{oy}(z) \langle C_0(z) \rangle \, \mathrm{dz} \tag{23}$$

where

$h$ is the water height and velocities $U_{ox}^{-}(z)$ and $U_{oy}(z)$ are the velocities of a current induced in the nearshore zone of the sea, respectively to the reverse current (offshore) and longshore current, generated by waves which propagate at an angle to the shore's transverse profile.

## Conservation of the sediment mass

The equation derived from the principle of mass conservation, as shown in detail in Part 1, can be presented for a 2-D case in the following form:

$$\frac{\partial z_m}{\partial t} + \frac{1}{(1 - n_p)} \left( \frac{\partial q_x^{+}}{\partial x^{+}} + \frac{\partial q_x^{-}}{\partial x^{-}} + \frac{\partial q_y}{\partial y} \right) = 0 \tag{24}$$

where:

$q_y$ – the longshore sediment transport rate,

$q_x^{+}$ – the onshore sediment transport rate,

$q_x^{-}$ – the offshore sediment transport rate,

$z_m$ – thickness of the layer of sediment grains being in motion and densely packed.

In the space $(j,k)$ of the shore's transverse profile, the layer $z_m$ changes over a time period $\Delta t$. Erosion or accumulation happens because of changes in the transport rates $q_x^{+}$ and $q_x^{-}$ over the length $\Delta x$ as well as $q_y$ over the length $\Delta y$. Taking advantage of the upwind numerical scheme, equation (24) can be written with the help of finite differences in the form:

$$\Delta(z_m)_{j,k} = \frac{\Delta t}{\Delta x} [(q_x^{+})_{j,k} + |(q_x^{-})_{j,k}| - (q_x^{+})_{j-1,\mathrm{k}} - |(q_x^{-})_{j+1,\mathrm{k}}|]$$

$$+ \frac{\Delta t}{\Delta y} [|(q_y)_{j,k}| - |(q_y)_{j,k-1}|] =$$

$$= \frac{\Delta t}{\Delta x} [(q^+_{bx})_{j,k} + (q^+_{cx})_{j,k} + |(q^-_{bx})_{j,k}| + |(q^-_{cx})_{j,k}| + |(q^-_{0x})_{j,k}| - (q^+_{bx})_{j-1,k}$$

$$- (q^+_{cx})_{j-1,k} - |(q^-_{bx})_{j+1,k}| - |(q^-_{cx})_{j+1,k}| - |(q^-_{0x})_{j+1,k}|] \qquad (25)$$

$$+ \frac{\Delta t}{\Delta y} [|(q_{0y})_{j,k}| - |(q_{0y})_{j,k-1}|]$$

where

$q_{bx}$ – rate of the transport of sediment in the bedload layer in the direction of axis $x$ [m$^2$/s],

$q_{cx}$ – rate of the transport of sediment in the contact layer in the direction of axis $x$ [m$^2$/s],

$q_{0x(y)}$ – rate of the transport of suspended sediment in the direction of axis $x(y)$ [m$^2$/s].

Dependence (25) describes the thickness of the eroded or accumulated layer without including the porosity of sediments. Hence, in order to account for the effect of porosity on the thickness $\Delta(z_m)_{j,k}$ the result obtained should be multiplied by $(1\text{-}n_p)$.

All the transport rate components in equation (25) have positive values, which means that the values with a preceding „+" sign cause an increase in the depth, i.e. erosion of the bed, whereas the ones preceded with a „–" sign cause a decrease in the depth, that is the accumulation of the bed in a given point $(j,k)$.

The thickness of the eroded or accumulated layer for each i-*th* fraction of the sediment of which the bed is composed can be determined in the following way:

$$\Delta(z_m)^i_{j,k} = \frac{\Delta t}{\Delta x} [(q^+_{bx})^i_{j,k} + (q^+_{cx})^i_{j,k} + |(q^-_{bx})^i_{j,k}| + |(q^-_{cx})^i_{j,k}| + |(q^-_{0x})^i_{j,k}| - (q^-_{bx})^i_{j-1,k}$$

$$- (q^+_{cx})^i_{j,k} - |(q^-_{bx})^i_{j+1,k}| + |(q^-_{cx})^i_{j+1,k}| - |(q^-_{0x})^i_{j+1,k}|] \qquad (26)$$

$$+ \frac{\Delta t}{\Delta y} [|(q_{0y})^i_{j,k}| - |(q_{0y})^i_{j,k-1}|]$$

where

$$\Delta(z_m)_{j,k} = \sum_{i=1}^{N} \Delta(z_m)^i_{j,k} \qquad (27)$$

Predictably, in time $\Delta t$, the grain size distribution in the control volume changes from $n^i_{j,k}$ ($\sum n^i_{j,k} = 1$) to $m^i_{j,k}$ ($\sum m^i_{j,k} = 1$) according to the formula (cf. KACZMAREK et al. 2004):

$$m^i_{j,k} = \frac{n^i_{j,k}\ (h_m)_{j,k} - \Delta(z_m)^i{}_{j,k}}{(h_m)_{j,k} - \Delta(z_m)_{j,k}} \tag{28}$$

In equation (28), the value of $(h_m)_{j,k}$ describes the thickness of the mixing layer, as it is in that layer that the grain size distribution changes from $n^i_{j,k}$ to $m^i_{j,k}$ in time $\Delta t$. This change is induced by the difference in the rates of transport of particular fractions over the lengths $\Delta x$ and $\Delta y$.

As has been demonstrated by KACZMAREK et al. (2004), we can assume that in the time period $\Delta t$ half the transported sediment over a plane seabed in the analyzed area is scattered and suspended in the contact and outer flow layers ($j,k$), whereas the other half remains in the bedload layer and is mixing with the sediments flowing from other areas. The thickness of the mixing layer, therefore, takes the form of:

$$(h_m)_{j,k} = 2\,\frac{\Delta t}{\Delta x}\,[(q^+_{bx})_{j,k} + (q^+_{cx})_{j,k} + |(q^-_{bx})_{j,k}| + |(q^-_{cx})_{j,k}| + |(q^-_{0x})_{j,k}|] + 2\,\frac{\Delta t}{\Delta y}\,[|q_{0y})_{j,k}|] \tag{29}$$

The laboratory tests have revealed that the thickness of the mixing layer ranges from 2 cm (SISTERMANS 2001) to 5 cm (CHATELUS et al. 1998).

It is interesting to note that a change in the grain size distribution in the mixing layer $(h_m)_{j,k}$ from $n^i_{j,k}$ to $m^i_{j,k}$ does not depend on discretization in time or space because (assuming for simplification that $\Delta x = \Delta y$) all the components in the numerator and denominator in equation (28) contain the element $\Delta t/\Delta x$. A change in the grain size distribution, therefore, depends exclusively on the difference in the sediment transport rates.

Let us assume that in the analyzed control volume of the thickness $(h_m)_{j,k}$ erosion occurs (the value of $\Delta(z_m)_{j,k}$ is positive), then after the time $\Delta t$, it will cause the appearance of "a carpet of sediments" of the thickness $a_{j,k}$:

$$a_{j,k} = (h_m)_{j,k} - \Delta(z_m)_{j,k} \tag{30}$$

Thus, after the time $\Delta t$, i.e. during the time moment $(t + \Delta t)$, a new grain size distribution in a new mixing layer of the thickness $(h_m)_{j,k}$ will be composed of the distribution $m^i_{j,k}$ found in the „carpet of sediments" of the thickness $a_{j,k}$ and the distribution $(n_s)^i_{j,k}$, which characterizes the sediment in the parental bed:

$$[n(t + \Delta t)]^i_{j,k} = \frac{a_{j,k} m^i_{j,k} + [(h_m)_{j,k} - a_{j,k}](n_s)^i_{j,k}}{(h_m)_{j,k}} \tag{31}$$

If, in contrast, we assume that sediment is accumulated in the analyzed area (the value $\Delta(z_m)_{j,k}$ is negative), then the new grain size distribution after the time $\Delta t$ will be composed of the distribution $m^i_{j,k}$ alone, which will be found in the „carpet of sediments" of the thickness $a_{j,k}$.

Finally, by taking into consideration the porosity of sediment $n_p$, we obtain the dependence which describes changes in the bed bathymetry in time and space, which take into account changes in the grain size distribution of the sediments of which the seabed is composed:

$$h_{j,k}( t + \Delta t) = h_{j,k}(t) + \frac{1}{1 - n_p} \Delta(z_m)_{j,k} \tag{32}$$

## Delft Hydraulics laboratory experiment

Some of the laboratory measurements have been carried out at the Delft Hydraulics (VAN RIJN 1986), a research centre situated in northern Holland, in a flume canal 17.0 m long, 0.3 m wide and 0.5 m deep (Fig. 1). The aim of the experiment was to determine changes in bathymetry under given wave and current conditions and to establish vertical profiles of suspended sediment concentrations. The total duration of the trial was 10 hours.

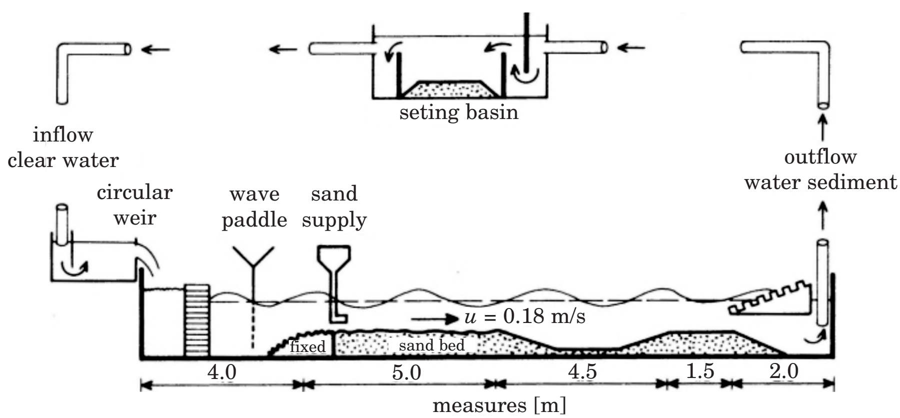


Fig. 1. The design of the experiment

Source: VAN RIJN (1986).

The bed consisted of a layer of fine sand ($d_{50}$ = 100 μm, $d_{90}$ = 130 μm) about 0.2 m thick. The incline of the canal's edge was 1:10. Sediment was supplied evenly from the inflow side of the canal so as to maintain constant (balanced) conditions. Regular waves of the height of 0.08 m and wave period 1.5 s were generated, and the current was set according to the direction of wave propagation. During the experiment, concentrations of sediment were measured in vertical profiles at five sites located along the canal (Fig. 2). Table 1 presents some of the basic parameters of the experiment.

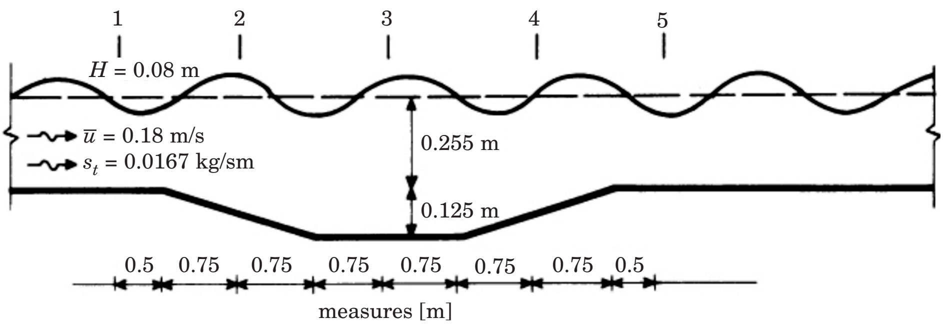


Fig. 2. Location of the measurement sites along the canal
Source: VAN RIJN (1986).

Table 1

Basic parameters of the experiment at the Delft Hydraulics

| Parametr | Value |
|---|---|
| Water depth $h_0$ | 0.255 [m] |
| Mean current velocity $u_0$ | 0.18 [m/s] |
| Wave height $H$ | 0.08 [m] |
| Wave period $T$ | 1.5 [s] |
| Transport of suspended sediment $q_{s.o}$ | 0.167 [kg/sm] |
| Diameter of grains of the sediment in bed $d_{50}$; $d_{90}$ | 0.10; 0.13 [mm] |
| Diameter of grains of the suspended sediment $d_s$ | 0.08 – 0.011 [mm] |
| Fall velocity of suspended grains $w_s$ | 0.005 – 0.010 (mean: 0.007) [m/s] |
| Density of sediment $\rho_s$ | 2650 [kg/m$^3$] |
| Density of fluid $\rho_w$ | 1000 [kg/m$^3$] |
| Porosity of sediment $n_p$ | 0.4 [–] |

## Model results versus laboratory data

### Modelling distributions of the vertical concentration of sediments

The scope and course of the experiment (cf. Figs 1 and 2) conducted at the Delft Hydraulics (VAN RIJN 1986) implicates a theoretical solution to a two-dimensional problem, i.e. it forces researchers to add the simplification $(q_{0y})_{j,k} = (q_{0y})_{j,k-1}$ to differential equations (25) and (29) due to the lack of a longshore current in the above experiment. Besides, in equations (25) and (29), the values $(q^-_{0x})_{j,k}$ and $(q^-_{0x})_{j+1,k}$ need to be substituted, respectively, with the values $(q^+_{0x})_{j,k}$ and $(q^+_{0x})_{j+1,k}$ because in the above experiment the current in the outer layer was directed in accordance with the propagation of waves, whereas equations (25) and (29) are based on the assumption that the current flows offshore, as is typical of the shore zone.

In the mathematical modelling of the vertical concentration of suspended sediments, two available parameters, $d_{50}$ and $d_{90}$, were used as input data for analyzing the grain size distribution of sediment. By knowing these data, it was possible to draw a grain-size curve, presented in Fig. 3. Due to the very narrow range of diameters of the fine sediment used in the experiment, three representative diameters were taken, which correspond to the lower values of the diameter ranges, shown in Fig. 3 as a histogram. The lower values of these ranges correspond to the mesh in the control sieves.

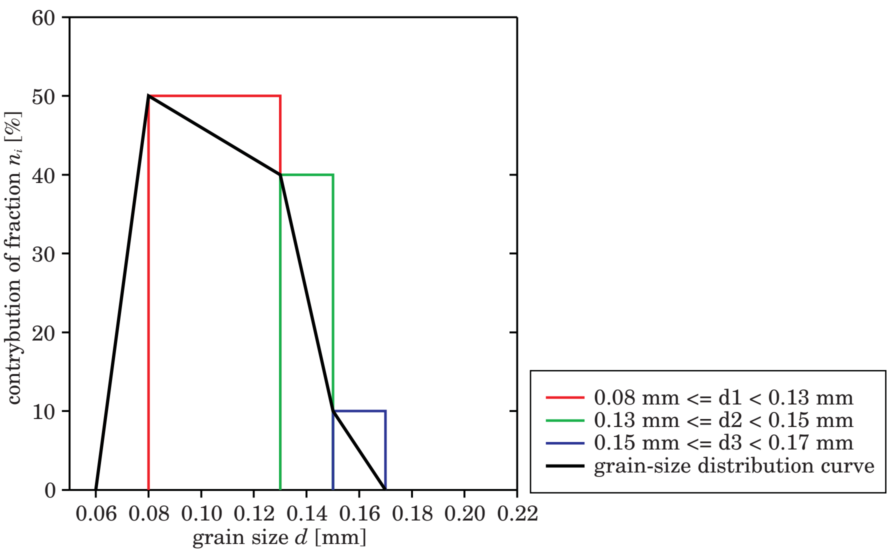


Fig. 3. The histogram of sediment grain size distribution used for calculations, acc. to parameters $d_{50}$ and $d_{90}$

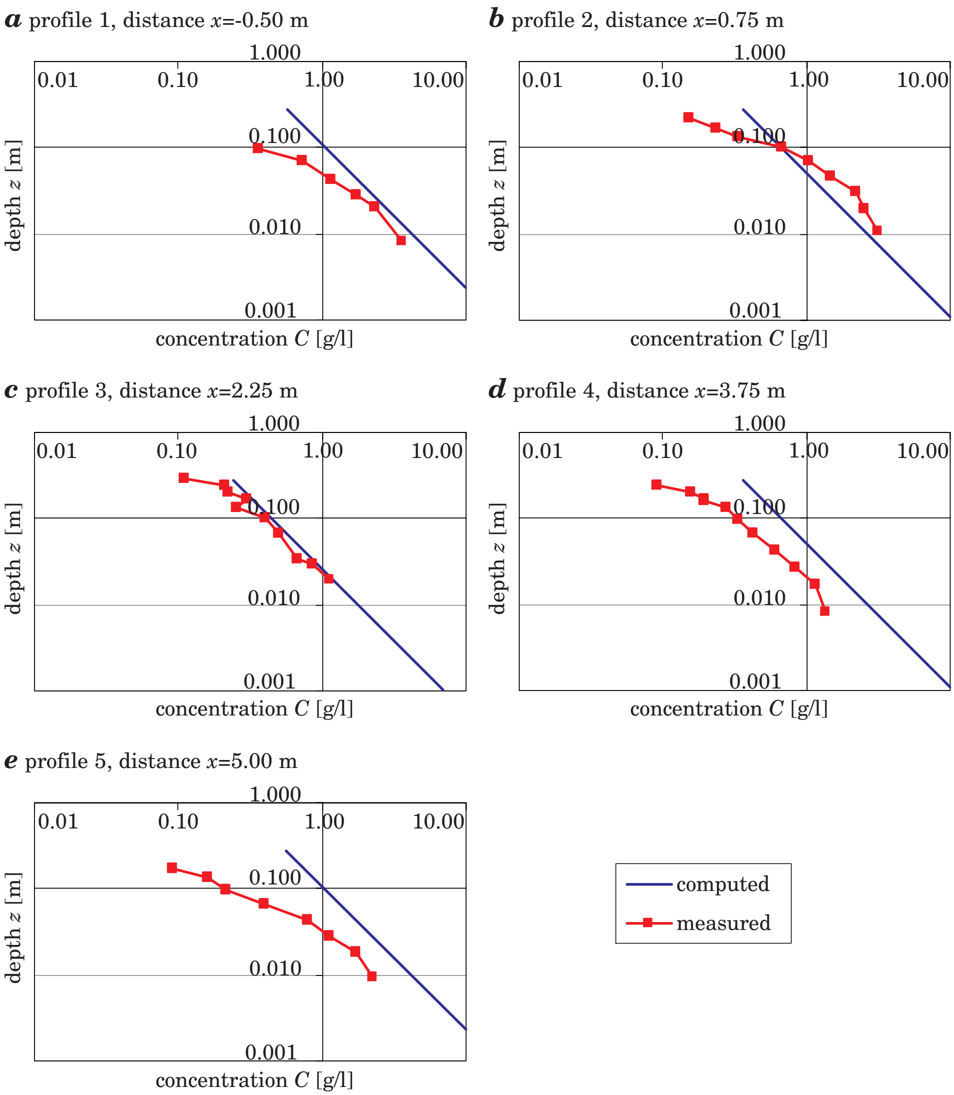


Fig. 4. Comparison of computed vertical profiles of suspended sediment concentrations for a regular wave with the ones measured during the experiment in five profiles set along the canal

The mathematical modelling of the vertical concentration of suspended sediments was completed for a symmetrical, regular (sinusoidal) wave. The calculated vertical concentration distributions were then compared with the results obtained during the experiment at five cross-sections located along the canal (Fig. 2). The measurement data from the experiment enabled us to analyze the vertical concentration distributions within the range of weak hydrodynamic conditions, where the dimensionless friction (Shield's parameters – see Eq. 6) ranged between $0.13 < \theta_{2.5} < 0.23$. The values of Shield's parameter indicate that the bed was rippled.

Figure 4 compares graphically the computed vertical profiles of suspended sediment concentrations for a regular wave with the data obtained from experimental measurements taken in five perpendicular profiles set along the canal.

Figure 4 shows that at a distance very close to the bed, the concentration of sediments was not measured because the measuring devices were set up at some further distance from the bed due to the ripples between 0.01 and 0.02 m high (VAN RIJN 1986), which covered the bed.

The coincidence of the results of computations attained with the proposed theoretical model with the experimental data can be said to be satisfactory.

## Modelling changes in the bathymetry and grain size distribution of sediments which made up the bed

Changes in the bed bathymetry were modelled in two stages, i.e. excluding changes in the grain size distribution induced by sediment transport from the considerations and taking such changes into account.

The results of modelling changes in the bathymetry without taking into consideration changes in grain size distribution are shown in Figure 5. It was assumed that the bed consisted of uniform sediment. Calculations were performed for two cases, i.e. when the bed consisted exclusively of grains 0.08 mm in diameter and when the bed was composed of grains of the diameter 0.13 mm. Because finer fractions are more intensively transported than coarser ones, the result of modelling finer sediment "escapes" beyond the scope of measurements. For coarser fractions, a change in the bathymetry is evidently slower. Let us take a representative diameter, e.g. $d_{50}$ = 0.10 mm, then the consequent result of modelling will be comprised within the range of solutions obtained for the fractions 0.08 and 0.13 mm but will not agree with the experimental results, either.

Attempts to calibrate own computational models in order to "adjust" results of modelling with experimental results have been made by various research centres involved in the international project SANDPIT (VAN RIJN 2005). Models have been scaled for two approaches, *A* and *B* (WALSTRA 2005). The A problem dealt with attempts to adjust results of computations based on the measured values of transport loads, whereas the *B* approach relied on observations of the morphology of the bed. Results of modelling are presented for both approaches in Figure 6 (*A* approach) and Figure 7 (*B* approach).

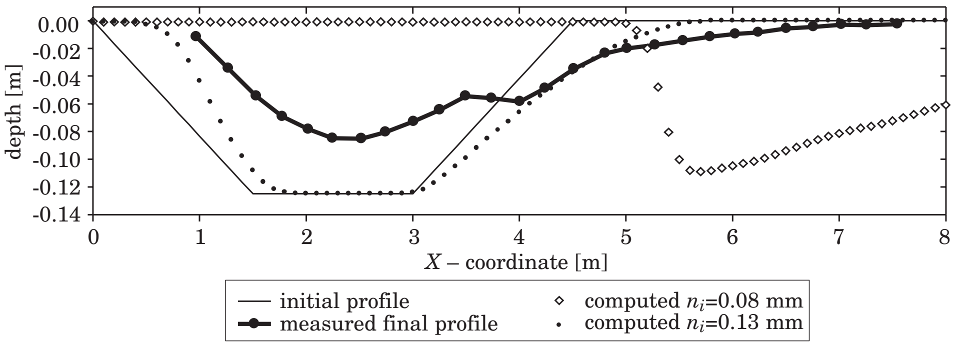


Fig. 5. Changes in the bed profile after time $T$ = 10 h. Comparison with the results of modelling for non-uniform sediment

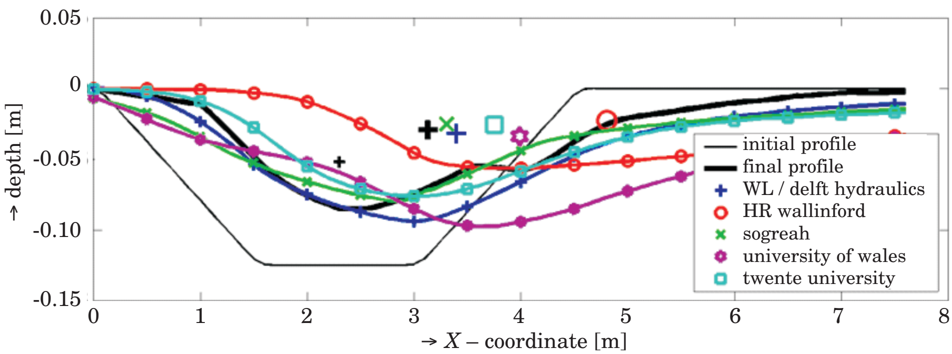


Fig. 6. Changes in the bed profile in time $T$ = 10 hrs. A approach – calibration of models based on measured transport

Source: WALSTRA (2005).

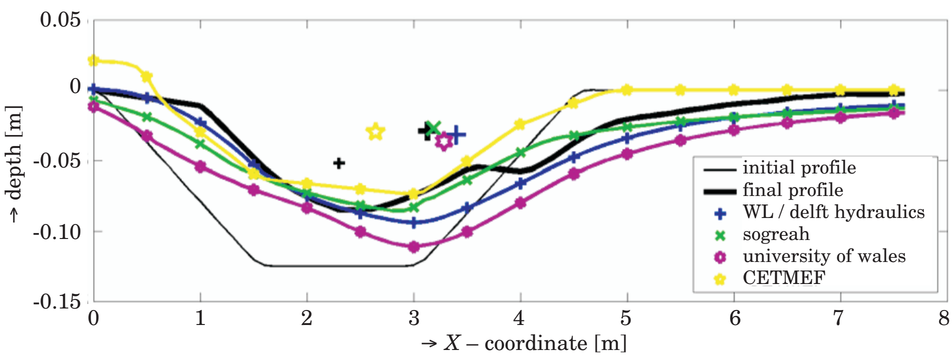


Fig. 7. Changes in the bed profile in time $T$ = 10 hrs. B approach – calibration of models based on observations of the bed's morphology

Source: WALSTRA (2005).

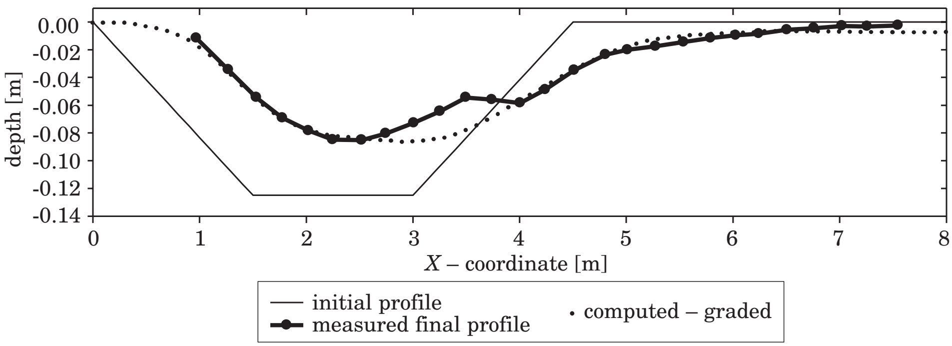


Fig. 8. Changes in the bed profile after time $T$ = 10 hrs. Comparison of the results of modelling for non-uniform sediment (including changes in the grain size distribution) with measurements

Comparison of the results of modelling changes in the bathymetry according to the theoretical model presented in this paper for non-uniform sediment, including changes in the grain size distribution is shown graphically in Figure 8. The calculations were performed for three diameters (Fig. 3). As seen from the diagram, the coincidence between modelling the results and measurements was very close.

Figure 9 shows calculated changes in the grain size distribution in the bed sediment after time $T$ = 10 h, in a longitudinal profile. In sites where sediment accumulated, a high increase in the share of finer fractions is observable, alongside a decrease in the coarser ones. the situation is reverse in eroded areas. This fact can be explained by more intensive transport of fine than coarse fractions.

Based on Figs 5, 8 and 9, a fundamental difference can be noticed in the reconstruction of the bed's profile due to transport of non-uniform and

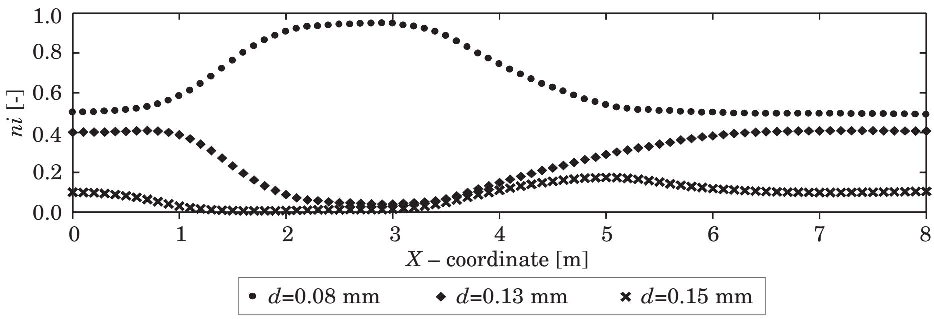


Fig. 9. Results of the modelling of grain size distribution changes after time T = 10 hrs

uniform sediments, including the influence of changes in the grain size distribution of sediments along the whole canal. It becomes evident that grading of sediments plays an important role in both sediment transport and sedimentation of a canal, not only in respect of the rate of sedimentation but also the character of changes in the canal's structure.

The agreement between the computational and experimental results can be treated as satisfactory, which proves that the computational model reflects well changes in the bed bathymetry and describes changes in grain size distribution of sediments. This, in turn, implies that the model enables us to predict accurately sedimentation of waterways.

## Summary

The purpose of this paper has been to compare the results of modelling changes in the bed bathymetry around and inside a canal, constructed previously, during its sedimentation with the results yielded by a laboratory experiment, conducted in a wave flume under wave and current conditions. The calculations concerning the bed's morphology accounted for the time and space-related changeability of the grain size distribution of transported sediments. Further, the results of computations of the distribution of vertical concentrations of suspended sediments at different depths were compared. It was demonstrated that the effect of changes in the grain size distribution of sediments occurring during the process of sedimentation of waterways is extremely important. It was also revealed that the process of sorting out of sediments should be included in the modelling of changes in the bed bathymetry occurring in time and space. The mathematical model used in the calculations was a three-layer model of transport of sediments composed of non-uniform particles, derived by Kaczmarek (1999) according to the principle of conservation of water and sediment motion amounts in the nearbed layer.

The results of modelling of the changes on the bed bathymetry, presented in this paper, alongside vertical profiles of concentrations of suspended sediments agreed very well with the results produced by a laboratory experiment. Thus, the presented model of sediment transport can be recommended as a useful tool, which seems applicable to making prognoses about changes in the bed's bathymetry within waterways or to determination of grain size distribution of sediments which fill up a waterway as well as the rate of sedimentation of approach fairways to harbours.

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