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STATISTICAL ANALYSIS OF THE FOURTH PRECISE LEVELLING CAMPAIGN IN POLAND

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K e y w o r d s: levelling networks, random errors, systematic errors.

A b s t r a c t

Paper presents statistical evaluation of accuracy of levelling network measured in Poland in years 1999–2001. The analysis was done using 16 150 misclosures from the double levelling of the sections, 382 misclosures from the double levelling of the lines and 133 loops misclosures. The statistical analysis was conducted by the regression method, correlation method and the analysis of variance. It results that the measured height differences have various accuracy (analysis of variance), and that systematic errors are changing according to the value and sign. The existence of systematic errors causes that the successive neighboring sections of some levelling lines are correlated. The correlation in the majority of the lines is not statistically essential.

STATYSTYCZNA OCENA DOKŁADNOŚCI CZWARTEJ KAMPANII NIWELACJI PRECYZYJNEJ W POLSCE

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K e y w o r d s: wyrównanie sieci, błędy przypadkowe, błędy systematyczne.

A b s t r a k t

W pracy przedstawiono statystyczną ocenę dokładności sieci niwelacji precyzyjnej pomierzonej w Polsce w latach 1999–2001. Do analizy wykorzystano 16 150 odchyłek z podwójnej niwelacji odcinków, 382 odchyłki z podwójnej niwelacji linii oraz 133 odchyłki zamknięć poligonów. Analizę przeprowadzono metodą regresji, korelacji i metodą analizy wariancji. Z przeprowadzonych analiz wynika, że pomierzone różnice wysokości mają zróżnicowaną dokładność (analiza wariancji) oraz że błędy systematyczne są różne co do wartości i znaku. Istnienie błędów systematycznych powoduje, że kolejne sąsiednie odcinki niektórych linii niwelacyjnych są skorelowane. Korelacja w większości linii nie jest statystycznie istotna.

Introduction

A priori accuracy of a levelling network traditionally is estimated by the Lallemand's and Vignal's formulas. Such estimation of all four levelling campaigns in Poland was conducted in the work (ŁYSZKOWICZ, BERNATOWICZ 2010). Since 1950s to analyze levelling data the statistical methods were employed, mainly analysis of variance. The main goal of this work is applying the statistical analysis to better understanding the behavior of the structure of the errors in the Polish precise levelling network measured in 1999–2002.

Analyzing systematic and random errors in the levelling networks one suppose that the misclosures ρ_i from forward and backward levelling can be express as a sum

$$\rho_i = \Delta_i + \varepsilon_i \tag{1}$$

where Δ_i is a systematic error and ε_i is a random error.

Random errors vary from setup to setup of a instrument and their stochastic character can be express by

$$E(\varepsilon_i) = 0, \quad E(\varepsilon_i^2) = \sigma_\varepsilon^2, \quad E(\varepsilon_i \varepsilon_j) = 0 \tag{2}$$

The systematic errors are equally or similarly contained in some or all heights differences. If these errors are also considered as a stochastic values, their characteristics can be described as

$$E(\Delta_i) = \mu_\Delta, \quad E(\Delta_i^2) = \sigma_\Delta^2, \quad E(\Delta_i \Delta_j) = \mathrm{cov}(\Delta_i \Delta_j) \neq 0 \tag{3}$$

According to equations (2), (3) the main difference between systematic and random errors is in the covariance does not equal zero. Systematic errors lead to correlations between height differences.

This paper presents a comprehensive discussion of regression analysis, analysis of variance and analysis of correlation applied to the fourth precise levelling campaign in Poland. The regression analysis was used to show how systematic errors affecting levelling lines. Analysis of variance shows that the systematic effects are different for each line and analysis of correlation was used to show the degree of correlations between the neighboring sections.

First time the analysis of variance was applied to examination of the levelling network in the Nile Delta (WASSEF 1955) and concerned rather small network consisting of 2450 km. In Poland analysis of variance first time was applied to analysis of levelling network by (TYRA 1983). He conducted analysis of levelling network measured in 1953–1955 which was composed of 10 303 sections. The basic defect of this work is that the author analyzed misclosures $\frac{\rho}{l}$, which have different accuracy and do not fulfill the assumption of analysis of variance.

In the paper (LEWANDOWICZ 1994) the estimation of the sources of errors and their influence on the results of levelling measurements was done. The evaluation of the observation was done with the use of statistical methods such as: the method of empirical moments, statistical tests and the method of the estimation of empirical distribution function. In practical calculations the results of the levelling measurements from years 1974–1979 were used. Similarly as in the previous work in this study the misclosures $\frac{\rho}{l}$ were used, which have no the same accuracy. Similar defect is in the paper (ŁYSZKOWICZ, LEOŃCZYK 2005), where to the levelling network measured in 1999–2001, in the analysis of variance, the misclosures $\frac{\rho}{l}$ were used incorrectly.

Below is described such a method of normalization of the misclosures ρ, λ and φ which removes the dependence from the length, and thus gives the misclosures with the same accuracy.

If we assume that the result of forward δH^g and backward δH^p measurement of the levelling section is a stochastic variables with the normal distribution $N(\mu, \sigma)$ then the expected value of measured heights differences is equal:

$$E(\delta H^g) = E(\delta H^p) = \mu \tag{4}$$

and the variance of height difference is:

$$\mathrm{var}(\delta H^g) = \mathrm{var}(\delta H^p) = \sigma_o^2 l \tag{5}$$

where l is a length of levelling line and σ_o^2 is a variance of height differences of the section1 km long from the survey in one direction.

If we consider the misclosures ρ which is the difference from forward and backward measurement, then its expected value is equal:

$$E(\rho) = 0 \tag{6}$$

and variance of such stochastic variable ρ is:

$$\mathrm{var}(\rho) = \mathrm{var}(\delta H^g) - 2\ \mathrm{cov}\ (\delta H^g,\ \delta H^p) + \mathrm{var}(\delta H^p) \tag{7}$$

where cov $(\delta H^g, \delta H^p)$ denoted covariance of the variables δH^g and δH^p.

If we assume that the random variables are independent then the covariance will be equal zero and all variances will be identical and the equation (7) will be reduce to the form:

$$\mathrm{var}(\rho) = 2\ \mathrm{var}(\delta H) = 2\sigma_o^2 l \tag{8}$$

To compare misclosures ρ counted from sections measurements with various lengths, one should standardize them in the following way:

$$\rho_u = \frac{\rho}{\sqrt{2l}} \tag{9}$$

since the expected value of such standardized misclosures ρ_u is equal zero:

$$E(\rho_u) = 0 \tag{10}$$

while the variance is:

$$\mathrm{var}(\rho_u) = \frac{1}{2l}\ \mathrm{var}(\rho) = \frac{1}{2l}\ 2\sigma_o^2 l = \sigma_o^2 \tag{11}$$

for all the lengths l of the sections of the levelling network.

In a case of the line misclosures λ, which is a sum of section misclosures ρ consisting of given line, the variance of λ is the sum of all variances ρ:

$$\mathrm{var}(\lambda) = 2\sigma_o^2 l_1 + 2\sigma_o^2 l_2 + \ldots 2\sigma_o^2 l_n = 2\sigma_o^2 L \tag{12}$$

where L is a line length.

Standardized discrepancy λ_u is defined in the following way:

$$\lambda_u = \frac{\lambda}{\sqrt{2L}} \tag{13}$$

because only variance of such random variable is equal σ_o^2.

If a few lines create the loop then the sum of mean height differences, corrected due to gravity and other systematic factors should be equal zero. However in practice, one receives the certain loop misclosures which can be expressed by the formula:

$$\varphi = \sum_{i=1}^{n} \Delta H_i^{śr} \tag{14}$$

where n is a number of levelling traverses. Variance of a random variable φ be:

$$\mathrm{var}(\varphi) = \frac{1}{2}\,\sigma_o^2 l_1 + \frac{1}{2}\,\sigma_o^2 l_2 + \ldots = \frac{1}{2}\,\sigma_o^2 \,\Sigma l_i = \frac{1}{2}\,\sigma_o^2 F \tag{15}$$

where F is the length of levelling traverse and σ_o^2 is variance of height differences 1 km length measured in one direction. Standardized misclosures φ_u is defined in the following way:

$$\varphi_u = \varphi\sqrt{\frac{2}{F}} \tag{16}$$

since the variance of such variable is equal σ_o^2 for all length F of levelling traverses.

The fourth precise levelling campaign in Poland

The fourth precise levelling campaign in Poland started in 1999 and was finished in 2003 (Fig. 1). The network consists of 16 150 sections with average length 1.1 km, 382 lines with average length about 46 km, 135 loops, and 245 nodal points. Total length of levelling lines is 17 516 km. The levelling lines were measured with Zeiss Ni002 (66% measurements), Zeiss DiNi 11 (31% measurements), Topcon NJ (3% measurements) e.g. (PAŻUS 2001). Rod scale corrections, rod temperature corrections, tidal corrections and normal Molodensky corrections were introduce to the raw measured of height differences.

From the field measurements we have 16 150 section discrepancies ρ, 382 line discrepancies λ and 133 loop misclosures φ. In table 1 is given statistical character of these misclosures.

On the basis of the misclosures ρ, λ and φ the accuracy of the fourth levelling campaign was evaluated using so called "old" formulas and the

Fig. 1. The levelling network of the fourth campaign

following results were obtained. The root mean square error m_1 of height differences counted from misclosures ρ is ±0.278 mm, the root mean square error m_2 of height differences counted from misclosures λ is ±0.519 mm and the root mean square error m_3 of height differences counted from misclosures φ is ±0.826 mm (BERNATOWICZ 2010).

Table 1

Statistical character of misclosures ρ, λ, φ

Specification	ρ	λ	φ
Number of discrepancies	16 150	382	133
Mean [mm]	+0.07	2.74	0.27
Std dev. [mm]	±0.58	6.89	12.54
Min [mm]	-1.83	-20.41	-31.49
Max [mm]	1.82	20.83	28.83
Skewness	0.08	-0.40	0.21
Kurtosis	-0.01	0.30	-0.10

The first, the simplest assessment of a successful network adjustment of the fourth campaign is described in (ŁYSZKOWICZ, JACKIEWICZ 2005). The adjustment of the network was done as the minimally constrained adjustment and the standard deviation of height differences equal ±0.88 mm was obtained. Identical evaluation of the accuracy of the campaign IV was obtained in the network adjustment carried out in the study (GAJDEROWICZ 2005).

Statistical analysis of standardized discrepancies ρ_u, λ_u and φ_u

In order to perform the statistical analyses, the discrepancies ρ, λ and φ were normalized according to formulas (9), (13) i (16). The results of calculations are given in table 2.

Table 2

Statistical characteristic of normalized discrepancies ρ_u, λ_u i φ_u (BERNATOWICZ 2010)

Specification	ρ_u	λ_u	φ_u
Number of discrepancies	16 150	382	133
Mean [mm]	0.05	0,28	0.02
Std dev. [mm]	±0.39	±0.68	±1.17
Min [mm]	-1.06	-1.86	-2.62
Max [mm]	1.13	1.55	2.64
Skewness	-0.10	-0.58	0.18
Kurtosis	-0.66	-0.06	-0.16

Statistical hypothesis test was conducted to verify if the mean of discrepancies is not significantly different from zero. To see whether the mean of the discrepancies is significantly different from zero, the student t test was applied to each mean at the 0.05 level of significance to test the null hypothesis H_0: $\mu = \mu_0$ against alternative hypothesis H_1: $\mu \neq \mu_0$. The variable t_0 may be computed from (MIKHAIL 1976):

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \tag{17}$$

Rejection of the hypothesis H_0 on the level of significance α on the advantage of the alternative hypothesis H_1: $\mu \neq \mu_0$, follows when $|t_0| \geq t_{\alpha/2}$.

In the case of testing the significance of the mean value of the discrepancies ρ_u we have: $\bar{x} = 0.05$ mm, $s = \pm 0.39$ mm, $n = 16\ 150$, $\mu = 0$. Computed value of

t_0 is 16.29 and critical value of t at α = 0.05 and 16 149 degree of freedom is 1.96, therefore the hypothesis that the mean value is equal 0.05 mm should be accepted. In the case of mean value of discrepancies λ_u we have t_0 = 8.04 and critical value of t is 1.97 what indicate that the hypothesis that mean value is equal 0.28 mm should be accepted. In the last case, if we consider the mean value of discrepancies φ_u we have t_0 = 0.20 and because the critical value of t is 1.98 the hypothesis that the mean value is zero should be accepted.

Table 3

The results of goodness fit of the empirical expansion of discrepancies ρ_u with normal expansion

Line	Test χ^2			Line	Test χ^2		
	χ^2 practical	degree of freedom	χ^2 theoretical		χ^2 practical	degree of freedom	χ^2 theoretical
29	9.41	3	7.81	141	4.05	4	9.49
32	7.44	4	9.49	147	2.22	4	9.49
36	0.42	3	7.81	154	7.66	5	11.07
41	2.10	2	5.99	170	5.17	6	12.59
44	3.79	6	12.59	192	2.35	4	9.49
54	2.42	5	11.07	196	3.67	5	11.07
92	7.49	3	7.81	198	3.45	5	11.07
97	5.10	5	11.07	206	3.10	4	9.49
118	7.88	6	12.59	217	4.65	5	11.07
119	3.82	7	14.07	225	1.35	2	5.99
121	2.37	7	14.07	236	2.67	6	12.59
131	2.73	4	9.49	284	2.70	4	9.49
133	7.07	7	14.07	319	1.19	3	7.81
135	3.61	7	14.07	335	4.13	5	11.07
137	2.93	6	12.59	337	7.42	6	12.59

Analysis of variance assumed that discrepancies ρ_u of a levelling line should have normal distribution. The investigation of the goodness of fit of the empirical expansion of discrepancies ρ_u with the normal expansion was conducted using the χ^2 test. The results of computation are given in table 3. From table 3 is seen that practical statistics χ^2 are always smaller than the theoretical (with except of the line 29). This means that discrepancies ρ_u of examine levelling lines have the normal expansion and that main assumption of the analysis of variance is fulfilled.

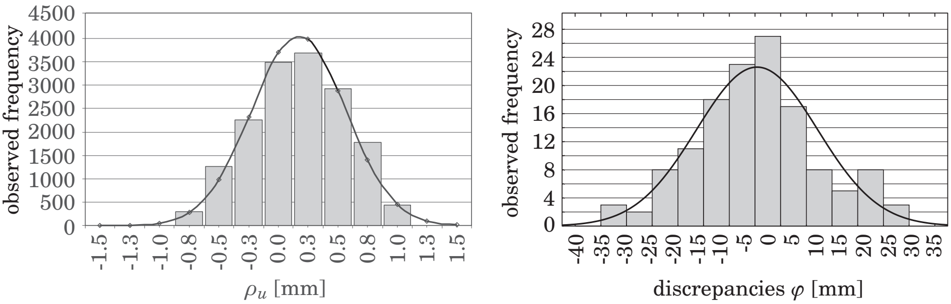

Fig. 2. Histogram of 16 150 standardized discrepancies ρ_u (χ^2_{pr} = 263.0, χ^2_{teor} 12.6), histogram of 133 standardized discrepancies φ_u ($\chi^2{}_{pr}$ 8.52, $\chi^2{}_{teor}$ 12.6)

The computed values of skewness and excess (table 2) show that the data sets of discrepancies ρ_u and λ_u created for the whole levelling network do not have the normal expansion, while the set of discrepancies φ_u should rather have the normal expansion. Investigations conducted with the use of the χ^2 test confirm these presumptions. The lack of the fit of the sets of empirical distributions of ρ_u and λ_u with the normal expansion means that discrepancies ρ_u and λ_u are apparently affected by systematic errors. Only empirical distribution of discrepancies φ_u fits to the normal distribution (Fig. 2).

Linear regression

Linear regression can be used to find the function relating two or more variables. In our case the values of d (distance) are regarded as fixed (error free) and the values of successive $\Sigma\rho$ are the measured values (see Fig. 3).

From this drawing results, that the empirical graph of successive sums can be approximated by the straight the line. It inclination gives evaluation of the systematic error on 1 km of the levelling line. In the considered case this systematic error is 0.09 mm/1 km.

In the way described above the systematic errors were computed for 30 lines (Tab. 4). The lines chosen to the analysis have the length more than 75 km. The average length of the lines is 82 km, and the average number of sections in a line is 70. Average systematic error computed for this set of lines is 0.09 mm/1 km and accuracy estimation of this error is very credible because its value is 10 times smaller than the value of the error itself.

From (Tab. 4) results, that the systematic errors of the measured height differences consisting of the levelling line are not typical systematic errors and they are changing in a size as in sign. One can conduct the similar analysis for discrepancies λ, but the investigation of the sum of discrepancies λ have no sense and therefore the direct dependence of discrepancies λ are studied in the respect to a length L of a line.

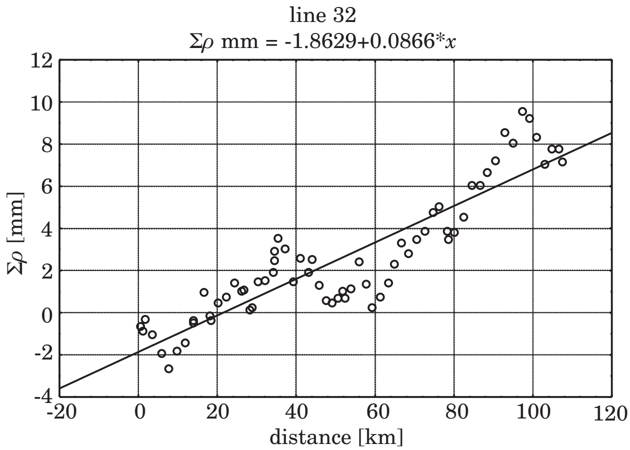

Fig. 3. Increasing, along the line 32, sum of the discrepancies ρ

Table 4

Computed systematic errors for the selected 30 lines of the levelling network (see Fig. 1)

Line	Number of section	Length of line in km	Systematic error and its accuracy in mm		Line	Number of section	Length of line in km	Systematic error and its accuracy in mm	
			δ	m_δ				δ	m_δ
29	68	82.03	0.17	0.01	141	79	95.29	0.07	0.02
32	67	107.41	0.09	0.01	147	100	83.13	0.09	0.01
36	59	76.57	0.04	0.005	154	69	76.07	0.06	0.00
41	60	82.53	0.10	0.004	170	102	92.32	0.11	0.00
44	75	91.93	0.18	0.01	192	76	79.57	0.15	0.00
54	56	77.51	-0.08	0.01	196	65	81.82	0.14	0.00
92	58	80.84	0.05	0.01	198	61	98.99	0.11	0.01
97	67	71.59	0.10	0.01	206	66	88.49	-0.29	0.01
118	67	83.73	0.15	0.004	217	59	83.85	0.11	0.01
119	82	96.31	0.21	0.01	225	76	79.01	0.03	0.01
121	83	83.53	0.10	0.01	236	66	82.18	0.24	0.01
131	52	63.81	0.11	0.01	284	76	87.55	0.13	0.01
133	87	88.56	-0.15	0.003	319	60	76.79	0.23	0.01
135	76	85.07	0.14	0.01	335	78	87.60	-0.13	0.00
137	77	90.48	0.17	0.01	337	78	79.26	0.21	0.01

On the figure 4 is shown the graph of the 383 discrepancies λ in respect to the length of the levelling lines. From figure results that the longer is line the larger value of discrepancies it has, which can be interpreted as accumulation of the successive systematic errors. In our case the estimated systematic error from discrepancies λ, evaluated by the regression line, is (0.07±0.02) mm/km.

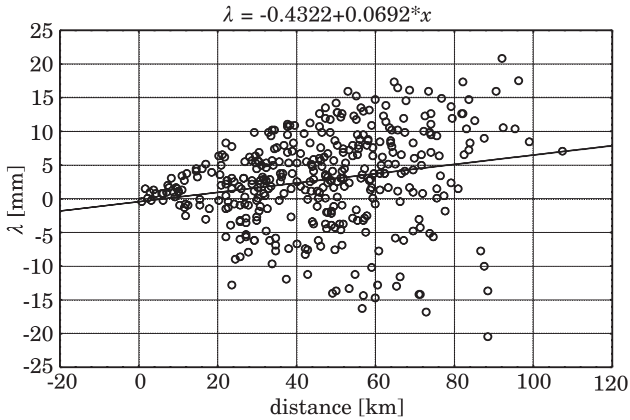

Fig. 4. Graph of the 383 discrepancies λ in respect to the length of the levelling line

The systematic error of the levelling network also can be estimated the by the method of linear regression from the discrepancies φ of the loop misclosures (Fig. 5).

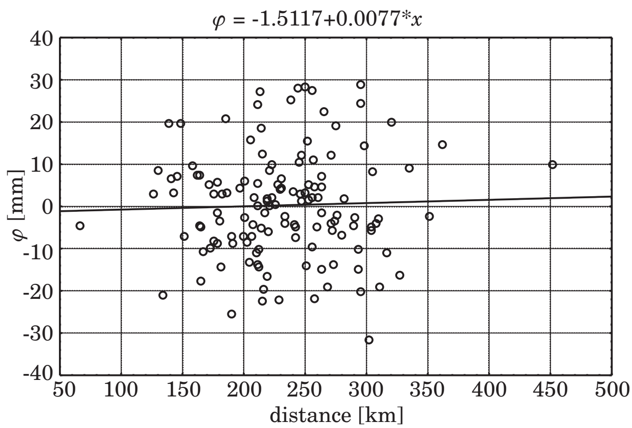

Fig. 5. The graph of 133 discrepancies φ in respect to the length of the loop

From figure 5 results that there is no practically dependence between the length of the loop and the discrepancies φ.

From conducted in the present chapter investigations results, that observations of height differences of a sections (average length of a section 1.1 km) are affected by the systematic errors considerably. Observations of height differences of a line (average length of the line 46 km) are affected less by systematic errors and the levelling loops where the average length is 232 km, do not practically are affected by the systematic errors. This means that systematic errors behave initially as traditional systematic errors and than they increase and after crossing of certain length of the levelling line they behave as accidental errors and in adding up give zero.

Correlation

It was showed in the previous chapters that the measured height differences of a sections were affected by the systematic errors. These errors can cause correlations between the successive sections of a line what results from the formula (3). Determination of these correlations will be conducted below.

In the work (LUCHT 1983) and (BERNATOWICZ 2010) the correlation of neighboring sections is defined in the following way:

$$r_t = \frac{\sigma_{Di}^2}{2\sigma_\rho^2} - 1 \tag{18}$$

where variances σ_{Dt}^2 i σ_ρ^2 were estimated as:

$$s_{Dt}^2 = \frac{\Sigma D_t^2}{n_{Dt}} \qquad s_\rho^2 = \frac{\Sigma \rho^2}{n} \tag{19}$$

and D_t is define as

$$D_t = \rho_i + \rho_j \quad \text{dla} \quad t = j - i = 1,2,\ldots, n - 1 \tag{20}$$

In the present work the coefficient of the correlation r_1 was computed for 30 test lines given in table 4. The analysis was conducted for the normalized discrepancies ρ_u according to the formula (9). On the basis of so normalized discrepancies the sums describe by the formula (20) were created and empirical standard deviations s_{Dt}^2 and s_ρ^2 were computed and then the empirical coefficients of the correlation r_1.

Table 5

Computed coefficients of correlation r_1 for the individual levelling lines

Line	r_1	z	Line	r_1	z	Line	r_1	z
29	**0.24**	**1.97**	121	0.13	1.17	196	-0.14	-1.11
32	-0.01	-0.08	**131**	**0.31**	**2.24**	198	-0.13	-1.00
36	0.19	1.44	**133**	**0.31**	**2.94**	**206**	**0.44**	**3.75**
41	-0.18	-1.37	**135**	**0.25**	**2.18**	217	0.004	0.03
44	0.14	1.20	**137**	**0.24**	**2.11**	225	0.09	0.77
54	-0.13	-0.95	**141**	**0.33**	**2.99**	**236**	**0.33**	**2.72**
92	0.10	0.74	147	0.13	1.29	**284**	**0.27**	**2.37**
97	-0.17	-1.37	**154**	**-0.28**	**-2.34**	319	0.16	1.22
118	0.19	1.54	170	0.18	1.81	335	-0.15	-1.31
119	0.10	0.89	192	-1.79	-0.21	337	0.01	0.09

The question comes into being or the counted correlation coefficients r_1 are essential from the statistics point of view. To check this condition the test described in (MIKHAIL 1976) was applied and the hypothesis H_0: $r_1 = 0$ in respect to the hypothesis H_1: $r_1 \neq 0$ was applied. For large n variable

$$z = \frac{\sqrt{n-3}}{2} \ln\left(\frac{1+r}{1-r}\right) \tag{21}$$

has the normal distribution and the hypothesis H_0 is accepted when

$$-z_{\alpha/2} < z < z_{\alpha/2} \tag{22}$$

When H_0 is rejected the two random variables in question are said to be significantly correlated at the level α. In our case variable z should be inside the interval -1.93 and +1.93 if the level $\alpha = 0.05$ is assumed. From the table 5 it is seen that in the case of the ten lines (bold letters) the correlation coefficients are between 0.25 and 0.44 and are statistically significant. The remaining twenty lines has correlation coefficients less than 0.25 and they are no significant.

Analysis of variance

Various systematic factors act on the measurements of the height differences of the different lines and one can conclude about it from the analysis variance of discrepancies ρ_u. The analysis of variance serves to the verification of the hypothesis about the equality of a mean value of a tested samples by comparison their variances. In this method the tested sample of the size n is divided into m groups. In our case tested sample contains all discrepancies ρ_u of the forward and backward levelling of a sections in the network and the levelling lines form a groups.

One verifies the hypothesis about the equality of a mean values compute for each group (levelling line). If this hypothesis is true, then all tested samples come from the same expansion (BRANDT 2002) what in the case of levelling means that the same systematic error are present in all measurements.

Computational diagram given in (BRANDT 2002, p. 456) for 382 levelling line was applied to verify the hypothesis about equality of mean value of forward and backward levelling of a levelling sections. Discrepancies were normalized according to the formula (9). The whole calculations was executed in Excel. Suitable numerical values are written down in table 6.

Table 6

Comparison of the results obtained from the analysis of variance for discrepancies ρ_u

Variance [mm²/km]		Sum of squares	Degree of freedom	$F > F_{1-\alpha}$
Between lines	0,434	165,47	381	3,02 > 1,12
Within lines	0,144	2268,85	15 768	
Total	0,151	2434,32	16 149	

From table 6 results, that for the studied sample of discrepancies ρ_u on the level of significance $\alpha = 0.05$, statistic F is 3,018 while its theoretical value $F_{1-\alpha}$ for $k_1 = 380$ and $k_2 = 15\ 767$ is 1,124. This means, that the variances are not equal, because of different systematic errors presented in the analyzed observations. The results of analyses, show on inhomogeneity of the accuracy of the analyzed network. Such heterogeneity of the network also shows that studied sample of all 16 150 discrepancies does not possess normal expansion (see chapter 3).

Summary

Discrepancies ρ, λ and φ from the double levelling of a sections, lines and the loop closures have no the same accuracy because they are dependent on the length. Therefore one should apply formulas (9), (13) and (16) to normalized them. The empirical expansion of discrepancies ρ_u consisting of individual levelling lines is characterized by normal expansion (chapter 3) which make possible applying the analysis of variance to study the levelling network. However the empirical expansion of the discrepancies ρ_u of the whole levelling network does not show the normal expansion. Similarly the empirical expansion of all discrepancies λ_u have no the normal expansion, while the empirical expansion of all deviations φ_u has the normal expansion.

From the conducted linear regression results, that lines clearly are affected by systematic errors, but these errors are not typical systematic errors and they are changing to size and sign.

The occurrence of systematic errors causes that the successive measurements of the height differences of the sections are correlated (chapter 5). It results that the degree of the correlation is different to the value (from 0.01 to 0.44) and the sign and, that the majority of the correlations are not essential from the statistical point of the view.

The analysis of variance confirms that the accuracy of the measurements of the height differences in the whole network are not homogeneous what means various systematic factors in the individual lines.

From the carried out analysis results that in a *a priori* estimation of accuracy of levelling network the largest difficulty is proper definition of the systematic errors and their influence on the accuracy of a levelling networks.

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