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DESCRIBING OF GENERALIZED DRYING KINETICS WITH APPLICATION OF EXPERIMENT DESIGN METHOD

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Abstract

The purpose of this article is an application of experiment design method for describing of generalized drying kinetics. Generalized kinetic equation for the first drying period as the linear dependence of the material moisture content vs. drying time and temperature is obtained. For determination of generalized kinetic equation the limited number of experimental data (only two moisture contents of a material for two temperatures of drying agent) is needed. The comparison of calculations of generalized kinetic equation with experimental data for convection drying of sludge and for convectional drying of cotton fabric in the first drying period are fulfilled. The mathematical modification of the two-period model of drying kinetics over the entire drying process is obtained. This modified model permits to avoid of the determination of a characteristic drying time. The comparison of drying kinetic calculations with experimental data for convection drying of sludge in the first and second drying periods for four temperatures using of experiment design method was fulfilled. As follows from calculations, the experimental data of sludge correspond well to the lines obtained through the factorial design which means that this method can successfully be used to determine constant drying rates, total kinetics and the influence of temperature in drying process.

Nomenclature

E – Dimensionless drying time
N_0 – Initial drying rate
t – Drying air temperature
X_0 – Initial moisture content (dry basis)
$X(\Theta)$ – Moisture content (dry basis)
X_{eq} – Equilibrium moisture content

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Greek letters

θ – Drying time

θ_f – Total (final) drying time

σ – Characteristic drying time

Introduction

For the description of drying kinetics it is required to establish an appropriate mathematical model and to find numerical values for the model parameters. Drying of different materials is almost always treated empirically in the primary literature, usually by regression on experimental data or by empirical equations, par example, in exponential form (so-called Page model) (CHEN, SCHMIDT 1990, PERKIN 1990, KEMP et al. 2001, MUJUMDAR 2007). Several equations available in the literature for explaining drying behavior of different products have been used (FROLOV 1987, CHEN, SCHMIDT 1990, PERKIN 1990, EFREMOV 1998, 2000, 2001, 2002, KEMP et al. 2001, CHEN et al. 2002, BENALI, KUDRA 2002, REYES et al. 2004, MUJUMDAR 2007).

Another approach to determine the drying kinetics includes the using of experiment design method (BOX et al. 2005, EFREMOV, KUDRA 2011, EFREMOV 2012, MONTGOMERY 2012). As an initial model, the results of experiment often serve that represent a set of the several measurements executed according to a certain plan. In the elementary case this plan is built on the description of conditions to perform measurements that is the set of values of entrance parameters (factors).

In statistics, a full factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible values or ;levels;, and whose experimental units take on all possible combinations of these levels across all such factors (BOX et al. 2005, MONTGOMERY 2012). Such an experiment allows studying the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable.

In this study an application of experiment design method for describing of drying kinetics of sludge and cotton fabric is considered. Generalized kinetic equation for the first drying period as the linear dependence of the material moisture content vs. drying time and temperature is obtained. For determination of coefficients of this equation the limited number of experimental data (only two moisture contents of a material for two temperatures of drying agent) is needed. The comparisons of calculations of generalized kinetic equation with experimental data for convection drying of sludge and for convectional drying of cotton fabric in the first drying period are considered. The mathematical modification of the two-period model of drying kinetics

(Efremov 1998, 2000, 2002, Chen et al. 2002) over the entire drying process is also proposed.

The comparison of drying kinetic calculations with experimental data for convection drying in the first and second drying periods for four temperatures using experiment design method was fulfilled. The comparison of calculations with experiments shows that the deviations of calculations from experience did not exceed 3.8%.

Materials and Methods

Kinetics of drying

Drying kinetics in the first period is described by linear dependence of the volume-averaged moisture content of a material X from time of the process θ (Frolov 1987, Chen, Schmidt 1990, Perkin 1990, Efremov 1998, 2000, 2001, 2002, Kemp et al. 2001, Chen et al. 2002, Benali, Kudra 2002, Reyes et al. 2004, Mujumdar 2007). Linear dependence of moisture content is successfully applied to describe convective and microwave drying of hygroscopic (Chen, Schmidt 1990) and non hygroscopic (Perkin 1990) capillary-porous materials, such as polymer pellets, glass beads and alumina spheres. Linear drying kinetics in the first period is used also for description of drying of fine-dispersed macro porous materials (Frolov 1987). The proposed two-period model (Efremov 1998, 2000, 2002) is suitable when drying takes place in both constant rate (first) and falling rate (second) periods of drying. Though the model was developed for convective drying of capillary-porous materials such as leather, paper, cotton, fibre and peat (Efremov 2000) it was successfully applied to describe drying of organic materials (Efremov 1998, 2002, Chen et al. 2002, Benali, Kudra 2002). Two-period model was obtained as summation of two solutions of diffusion process (isotropic flat material with uniform distribution of initial moisture content – Efremov 2000). First solution of the diffusion process was obtained on the basis of constant initial drying rate N_0 and the second solution was obtained in the form of function of integral of errors with modification for volume-averaged moisture content (Efremov 1998, 2000). This two-period model in form of ration of time-dependant and initial volume-averaged moisture content is describing by the following equation (Efremov 1998, 2000, 2001, 2002, Chen et al. 2002, Benali, Kudra 2002, Reyes et al. 2004):

$$\frac{X(\theta)}{X_0} = \left(1 - N_0 \frac{\theta}{X_0}\right) + \frac{N_0 \sigma \sqrt{\pi}}{2X_0} \operatorname{erfc}\left(\frac{\theta_f - \theta}{\sigma}\right) \tag{1}$$

where: X is the volume-averaged moisture content (kg/kg, db), X_0 – the initial moisture content, θ – time of drying, θ_f – the elapsed time of drying, σ – the characteristic drying time which is constant for given process conditions.

The first term in Equation (1) represents the relatively long first drying period characterized by a constant drying intensity $N_0 = \mathrm{d}X/\mathrm{d}\theta$. The second term reflects the macroscale nonstationary diffusive moisture transport in a drying material, obtained by applying the Laplace transform method to the equation of isotropic diffusion with boundary conditions in the form of a constant concentration on the material surface (FROLOV 1987, EFREMOV 1998, 2000, REYES et al. 2004). Accepting that the moisture content at the end of drying approaches an equilibrium value X_{eq}, equation (1) becomes

$$\frac{X_{eq}}{X_0} = \left(1 - N_0 \frac{\theta}{X_0}\right) + \frac{N_0 \sigma \sqrt{\pi}}{2X_0} \tag{2}$$

Hence from Equation (2) the characteristic drying time is

$$\sigma = \frac{2}{\sqrt{\pi}} \left(\theta_f - \frac{X_0 - X_{eq}}{N_0}\right) \tag{3}$$

Using of equation (3) leads to the following mathematical modification of the two-period model (1):

$$\frac{X(\theta)}{X_0} = 1 - \frac{N_0}{X_0} \cdot \left[\theta - \left(\theta_f - \frac{X_0 - X_{eq}}{N_0}\right) \cdot \operatorname{erfc}\left(\frac{N_0\sqrt{\pi}}{2} \cdot \frac{\theta_f - \theta}{N_0 \cdot \theta_f - X_0 + X_{eq}}\right)\right] \tag{4}$$

The drying kinetics can be generalized using the following dimensionless equation (EFREMOV 2001):

$$K = \frac{X - X_{eq}}{\sigma N_0} = \frac{\theta_f - \theta}{\sigma} - \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{\theta_f - \theta}{\sigma}\right) = E - \frac{\sqrt{\pi}}{2} \operatorname{erf}\,(E) \tag{5}$$

where: $E = (\theta_f - \theta)/\sigma$ represents the dimensionless drying time.

Usually initial and equilibrium moisture content, elapsed time of drying are the experimentally determined drying parameters. The main problem is determination of N_0 – the constant drying rate in the first period of drying as a function of drying time θ and process temperature t.

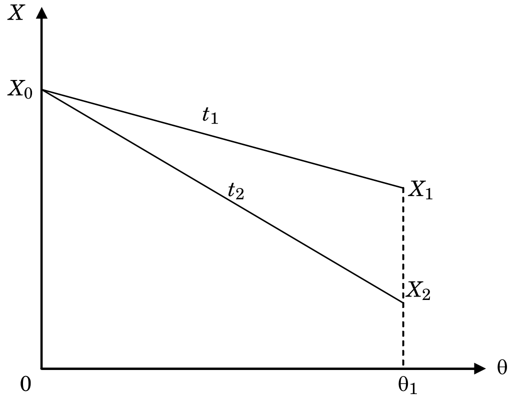

Fig. 1. Dependence of average moisture content of a material X from time θ in the first period of drying for two temperatures

Drying kinetics in the first period is described by linear dependence of moisture content of a material X from time of the process θ (FROLOV 1987, CHEN, SCHMIDT 1990, PERKIN 1990, EFREMOV 1998, 2000, 2001, 2002, KEMP et al. 2001, CHEN et al. 2002, BENALI, KUDRA 2002, REYES et al. 2004, MUJUMDAR 2007). In Figure 1 the example of such dependence for drying agent of two temperatures $t1$ and $t2$ is submitted. The lines of drying kinetics pass from the point of total initial moisture content $X0$ (at $\theta 0 = 0$) up to points corresponding of final values $X1$ and $X2$, appropriate to the minimal and maximal values of temperature of the drying agent $t1$ and $t2$ in experiment (Fig. 1).

Experiment design method

For mathematical description of the dependence of moisture content from time and temperature of the process can be used experiment design method (BOX et al. 2005, MONTGOMERY 2012). To obtain the mathematical description of such model the plan of first order can be used. For the vast majority of factorial experiments, each factor has only two levels. For example, with two factors each taking two levels, a full factorial experiment would have four treatment combinations in total, and is usually called a 2×2 (or 2^2) factorial design. Three factors, each assuming two levels give 2×3 (or 2^3) factorial design which generates 8 combinations and so on. If the number of combinations in a full factorial design is too high to be logistically feasible, a fractional factorial design may be used, in which some of the possible combinations (usually at least half) are omitted.

For an illustration of this process, the estimation of parameters is interesting from the practical point of view, the calculation of temperature dependence for the first period of drying can be considered also.

Let's consider at first the solution in a general view. The process of convective drying of a material at constant drying rate depends mainly on external mass transfer and it can be described by temperature of hot air t and time of process θ.

The necessary number of full factorial experiment would have four combinations $2^2 = 4$. The drying kinetics are describing by changing of moisture content from initial value X_0 (at $\theta_0 = 0$) up to final values X_1 and X_2, appropriate to the minimal and maximal values of temperature of the drying agent t_1 and t_2 (Fig. 1).

The matrix of full factorial experiment planning for coded variable of two factors in view of their double interaction (BOX et al. 2005, MONTGOMERY 2012). To save space, the points in a two-level factorial experiment are often abbreviated with strings of plus (+) and minus (–) signs. The strings have as many symbols as factors, and their values dictate the level of each factor; conventionally, "–" for the first (or low) level, and "+" for the second (or high) level. The plan of this two-level factorial experiment is presented entered in the Table 1. Parameter y represents the current material moisture content X; x_1 and x_2 are two coded variables of entrance parameters (factors) – temperature of hot air t and drying time θ; b_0, b_1, b_2 and b_{12} are the coefficients of regression equation (6). The value of temperature at the centre of the plan we shall designate t_0, interval of a temperature variation δ_1, accordingly the values for time of the process – θ_0 and δ_2.

Table 1

Matrix of full factorial experiment for coded variable of two factors

x_0	x_1	x_2	$x_1 \cdot x_2$	y
+	–	–	+	y_0
+	+	–	–	y_0
+	–	+	–	y_1
+	+	+	+	y_2

The appropriate regression equation has the following form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 \tag{6}$$

The coefficients of the regression equation (6) can be calculated from the formula (EFREMOV 2001, 2012, EFREMOV, KUDRA 2011, BOX et al. 2005)

$$b_i = \frac{\sum_1^4 x_i y_j}{4} \tag{7}$$

Then, the coefficients of regression in view of given matrices of planning (Table 1)

$$b_0 = \frac{y_0 + y_0 + y_1 + y_2}{4}, \ b_1 = \frac{-y_1 + y_2}{4}, \ b_2 = \frac{-y_0 - y_0 + y_1 + y_2}{4}, \ b_{12} = \frac{-y_1 + y_2}{4} \tag{8}$$

Let’s substitute values of the coefficients in the regression equation (6)

$$4y = (y_0 + y_0 + y_1 + y_2) + (y_1 + y_2)x_1 + (-y_0 - y_0 + y_1 + y_2)x_2 + (-y_1 + y_2)x_1x_2 \tag{9}$$

The transitions from coded variables to initial parameters are

$$x_1 = \frac{t - t_0}{\delta_1}, \ x_2 = \frac{\theta - \theta_0}{\delta_2}, \ y_0 = X_0, \ y_1 = X_1, \ y_2 = X_2, \ \theta_0 = \delta_2 \tag{10}$$

After transformations, finally the generalized kinetic equation for the first drying period can be obtained

$$X = X_0 - \frac{2X_0 - X_1 - X_2}{4} \cdot \frac{\theta}{\delta_2} - \frac{X_1 - X_2}{4} \cdot \frac{t - t_0}{\delta_1} \cdot \frac{\theta}{\delta_2} \tag{11}$$

It is necessary to note, that the influence of temperature on a drying process is taken into account only by the last member of double interaction of the influencing factors because the second term of right side of equation (6) equals to 0.

The first two experiments (Table 1) correspond to initial condition of a material ($\theta = 0$ and moisture content X_0). The plan of a two-level full factorial experiment indicates that it is necessary to carry out only 2 experiments (X_1 and X_2) to determine the linear dependence of the material moisture content vs. drying time and temperature.

Results and Discussion

The 2^2 factorial experiment is exemplified in this paper for the process of convection drying of sludge from a municipal sewage treatment plant in a laboratory drying tunnel with parallel airflow at different temperatures and at constant air velocity 0.65 m/s (REYES et al. 2004, EFREMOV, KUDRA 2011. The initial sludge had a paste-like consistency with 72.6% wb moisture content, equivalent to 2.65 kg water/kg dry matter (REYES et al. 2004). The experiments were performed in a standard drying tunnel with a cross section 0.2×0.2 m. The samples were held on sets of 0.1×0.1 m metal boxes with rim height $L = 0.005$ m.

As drying of sludge depends on two main parameters, namely temperature of hot air t and drying time θ, the equation of regression was using in the form (6), where y represents the current material moisture content X; x_1 and x_2 are two coded variables of entrance parameters (factors) – temperature of hot air t and drying time θ; b_0, b_1, b_2 and b_{12} are the coefficients of regression equation (6).

In this process the change of temperature t (variable x_1) is in an interval 80-112° and the change of time θ (variable x_2) is from 0 up to 80 minutes.

Value of air temperature at the centre of the plan was designated t_0, interval of a variation δ_1, accordingly for time of process – θ_0 and δ_2. The initial data for this drying process are shown in Table 2.

Table 2

The initial data for convective drying of sludge

Variable, x	Min. (–)	Max (+)	Center	Interval (δ)
x_1 – temperature t, °C	80	112	96	16
x_2 – time θ, min	0	80	40	40

The plan of two-level factorial experiment for convective drying of sludge is presented in Table 3. The first two experiments correspond to initial condition of a material ($\theta = 0$ and moisture content X_0). The plan indicates that it is necessary to carry out only 2 experiments to determine moisture content at the same temperature at time $\theta = 80$ minutes. As a result of the experiments, the appropriate values of moisture content are equal to 1.31 and 0.49. All values of moisture content are in a matrix of planning of full factorial experiment for convective drying of sludge (Table 3).

Table 3

Matrix of full factorial experiment for convective drying of sludge

x_0	x_1	x_2	$x_1 \cdot x_2$	y
+	–	–	+	2.65
+	+	–	–	2.65
+	–	+	–	1.31
+	+	+	+	0.49

After substitution data of moisture content (Table 3), the generalized equation for the first drying period (11) is

$$X = X_0 + 0.009125 \cdot \theta - 0.0003203 \cdot \theta \cdot t \tag{12}$$

The drying rate can be calculated by differentiation of equation (12) with respect to time θ:

$$N_0 = \frac{dX}{d\theta} = 0.009125 - 0.0003203 \cdot t \tag{13}$$

Figure 2 shows the comparison of calculations using equation (12) with experimental data for convection drying of sludge in the 1-st drying period for four temperatures (REYES et al. 2004). The straight lines are drawn through three black points that mark the values determined according to the full factorial experimental plan for these calculations. Method can successfully be used to determine the influence of temperature from limited number of experimental data. The plan of a two-level factorial experiment indicates that it is necessary to carry out only 2 experiments to determine the linear dependence of the material moisture content vs. drying time.

Table 4

Designation of experimental points in Figure 2 and 3

Temperature t, °C	80	90	100	112	80 (plan)
Run No (BENALI, KUDRA 2002)	9	1	10	5	
Symbol	○	◊	+	□	●

As follows from Figure 2, the experimental data correspond well to the lines obtained through the factorial design which means that this method can successfully be used to determine constant drying rates from limited number of experimental data.

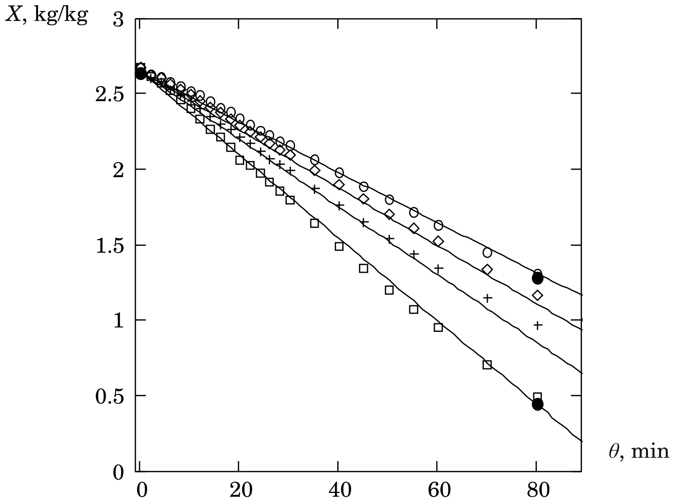

Fig. 2. Comparison of calculations of moisture content X vs. drying time θ using equation (11) with experiments on convection drying of sludge in the first period (Reyes et al. 2004) for four temperatures (Table 4)

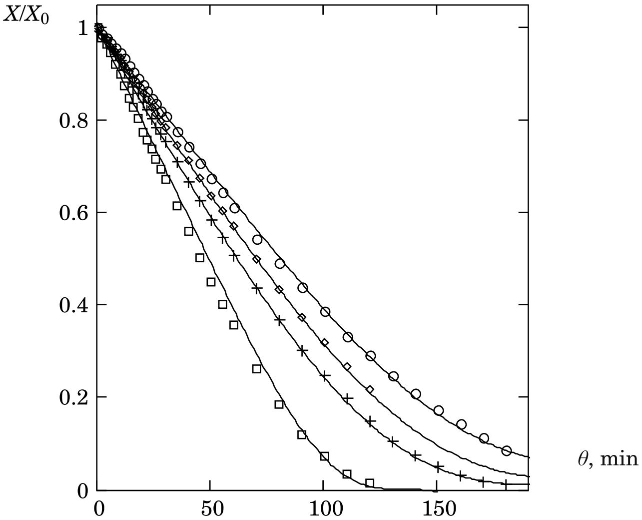

Fig. 3. Comparison of calculations using two-period model (4) with experiments
Source: Reyes et al. (2004).

Using the equation (13) for drying rate N_0 in the first drying period the calculation of moisture content for two-period model (4) may be fulfilled. Table 5 compiles the calculated values of drying parameters σ, N_0 and θ_f for two-period model whereas Figure 3 compares the results of calculations according to equation (4) with experimental data. For all run the equilibrium moisture content $X_{eq} = 0.03$ kg/kg. For clarity reasons only 4 curves for runs No 1, 5, 9 and 10 are shown in the Figures 2 and 3, but equally good fit was obtained for other experimental data (BENALI, KUDRA 2002).

Table 5

Experimental and calculated parameters for the two-period model (Drying kinetics of sludge)

Run No [10]	t, °C	N_0, 1/min	σ, min	θ_f, min
9	80	0.017	97	235
1	90	0.020	101	220
10	100	0.023	96.1	200
5	112	0.027	42.1	135

Good match of experimental points and model solution corresponds to sludge tests in Figure 3. It indicates that two-period equation (4) can be used to predict drying kinetics with possible extrapolation for different temperatures and air velocities.

Analogical calculations and comparison with experiments were fulfilled for convectional drying of cotton fabric for first and second periods in temperature interval 480–83°C and at change of time θ from 0 up to 18 minutes (EFREMOV 2012). The initial data for convective drying of cotton fabric are shown in Table 6.

Table 6

The initial data for convective drying of cotton fabric

Variable, x	Min. (–)	Max (+)	Center	Interval (δ)
x_1 – temperature t, °C	48	83	65.5	17.5
x_2 – time θ, min	0	18	9	9

The initial moisture content of cotton fabric was equivalent to $w_0 = 1.09$ kg water/kg dry matter. Accordingly of matrix of full factorial experiment (Table 3) the first two experiments correspond to initial condition of a material ($\theta = 0$ and moisture content X_0). The matrix indicates that it is necessary to carry out only 2 experiments to determine moisture content at temperature 48 and 83°S and time $\theta = 18$ minutes. As a result of the experiments, the appropriate values of moisture content were equal to 0.65 and 0.1. After substitution data of moisture content, the generalized equation for the first drying period (11) of cotton fabric is

$$X = X_0 + 0.01746 \cdot \theta - 0.000873 \cdot t \cdot \theta \tag{14}$$

Comparison of calculations with experiments (FILONENKO 1939) was fulfilled for four temperatures: 48°C, 56.3°C, 70°C and 83.5°C (Table 7, Figure 4). The deviations of calculations from experience did not exceed 3.8%.

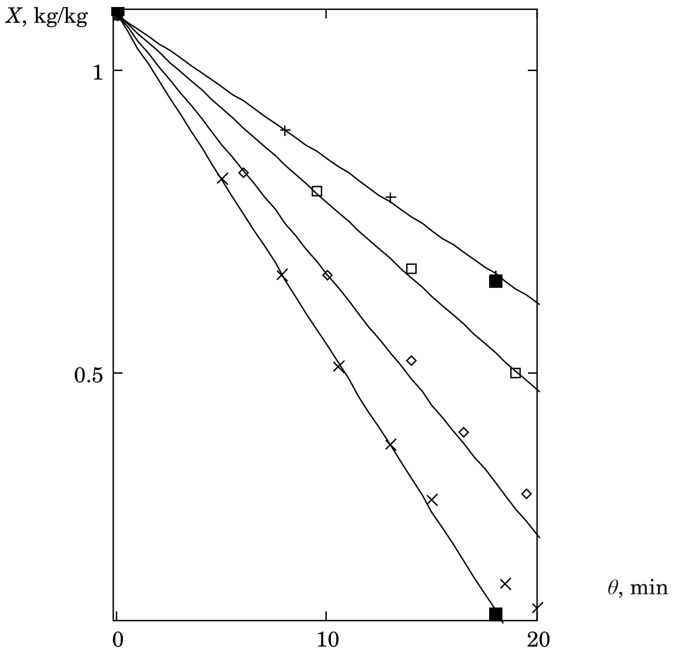

Fig. 4. Comparison of calculations of moisture content X vs. drying time θ using equation (14) with experiments on convection drying of cotton fabric (FILONENKO 1939) for four temperatures (Table 7)

Table 7

Designation of experimental points (FILONENKO 1939) in Figure 4

Temperature t, °C	48	56.3	70	83.5
Symbol	+	□	◊	×

Using the characteristic drying parameters listed in Table 5, the dimensionless number K (Equation 5) was calculated and plotted against the E parameter. Good match of the experimental points and the model solution in Fig. 4 indicates that generalized equation (5) can be used to predict drying kinetics with possible extrapolation for other temperatures and air velocities.

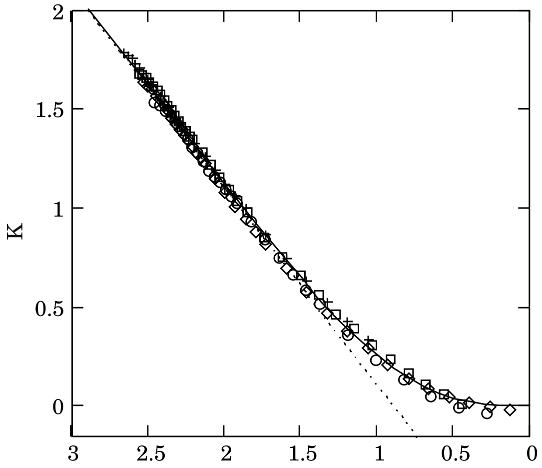

Fig. 5. Generalized drying curve for two-period model

It is necessary to note, that the obtained generalized kinetic equation (11) for the first drying period may be used for different types of drying and kind of materials. The drying rate N_0 for the first drying period can always be calculated by differentiation of the generalized equation with respect to time θ. Using the equation for drying rate N_0 in the first drying period the calculation of moisture content for two-period model may be fulfilled. The application of obtained mathematical modification of two-period model allows to avoid the application in calculations of the characteristic drying time which is constant for given process conditions.

Conclusion

The experiment design method (BOX et al. 2001, MONTGOMERY 2012) is a progressive method in application for convection drying in the first drying period and for two-period drying model. The comparison of calculations with experimental data for convection drying of sludge and cotton fabric for four temperatures using of experiment design method is fulfilled. As follows from calculations, the experimental data correspond well to the lines obtained through the factorial design which means that this method can successfully be used to determine drying kinetics for different types of drying and materials and to determine the influence of temperature from limited number of experimental data. The plan of a two-level factorial experiment indicates that it is necessary to carry out only 2 experiments to determine the linear dependence of the material moisture content vs. drying time.

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