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SENSITIVITY ANALYSIS OF KOZENY-CARMAN AND ERGUN EQUATIONS

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Abstract

In the article a sensitivity analysis of linear and nonlinear terms in the Kozeny-Carman and Ergun equations was shown. In the first case the impact of the porosity, tortuosity, specific surface of the porous body and the model constant was investigated. In the second case the porosity, the particle diameter and the sphericity function were taken into account. To express the model sensitivity by numbers, an earlier developed method was used. In this way the order and the importance of the impact of individual parameters was determined. The motivations to create this article were questions, which occurred during developing a novel investigation method, linking the Discrete Element Method and the CFD techniques. The first aim was to predict what will happen, if individual parameters will be set with an error: which data should be set as accurately as possible and which data are not very important for the result value. The second intention was to state which of parameters used in porous media investigations should be expressed by functions and which by constant values. The article may be treated as set of pointers helping in using of Kozeny-Carman and Ergun laws or as an example of research methodology based on the sensitivity analysis.

Introduction

A porous medium is a solid matrix containing a sufficiently high amount of free spaces interconnected, in which fluid flow is possible. The available spaces, also known as pores, form a complex and often irregular network of flow channels. The solid part of a porous medium may be in the form of a rigid skeleton (for example, limestone), a flexible frame (e.g. a layer of fabric) or a collection of particles: loose or in some way connected to one another (such as

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soil). Different parts or elements of the skeleton of a porous medium can be very different in shapes and sizes and may consist of one or a number of different materials (including inorganic and organic).

The spatial structure of a porous body is usually complicated, which leads to difficulties in mathematical description of such media. In the literature many parameters characterizing porous media: porosity, tortuosity, the specific surface on the solid body or particle diameter and sphericity coefficient in case of porous beds could be mentioned. By years many laws describing the fluid flows through porous media were developed, too. In these laws, as well as in other investigations in porous media area, the spatial parameters were introduced usually as constant values. Is it a good approach if the spatial structure can be so much complicated? The article is an attempt to answer to this question.

Basic macro-scale laws for flow through porous media

A fundamental law describing pressure drop in fluid flow through porous media is Darcy law (1856). It can be applied to flows of gases, liquids or mixtures. The Darcy's law may be written as follows (ANDERSON et al. 2009, HELLSTRÖM, LUNDSTRÖM 2006):

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{1}{\kappa} \cdot (\mu \cdot \vec{v}_f) \tag{1}$$

where:
p – pressure [Pa],
x – a coordinate along which the pressure drop occurs [m],
κ – permeability coefficient [m^2],
μ – dynamic viscosity coefficient of the fluid [kg/(m · s)],
$\vec{v}_f$ – filtration velocity [m/s].

For low velocity flows, Darcy's law adequately describes the flow in porous media (HELLSTRÖM, LUNDSTRÖM 2006). However, as velocities are becoming higher, discrepancies between experimental data and Darcy's law calculations appear. FORCHHEIMER (1901) linked this discrepancy to kinetic effects and suggested to add to equation (1) a term representing kinetic energy (ANDERSON et al. 2009, ANDRADE et al. 1999, EWING et Al. 2009, HELLSTRÖM, LUNDSTRÖM 2006):

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{1}{\kappa} \cdot (\mu \cdot \vec{v}_f) + \beta \cdot (\rho \cdot \vec{v}_f^2) \tag{2}$$

where:
β – Forchheimer coefficient (also known as non-Darcy coefficient, or β factor) [1/m],
ρ – density of the fluid [kg/m^3].

The equations (1) and (2) may be written in one general form:

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = A \cdot (\mu \cdot \overleftarrow{v}_f) + B \cdot (\rho \cdot \overleftarrow{v}_f^2) \tag{3}$$

where:
A and B are functions that presents several model parameters, including the permeability coefficient or the Forchheimer coefficient. If B is equal to 0, the formula (3) simplifies to the Darcy law (2), otherwise equation (3) represents the Forchheimer law.

In the literature many forms of formulas can be found for functions A and B (KEISHA 2008, SOBIESKI, TRYKOZKO 2001, VUKOVIC, SORO 1992). These formulas have been used and shown to be adequate for water flow through sand or gravel with small or average Reynolds number.

One of the most commonly used formulas for function A is the Kozeny-Carman equation. This formula, derived for calculating the permeability κ of well sorted sand (CARMAN 1937, FOURIE et al. 2007, NEITHALATH et al. 2009), can be written as follows:

$$A = \frac{1}{\kappa} = C_{\mathrm{KC}} \cdot \tau_f \cdot S_{0,\mathrm{Carman}}^2 \cdot \frac{(1-e)^2}{e^3} \tag{4}$$

where:
C_{KC} – Kozeny-Carman pore shape factor (a model parameter), which is suggested to be equal 5.0 [–] (CARMAN 1937),
τ_f – the tortuosity factor [m^2/m^2] defined as the square of the tortuosity τ [m/m],
$S_{0,\mathrm{Carman}}$ – the specific surface of the porous body [1/m],
e – the porosity [m^3/m^3].

Tortuosity is defined as the ratio of the actual length of flow path L_p and the length L_0 of the porous body:

$$\tau = \frac{L_p}{L_0} \tag{5}$$

The specific surface of the porous body is defined as follows – in Carman approach (CARMAN 1937)

$$S_{0,\mathrm{Carman}} = \frac{S_p}{V_p} \tag{6}$$

where:
S_p – the inner surface of the solid body [m^2],
V_p – the volume of the solid body [m^3].

Some researchers ignored the tortuosity in the Kozeny-Carman equation, probably due to difficulties in obtaining the adequate value of tortuosity (DUNN 1999, RAINEY et al. 2008), then equation (4) is simplified to:

$$A = C_{\mathrm{KC}} \cdot S^2_{0,\mathrm{Carman}} \cdot \frac{(1-e)^2}{e^3} \tag{7}$$

Some researchers went even further to omit the specific surface of the porous body from the Kozeny-Carman equation (BUYRUK et al. 2009)

$$A = C_{\mathrm{KC}} \cdot \frac{(1-e)^2}{e^3} \tag{8}$$

or replacing the specific surface by a simple parameter, such as particle diameter (OGILVIE et al. 2002)

$$A = C_{\mathrm{KC}} \cdot d^2 \cdot \frac{(1-e)^2}{e^3} \tag{9}$$

where:
d – is average diameter of the particle [m].

Similar form like (9) is used in the work (RESCH 2008):

$$A = C_{\mathrm{KC}} \cdot \frac{(1-e)^2}{d^2 \cdot e^3} \tag{10}$$

In next work, the author used not the diameter of the particle, but its radius R [m] (ROSSEL 2004):

$$A = C_{\mathrm{KC}} \cdot \frac{(1-e)^2}{R^2 \cdot e^3} \tag{11}$$

It should be mentioned that equation (9) is in fact incorrect due to the inconsistency of units. The function A should be expressed in [1/m^2]. A large discussion about the correct form of the Kozeny-Carman equation the Reader can find in the work (SOBIESKI 2014).

Review of the literature indicates that the Kozeny-Carman law is very general, but there are also problems with its use, forcing researchers to consider various simplifications. The Kozeny-Carman equation is the basis for creating other formulas. The ERGUN (1952) equation is one of the most commonly used equations (DUNN 1999, ERGUN 1952, NIVEN 2002). It takes the following form:

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \left[150 \cdot \frac{(1-e)^2}{e^3 \cdot (\phi \cdot d)^2}\right] \cdot (\mu \cdot \overleftarrow{v}_f) + \left[175 \cdot \frac{(1-e)}{e^3 \cdot (\phi \cdot d)}\right] \cdot (\rho \cdot \overleftarrow{v}_f^2) \tag{12}$$

where ϕ is the sphericity coefficient [–] and expressions in brackets are the A and B functions in equation (3), respectively. The sphericity coefficient in formula (12) is equal to 1 when the particles are spherical shape. In other cases the value is less than 1. The deviation from ideal sphericity can be calculated by using different formulas, e.g.: Wadell Roundness, Dobkins and Folk Roundness, Power's Roundness Classification Chart, Wadell Sphericity, Krumbein Sphericity, Sneed and Folk Sphericity, Shape Factor, Riley Sphericity and others. More details you can find in the work (SOBIESKI 2009).

The method of sensitivity analysis

The following impact (sensitivity) indicator is introduced to evaluate the degree of equilibrium in a system (the effect of the i^{th} input parameter on the j^{th} output parameter) (SOBIESKI, DUDDA 2014, SOBIESKI, TRYKOZKO 2011, SOBIESKI 2008):

$$I_{j,i} = \frac{\Delta\varphi_{j,i}^{\mathrm{out}}}{\Delta\varphi_i^{\mathrm{in}}} \tag{13}$$

where:

$I_{j,i}$ – indicator of the impact of the i^{th} input parameter on the j^{th} output parameter,

$\Delta\varphi_i^{\text{in}}$ – change in the value of the i^{th} input parameter,
$\Delta\varphi_{j,i}^{\text{out}}$ – change in the value of the j^{th} output parameter caused by a change in the value of the i^{th} input parameter.

Increments in formula (13) are defined as follows:

$$\Delta\varphi_i^{\text{in}} = \varphi_i^{\text{in}} - \overline{\varphi}_i^{\text{in}} \tag{14}$$

and

$$\Delta\varphi_{j,i}^{\text{out}} = \varphi_{j,i}^{\text{out}} - \overline{\varphi}_{j,i}^{\text{out}} \tag{15}$$

where:
φ_i^{in} – current value of the i^{th} input parameter (for which the value of the j^{th} output parameter is estimated),
$\overline{\varphi}_i^{\text{in}}$ – base value of the i^{th} input parameter (from the base model),
$\varphi_{j,i}^{\text{out}}$ – value of the j^{th} output parameter determined for the current value of the i^{th} input parameter,
$\overline{\varphi}_{j,i}^{\text{out}}$ – base value of the j^{th} output parameter (from the base model).

A normalized indicator based on values cross-referenced with base values can be introduced to facilitate comparisons of different numerical models:

$$\hat{I}_{j,i} = \frac{\Delta\hat{\varphi}_{j,i}^{\text{out}}}{\Delta\hat{\varphi}_i^{\text{in}}} \tag{16}$$

where:
$\hat{I}_{j,i}$ – normalized indicator of the impact of the i^{th} input parameter on the j^{th} output parameter,
$\Delta\hat{\varphi}_i^{\text{in}}$ – change in the value of the i^{th} input parameter relative to its base value,
$\Delta\hat{\varphi}_{j,i}^{\text{out}}$ – change in the value of the j^{th} output parameter relative to its base value, caused by a change in the value of the i^{th} input parameter.

After normalization, formulas (14) and (15) take on the following form:

$$\Delta\hat{\varphi}_i^{\text{in}} = \frac{\Delta\varphi_i^{\text{in}}}{\overline{\varphi}_i^{\text{in}}} = \frac{\varphi_i^{\text{in}} - \overline{\varphi}_i^{\text{in}}}{\overline{\varphi}_i^{\text{in}}} \tag{17}$$

and

$$\Delta\hat{\varphi}_{j,i}^{\text{out}} = \frac{\Delta\varphi_{j,i}^{\text{out}}}{\overline{\varphi}_{j,i}^{\text{out}}} = \frac{\varphi_{j,i}^{\text{out}} - \overline{\varphi}_{j,i}^{\text{out}}}{\overline{\varphi}_{j,i}^{\text{out}}} \tag{18}$$

A normalized impact indicator can be expressed as:

$$\hat{I}_{j,i} = \frac{\hat{\varphi}_{j,i}^{\mathrm{out}}}{\hat{\varphi}_{i}^{\mathrm{in}}} \tag{19}$$

where the normalized value of deviation of the i^{th} input parameter is:

$$\hat{\varphi}_{i}^{\mathrm{in}} = \frac{\varphi_{i}^{\mathrm{in}}}{\overline{\varphi}_{i}^{\mathrm{in}}} \tag{20}$$

and the normalized value of deviation of the j^{th} output parameter is:

$$\hat{\varphi}_{j,i}^{\mathrm{out}} = \frac{\varphi_{j,i}^{\mathrm{out}}}{\overline{\varphi}_{j,i}^{\mathrm{out}}} \tag{21}$$

A normalized impact indicator has the following characteristics:

– the greater the impact of the i^{th} input parameter on the j^{th} output parameter, the higher the value of the impact indicator,

– the indicator determines which direction of deviation from the base value of the i^{th} input parameter produces greater changes in the j^{th} output parameter (decrease or increase in output parameter),

– a negative impact indicator implies that an increase in the value of the i^{th} input parameter decreases the value of the j^{th} output parameter.

Calculations of the sensitivity indicators

The sensitivity calculation of the linear and non-linear terms in the Kozeny-Carman and Ergun equation consists of three steps. They are as follows:

– Calculation of the spatial structure of the porous bed. This step was reached by using the Discrete Element Method. In the current case the PFC^{3D} numerical code was used. As the result the data with locations and sizes of all particles in the porous bed were obtained. More details about the DEM model may be found in works LIU et al. (2008a and 2008b).

– Calculation of all needed geometrical parameters. Here an own numerical code with the name PathFinder was used (SOBIESKI, LIPINSKI 2014, SOBIESKI et al. 2012, The PathFinder Project 2013). This software gives information about the average particle diameter, the porosity, the tortuosity and the specific surface of the porous body. These values are shown in the Table 1, as well as the values of the Kozeny-Carman constant and the sphericity coefficient. All these values define the base model for which the sensitivity analysis is performed.

– Performing the sensitivity analysis. In this step other own software was used (so called „Sensitivity calculator" available as a complementary tool in the PathFinder project (SOBIESKI, LIPINSKI 2014)). This tool calculates many times A and B terms for the equation (3) and the base model, but in every loop run one of these parameters is changed by a normalized deviation of ± 1, 3, 5, 10, 15, 20, 25 or 30%. Results were collected in simple text files and next were

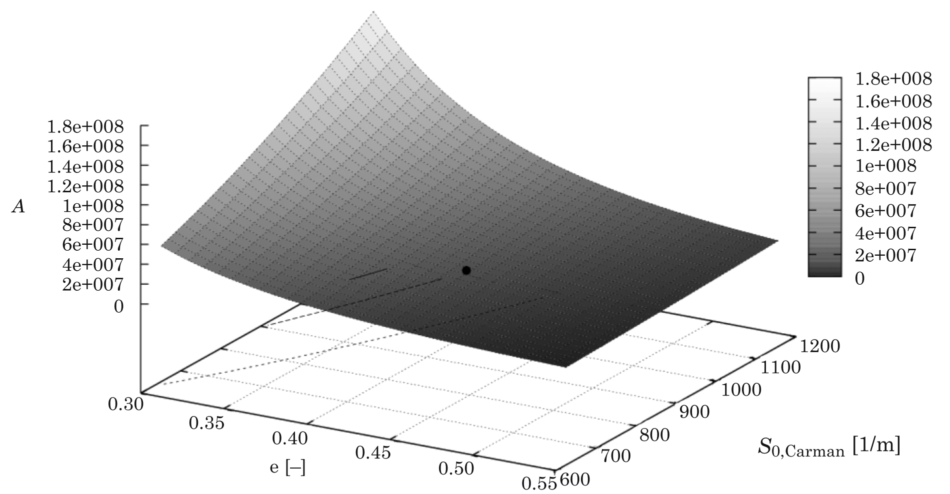

Fig. 1. Influence of parameters e and $S_{0,\mathrm{Carman}}$ for Kozeny-Carman A function (τ = const.)

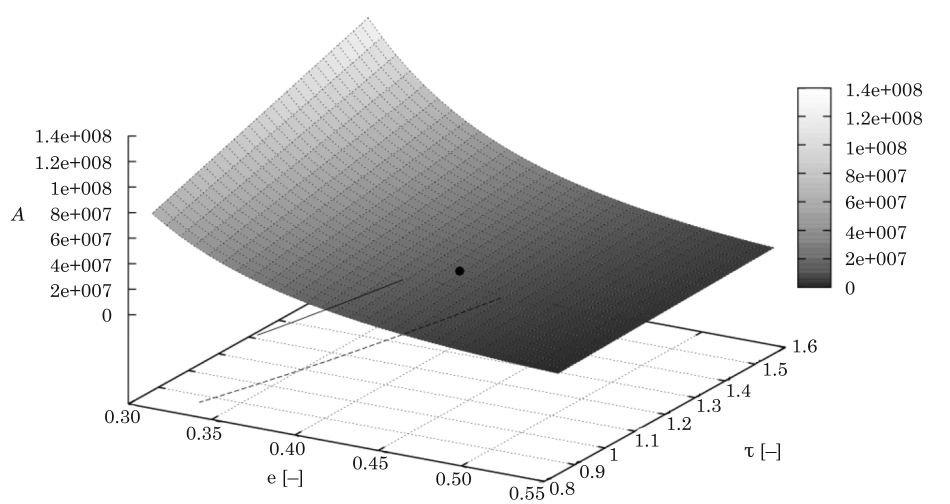

Fig. 2. Influence of parameters e and τ for Kozeny-Carman A function ($S_{0,\mathrm{Carman}}$ = const.)

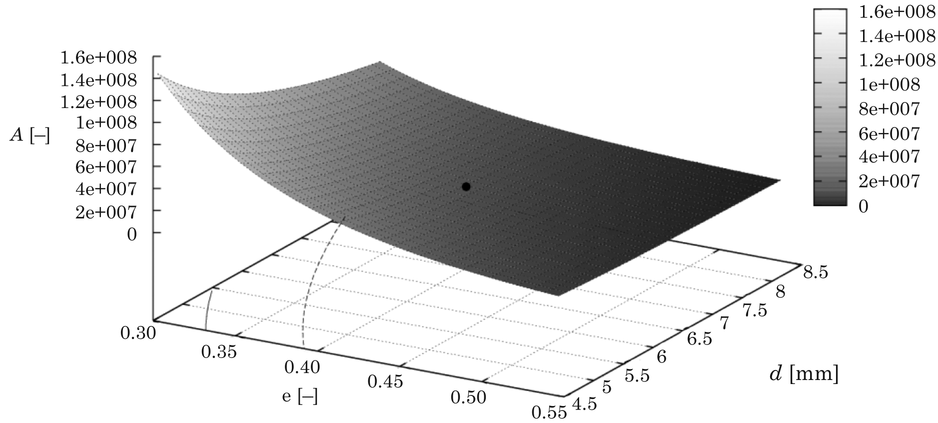

Fig. 3. Influence of parameters e and d for Ergun A function

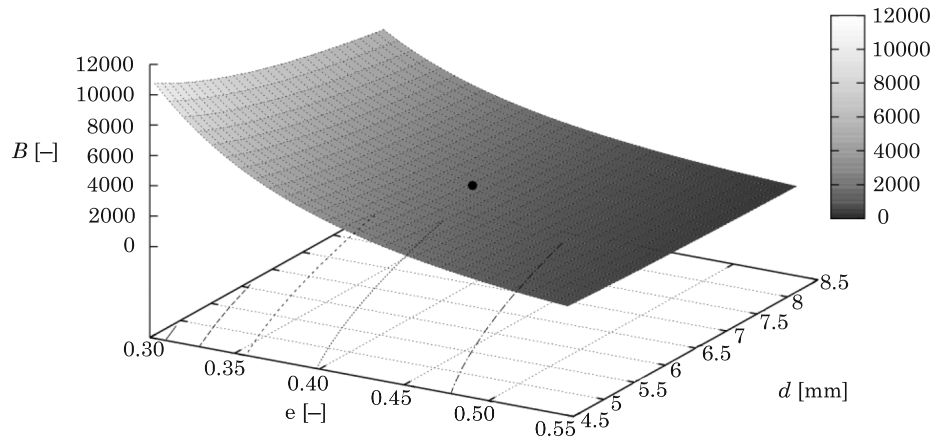

Fig. 4. Influence of parameters e and d for Ergun B function

visualized by using Gnuplot scripts (Figures 1–4 were made in such way). Sensitivity indicators were calculated by using Calc, spreadsheet from the LibreOffice software.

Note that in the context of this article the value of velocity and fluid properties like density and viscosity are not important.

Both functions A and B are most sensitive to porosity variations. It can be seen in Figures 5–7, in which the porosity curve differs significantly to the other curves in whole range of the deviation value. This result agrees with that observed by many researchers – the pressure drop in fluid flow through porous media depends in first order of porosity (see e.g. the review of equations in the

Table 1

The data of the basic model

Parameter	Symbol	Value	Unit
Particle diameter	d	6.45	[mm]
Porosity	e	0.42	[m^3/m^3]
Tortuosity	τ	1.2376	[m/m]
Specific surface of the porous body	$S_{0,\mathrm{Carman}}$	915.92	[m^2]
Kozeny-Carman constant	C_{KC}	5.0	[–]
The sphericity coefficient	ϕ	1	[–]

work (SOBIESKI, TRYKOZKO 2011) or (SOBIESKI 2014)). Furthermore, the high sensitivity to porosity indicates that even small changes in porosity would result in different results in pressure drop. It is important that both spatial and temporal changes in porosity (in cases when the porosity is defined as function of these factors) should be accurately determined for predicting pressure drops. It should be emphasized that the impact of porosity changes is in all cases nonlinear. Both models are more sensitive to porosity changes at lower porosity. This means that local compaction may play a greater role than dilution in affecting pressure drop.

Table 2

Sensitivity indicators for A and B functions in Kozeny-Carman and Ergun equations

	Kozeny-Carman A function				Ergun A function			Ergun B function		
	I_e	I_τ	I_{S_0}	$I_{C_{KC}}$	I_e	I_d	I_ϕ	I_e	I_d	I_ϕ
I^{-30}	–11.07	1.70	1.70	1.00	–116.25	–3.47	–3.47	–76.74	–1.43	–1.43
I^{-25}	–9.23	1.75	1.75	1.00	–41.82	–3.11	–3.11	–29.11	–1.33	–1.33
I^{-20}	–7.80	1.80	1.80	1.00	–18.76	–2.81	–2.81	–13.8	–1.25	–1.25
I^{-15}	–6.68	1.85	1.85	1.00	–10.29	–2.56	–2.56	–7.95	–1.18	–1.18
I^{-10}	–5.78	1.90	1.90	1.00	–6.75	–2.35	–2.35	–5.42	–1.11	–1.11
I^{-5}	–5.05	1.95	1.95	1.00	–5.17	–2.16	–2.16	–4.27	–1.05	–1.05
I^{-3}	–4.79	1.97	1.97	1.00	–4.79	–2.09	–2.09	–3.98	–1.03	–1.03
I^{-1}	–4.56	1.99	1.99	1.00	–4.56	–2.03	–2.03	–3.81	–1.01	–1.01
I^{1}	–4.34	2.01	2.01	1.00	–4.34	–1.97	–	–3.64	–0.99	–
I^{3}	–4.14	2.03	2.03	1.00	–4.04	–1.91	–	–3.42	–0.97	–
I^{5}	–3.95	2.05	2.05	1.00	–3.86	–1.86	–	–3.28	–0.95	–
I^{10}	–3.54	2.10	2.10	1.00	–3.08	–1.74	–	–2.68	–0.91	–
I^{15}	–3.18	2.15	2.15	1.00	–2.24	–1.63	–	–2.02	–0.87	–
I^{20}	–2.88	2.20	2.20	1.00	–1.52	–1.53	–	–1.42	–0.83	–
I^{25}	–2.63	2.25	2.25	1.00	–0.98	–1.44	–	–0.96	–0.8	–
I^{30}	–2.40	2.30	2.30	1.00	–0.62	–1.36	–	–0.62	–0.77	–

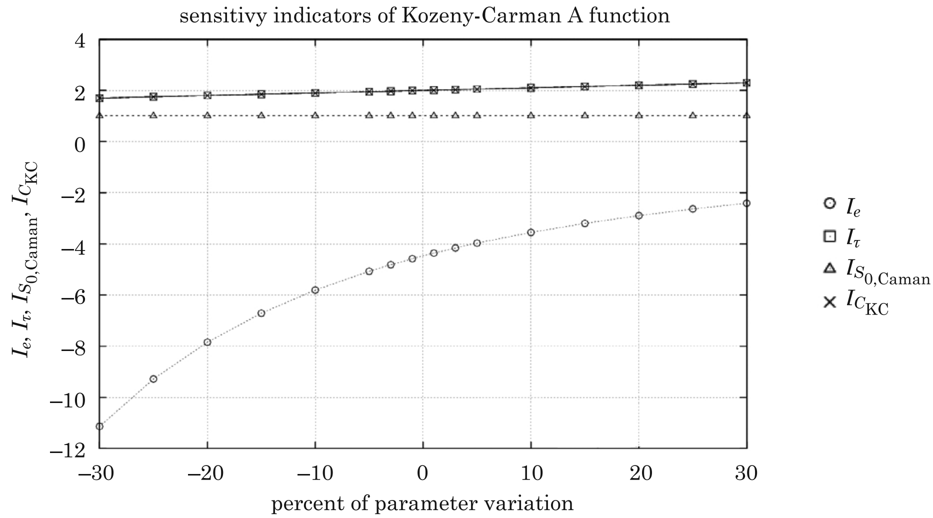

Fig. 5. Sensitivity indicators of Kozeny-Carman *A* function (Equation 4)

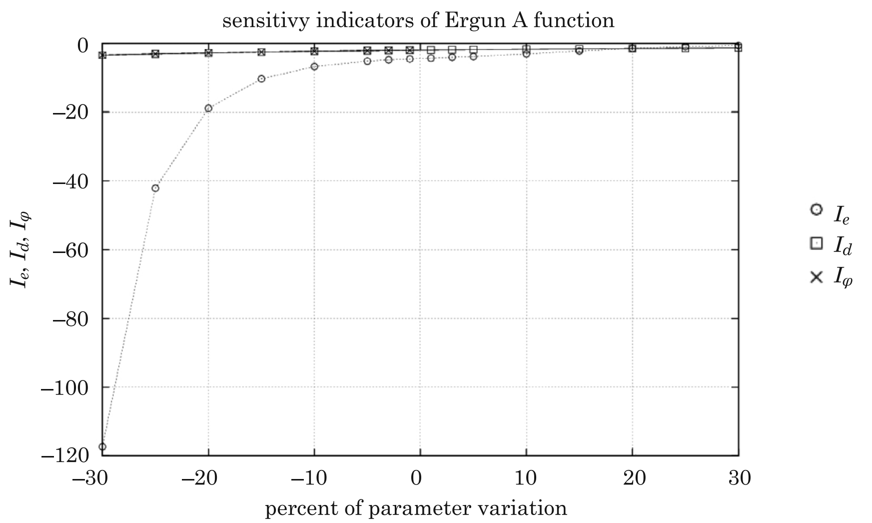

Fig. 6. Sensitivity indicators of Ergun *A* function (Equation 12)

Sensitivity analysis indicates that in the Kozeny-Carman model the tortuosity (the tortuosity factor) and specific surface have the same impact on the results. This is due to the fact that both of which are present in the formula (4) in the second power. The Kozeny-Carman constant has the weakest impact on the function what results from their linear relationship.

In the Ergun model significant discrepancies can be seen between porosity impact and impact of the other parameters. The scale of changes in the

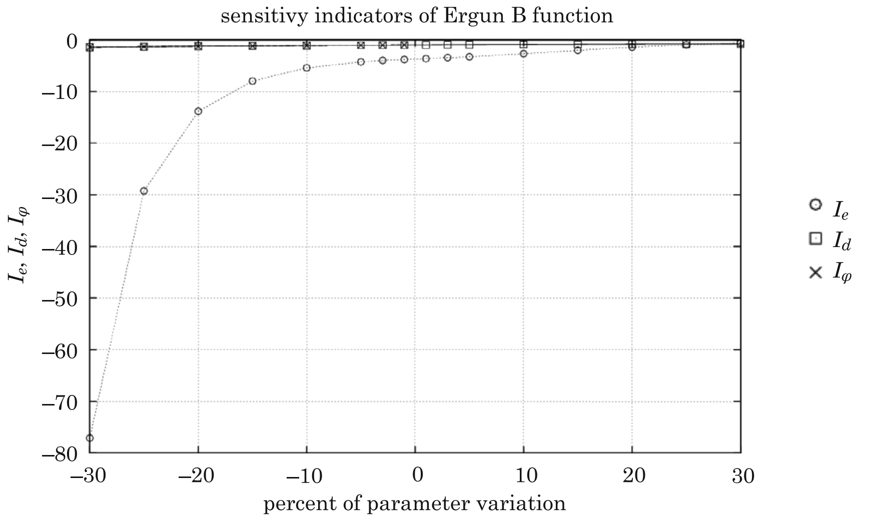

Fig. 7. Sensitivity indicators of Ergun B function (Equation 12)

B function is less than in the A function, what means that the nonlinear part of the Ergun equation plays smaller role as the linear part. It was observed already in the work (SOBIESKI, TRYKOZKO 2011). It should be added that the negative indicator value means that the increase of input parameter leads to decrease of the expected result.

The effect of various parameters on functions A and B are shown in Figures 1–4. The central dot on these figures denotes the result for the baseline parameter values. It can be seen again the predominant effect of porosity. Since the sensitivity indicator values for tortuosity, specific surface of the porous body, and the Kozeny-Carman constant are a few times smaller than sensitivity indicator for the porosity, one can conclude that the effect of these parameters (τ, $S_{0,\mathrm{Carman}}$, and C_{KC}) could be neglected in the cases studied here (typical). For small variations (in range of a few percents), the influence of all parameters is similar, including porosity. Although porosity effect is still dominant, the effect of tortuosity and specific surface is not negligible. This situation can be a reason why in the Ergun formula (created later than the Kozeny-Carman formula) these small factors are not included. In empirical investigations leading to general estimations this approach can be accepted, but when a physical side of phenomenon is on the first place, such a simplification should not take place.

An important feature of the Ergun equation is the use of particle diameter as a model parameter. In real systems this diameter may vary in certain range and the average diameter must be therefore used. As may be seen in Figures 3, 4 this parameter has impact on values of A and B functions, and in turn on the total pressure drop in the porous bed. It can be concluded that differences in particle size or particle distribution will influence on the result. Alternatively, a new function to the Kozeny-Carman or Ergun equations may be added: its value should be equal to one for porous beds consisting of particles with constant diameter and higher then one, if diameters are different.

Summary

The final conclusions are as follow:

– Porosity is the most important parameter in terms of its impact on the pressure drop in porous media, when the Kozeny-Carman and Ergun equations are used.

– The impact of the porosity is nonlinear for both Kozeny-Carman and Ergun equations; the impact is more pronounced at lower porosity.

– Functions A and B both Kozeny-Carman and Ergun equations are very sensitive to porosity, therefore, porosity should be treated as a function of location and time in a porous bed when predicting pressure drops.

– The impact level of the tortuosity and the specific surface on the result of Kozeny-Carman equation is the same, and the Kozeny-Carman constant has the least impact.

– The specific surface may be described as a function of location or time similar like the porosity function.

– The tortuosity did not have locally values due to the definition. For this reason an average value (calculated for many different paths) should be used in investigations. The application of a function is here limited, but it is possible to define a relationship between coordinates in the cross section of the porous bed in the plane perpendicular to the main flow direction an the tortuosity value.

– The particle diameter and the sphericity coefficient have the same effect on the result of the Ergun equation. In porous beds case, the porosity, tortuosity and other spatial parameters are dependent on the diameters distribution. For this reason diameter should be expressed with the use of more general formula

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