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INFLATION SIMULATION OF TRACTOR RADIAL TIRE

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Abstract

The possibility of prediction of some parameters of large-size pneumatic tire was presented. Author's computer program based on the finite element method with the use of axisymmetric tire model was utilized. Material constants required for the tire analysis were determined experimentally. The procedure for obtaining a simplified axisymmetric tire tread was described. The method used for simulation of tire inflation process allows obtaining the shape of an inflated tire, forces in cords and elastic energy density distribution. A test tire overall dimensions obtained in this way are close to measured values.

Introduction

Until recently, the work of pneumatic tires designer was more like the work of a shoemaker than the engineer-designer. In his activity was something of craft and even of art. One had to put a lot of work, because it was necessary to design the tire profile in the vulcanization form in such a way, that after tire mounting on the rim and inflating, the required overall dimensions, i.e. diameter and width, were being obtained. To realize the difficult task nomograms by BIDERMAN et al. (1967) were helpful. This method of tire design was based on a net model of tire where the influence of rubber on the shape of inflated tire was neglected. The huge advances in computer technology made it possible to speed up tire designers' work. However, in Poland only in the

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eighties there were attempts to automate this task (KONDEJ 1982). In the area of engineering activities, works by PELC and PETZ (1988, 1994) have provided effective, industry-verified design tools.

The finite element method enabled more advanced modeling of pneumatic tire deformation. Although its origins date back to the 1950s, in the field of pneumatic tires it has been applied only in the 70s (DUNN, ZOROWSKI 1970), while in Poland in the 90s (PELC 1992). This is due to the fact that from the view of the mechanics of deformable body, the pneumatic tire is a very complicated structure.

Today, the large-scale farms are being supplied with ever-larger power tractors. Pneumatic tires play a significant role in the effective use of these powers, determining the agriculture equipment productivity. Tire engineers have made a significant contribution to the process of increasing the efficiency of agricultural tractors through the development of radial tires. Compared to conventional tires, i.e. bias tires, radial ones are characterized by greater tread durability, less pressure on soil, more pulling power and a higher ride comfort (SKOCZEK 1995). Bearing in mind the reliability of the machines fitted with tires, tire manufacturers have been incorporated latest calculation methods into the tire design process. Due to the high efficiency in modeling of complex structures, the finite element method was mostly used (DEESKINAZI, RIDHA 1982, HEISE 1987, NANKALI et al. 2012).

The aim of the study is to examine the possibility of predicting some of the parameters of large-size tire using own computer program based on the finite element method and axisymmetric tire model. The way of the inflation process simulation for tractor radial tire is shown. A manner of simplified presentation of the geometrically complex actual tire tread, consisting of tread lugs smearing was proposed. The obtained results in the form of inflated tire shape, forces in cords, elastic energy density distribution are valuable for the tire designer allowing him to improve the structure. Overall dimensions of the test tire obtained from the simulation show good agreement with measurement results.

Characteristic of the modeled tire

Figure 1 shows a view of the actual tire for which the calculation model will be built and inflation simulation will be performed. This is the 14.9R28 radial agricultural tire, designed for rear drive wheels of tractors. The tire is a product of Stomil Olsztyn tire manufacturer. Tire parts within its profile are depicted in Figure 2. Nominal inflation pressure for the tire is equal to 160 kPa.

An inner tire profile in a mould and profile of the tire mounted on W12×28 rim are shown in Figure 3. It was assumed that, in the latter case, there was a pressure equal to zero in the tire. In fact, in this state, the pressure was equal to 10% of nominal one for the tire to ensure contact between the tire bead and the rim flange. During the simulation process the shape of tire profile was treated as its reference (initial) configuration.

Measurements of the overall dimensions of the tire, i.e. outer diameter and width, were performed on the randomly selected two prototype tractor tires. The diameter of the tire was determined by measuring its circumference. The

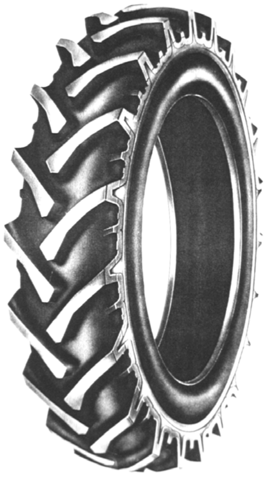

Fig. 1. Tractor radial tire 14.9R28

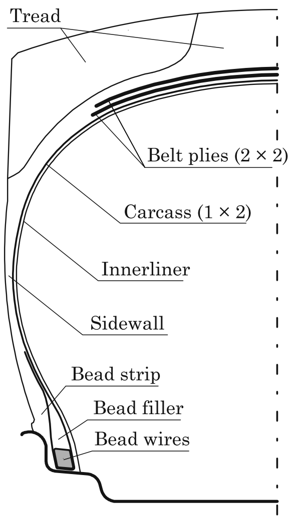

Fig. 2. Tire constituent elements within its cross-section

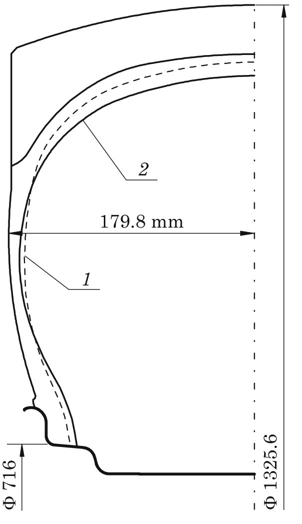

Fig. 3. Tire profile: *1* – in form (inner), *2* – after mounting on rim

Table 1

Measurement results of tire overall dimensions

Tire No.	Pressure kPa	Perimeter mm	Diameter mm	Width mm	Results of width measurement mm			
1	0	4161	1324.5	360.2	360.1	363.0	358.8	358.8
2	0	4168	1326.7	359.1	363.0	358.6	354.8	360.0
Average			1325.6	359.6				
1	160	4211	1340.4	364.6	364.7	365.0	364.6	364.2
2	160	4206	1338.8	369.1	367.5	369.5	369.7	369.8
Average			1339.6	366.8				
Changes in overall dimensions			14.0	7.2				

width of each tire was calculated as average value of the widths measured in four locations determined by quadrant points of the tire equator. The measurement results for the inflation pressure equal to zero and for the nominal pressure p = 160 kPa are shown in Table 1.

The tire carcass is made of two layers of polyamide cord, belts – four layers of rayon cord and the bead – 56 coils of steel wires. Young moduli of cord materials were determined on the basis of tensile tests:

– polyamide cord – E_p = 4428.8 MPa for effective yarn diameter d_p = 0.5 mm,

– rayon cord – E_r = 8383.6 MPa for effective yarn diameter d_r = 0.7 mm.

Shear moduli of purely rubber materials were determined by measuring their hardness. The relationship between hardness and Kirchhoff modulus has the form (PĘKALAK, RADKOWSKI 1989):

$$G = 0.086 \cdot 1.045^{\mathrm{HSh}} \text{ [MPa]} \tag{1}$$

were HSh – rubber hardness measured by Shore durometer (scale A).

The results of measurements and calculations are shown in Table 2.

Table 2

Hardness and shear modulus for tire rubbers

Tire part	Shore A hardness (HSh)	Kirchhoff modulus [MPa]
Tread	65	1.50
Sidewall	53	0.89
Bead strip	75	2.33
Bead filler	70	1.87
Innerliner	65	1.50

Computational tire model

Deformation of pneumatic tire treated as a deformable continuum is described by differential equations of mechanics of continuous media:

$$\operatorname{div} {}_0^t\mathbf{T}({}^0\mathbf{X}, {}_0^t\mathbf{X}) - {}_0^t\mathbf{f} = \mathbf{0} \tag{2}$$

where:

${}^t\mathbf{T}$ – the first Piola-Kirchhoff stress tensor in configuration t with respect to configuration 0 (initial configuration),

$^{0}\mathbf{X}$ – a material particle coordinate in initial configuration,
$^{t}\mathbf{X}$ – deformation gradient,
$^{t}\mathbf{f}$ – body force vector.

To obtain stresses and deformation of an elastic body the equations (2) is supplemented with strains-displacements and constitutive relations.

In the presented analysis of tire inflation by the use of the finite element method, an important role play a weak form of the problem:

$$\int_{^{0}V} \delta\, {}^{t+\Delta t}_{0}\mathbf{E} \cdot {}^{t+\Delta t}_{0}\mathbf{S}\, {}^{0}dV = {}^{t+\Delta t}L \tag{3}$$

where:
$\delta\, {}^{t+\Delta t}_{0}\mathbf{E}$ – variation of Green-Lagrange strain tensor,
${}^{t+\Delta t}_{0}\mathbf{S}$ – the second Piola-Kirchhoff stress tensor,
^{0}V – volume of the body in configuration 0,
${}^{t+\Delta t}L$ – virtual work of externally applied loads.

From (3) it follows FEM equation for determining the locations of model nodes in the consecutive states of balance for tire being inflated (BATHE 1982).

The main load carrying constituent elements of the pneumatic radial tire are shown in Figure 2. The deformation of the tire caused by internal pressure is characterized by moderate strains in its materials, and therefore a linear relationship between stresses and strains was assumed. Constitutive relationship for such a material written in the principal orthotropy axes has the form:

$$\mathbf{S} = \frac{1}{V}\begin{bmatrix} E_1(1-v_{23}v_{32}) & E_2(v_{12}+v_{13}v_{32}) & 0 & E_3(v_{13}+v_{12}v_{23}) & 0 & 0 \\ & E_2(1-v_{13}v_{31}) & 0 & E_3(v_{23}+v_{21}v_{13}) & 0 & 0 \\ & & V\cdot G_{12} & 0 & 0 & 0 \\ & & & E_3(1-v_{12}v_{21}) & 0 & 0 \\ & sym. & & & V\cdot G_{13} & 0 \\ & & & & & V\cdot G_{23} \end{bmatrix} \cdot \mathbf{E} \tag{4}$$

where:

$V = (1-v_{12}v_{21}-v_{13}v_{31}-v_{23}v_{32}-v_{12}v_{23}v_{31}-v_{21}v_{13}v_{32}$
$\mathbf{S}$ – vector of the second Piola-Kirchhoff stresses,
$\mathbf{E}$ – Green-Lagrange strains vector,
v_{ij} – Poisson ratio in ij plane: $i, j = 1, 2, 3$.

Indices 1, 2, 3 denote: the cord direction, axis perpendicular to the cord lying in the plane of the ply, the vector product of the unit vectors of 1 and 2 axes, respectively. Because the calculations are performed in a (x, y, φ) coordinate system, a constitutive matrix (4) is subject to appropriate transformations (cf. PELC 2007). The effective material constants for single ply E_1, E_2, G_{12} and υ_{12} is calculated using the HALPIN-TSAI (1969) equations. In addition, $\upsilon_{ij}E_j = \upsilon_{ji}E_i$, and it was admitted for the calculations that $E_3 = E_2$, $\upsilon_{13} = \upsilon_{12}$, $G_{13} = G_{12}$ and $G_{23} = 2G_{12}$. The determination method of constants takes into account the variability of cord volume fraction in cord-rubber composite, which is a result of tire forming process. Originally cord-rubber layers are arranged on a cylindrical building drum, wherein the cord density in each green layer remains constant (PELC 2009).

It should be noted that such a composite material model can be used to deformation analyses of tires with low inflation pressure (up to 250 kPa). At higher pressures, the tire model becomes numerically unstable.

The assumption was made that the material of rubber elements is homogeneous and isotropic, and almost incompressible (Poisson ratio $\upsilon = 0.48$).

The tractor tire tread possesses a fairly regular pattern of lugs in the circumferential direction, but complex geometry in general. Having the shape of the raw tread profile (Fig. 4) it is possible to smear the tire tread lugs both in the circumferential and meridional directions. Comparing the volume of rubber corresponding to the elementary increments of the arc coordinate s in the raw and cured treads, an outer contour line of the smeared tread can be obtained. Incompressibility of rubber was assumed, and therefore the treads are of equal volume.

With the smearing of the tread lugs in the circumferential direction henceforth the tire inflation problem can be treated as rotationally symmetrical. It was also assumed that the plane xz is the plane of symmetry of the tire model. To solve the problem, the finite element method was used. Taking into account the stratified nature of the tire its cross-section model has been discretized (Fig. 4.)

In order to reduce the number of elements in the model each of two adjacent layers were connected, i.e. adjacent belt plies with the cord angles $+\theta$ and $-\theta$ were modeled with one finite element in ply thickness direction. The innerliner was incorporated into the composite material of carcass. In result three orthotropic materials ($+\theta$ and $-\theta$ belts, carcass+innerliner and carcass turnup) and four homogeneous isotropic materials were specified.

On the axis of symmetry of the tire model the y-displacements of the nodes were blocked, and the nodes being in contact with the rim were fixed.

Due to the existence of geometric nonlinearity in the problem (large displacements) FEM equilibrium equations were written in Total Lagrangian

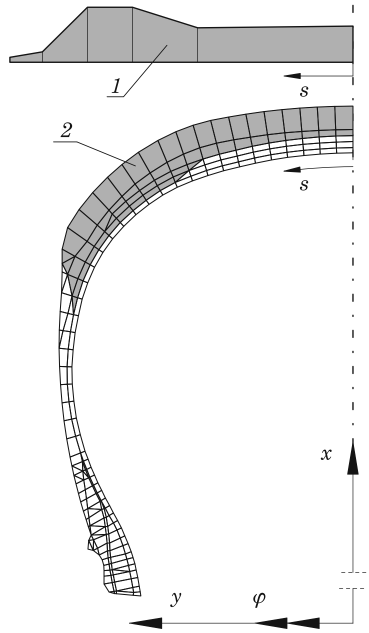

Fig. 4. Finite element mesh in reference tire configuration: *1* – raw tread profile, *2* – smeared tread profile of tractor tire

(TL) frame. (cf. PELC 2009). The nonlinear equilibrium path for the system was determined by modified Newton-Raphson method. To control the load increments, the modified constant arc length method suggested by CRISFIELD (1981) was utilized. The algorithm automatically increases the load increment if it is only possible due to the convergence rate of the iteration.

Results of simulation of tire inflation process

The tire model being in the initial configuration was loaded with an internal pressure equal to the nominal value p = 160 kPa. The procedure for automatic selection of load increment assured that in twelve steps the load has reached the final value. In each load step about three iterations have been performed.

The tire profile after deformation is shown in Figure 5. As a result of inflation the tire has increased both its width and diameter. It can be seen quite uniform displacements of the internal contour points of the tire toward the outside. This is the result of elongation of the carcass cords under the action of internal pressure.

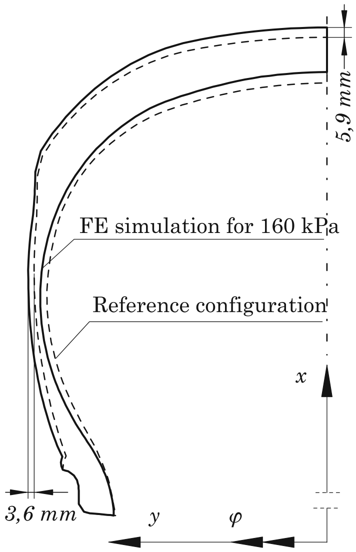

Fig. 5. Inflated tire profile

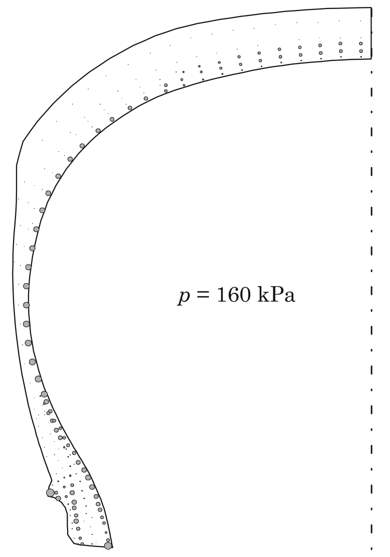

Fig. 6. Map of strain energy density in tire elements

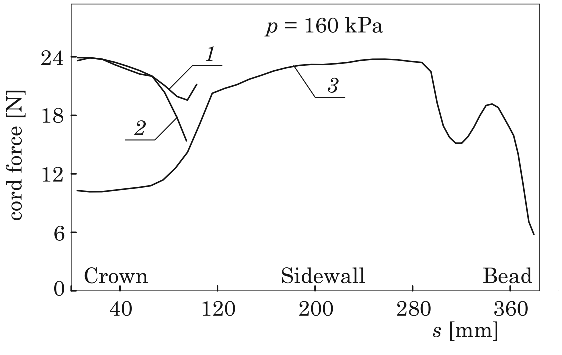

Fig. 7. Tensile forces in tire cords: *1* – 1st & 2nd belt, *2* – 3rd & 4th belt, *3* – carcass

The calculated increase in tire width is 7.2 mm and agrees well with the result of measurement equal to 7.2 mm. The measured increase in the tire diameter was 14.0 mm and the calculated – 11.8 mm and varies by 16%. The difference may be due to the fact that the smeared tread has greater circumferential stiffness than the tread built of lugs, and thus limits the radial displacements in tire.

Figure 6 shows the map of distribution of the elastic strain energy density within the inflated tire. The center points of circles are located in centroids of each finite element. Their diameters are proportional to the stored energy determined as the average of the sum of energies being occurred in Gauss

Points. It can be noticed high values of strain energy density in the layered materials, in particular within carcass ply below the largest width of the tire, also within the bead area as a result of the occurrence of considerable shear strains there.

Changes in cord forces of layered materials as a function of arc coordinate *s* running along the plies from the tire crown to the bead are shown in Figure 7. In the belt area the greater part of load is bearing by cords of belt plies. According to physics of the problem belt cord tensile force decreases towards the edges of the layer, because they ends are not stiffly fixed but embedded in a flexible rubber. The force does not fall to zero, as during the curing process the belts and the carcass plies are joined together with rubber. When the tensile force in the belt ply cord is decreasing the tensile force in the carcass cord is increasing, so that, within sidewall it is reaching twice the value occurred at the crown of the tire. Further, a local minimum can be seen resulting from the influence of the carcass layers turn-up. The existence of subsequent local maximum can be explained by the effect of cord-rubber layers bending around the edge of the rim. A further decline in the force value is due to increase in cord volume fraction of carcass and the impact of a rigid bead wires.

Conclusions

The proposed method of smearing of tractor tire tread lugs enables simulating of tire inflation by the use of axisymmetric model. Owing to this, obtained overall tire dimensions are in good agreement with the measurement results. This indicates a small influence of tread lugs on the overall shape of the inflated tire profile. Use of axisymmetric model in the tire inflation problem therefore has justification.

Finite element tire model makes it possible to determine the tensile force distribution between the carcass and the belt within belt zone of tire, which is not possible to achieve using the net tire model.

The simulation of pumping process results, especially information about the degree of effort of tire structural elements, can be utilized by designers of tires for their improvement.

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