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CHARACTERISTICS OF POROUS BEDS BASED ON FRACTAL PARAMETERS

***Mirosław Bramowicz*[1], *Sławomir Kulesza*[2], *Wojciech Sobieski*[3]**

[1] Department of Materials and Machine Technology
[2] Department of Relativistic Physics
[3] Department of Mechanics and Basics of Machine Construction
University of Warmia and Mazury in Olsztyn

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Abstract

The paper presents the results of a fractal analysis of the cross-sections of a porous mineral deposit consisting of spherical elements which formed a spatial system with varying porosity (0.4 to 0.95). The virtual deposit was generated using the Discrete Element Method in the YADE code by means of the so-called Radius Expansion Method. The fractal analysis was carried out using the structure function method, determining the fractal dimension (D), the topothesy (L) and the corner frequency (l) (MAINSAH et al. 2001). The conducted simulations have confirmed to a considerable extent the test results available in the literature involving the fractal analysis of mineral deposits with varying porosity. They clearly indicate that the fractal dimension does not change along with the porosity of the deposit, if the autocorrelation function or their transformations (e.g. structure function) methods are used. Moreover, based on the information available in the literature, it can be concluded that the value of the fractal dimension corresponds to mineral deposits with the specified geometric shapes of the elements which form them.

Introduction

The term "fractal" was introduced near the end of the 20th century by Benoit Mandelbrot; in Latin it means "broken, fractional" (MANDELBROT 1982). It is most commonly used to describe self-similar objects, meaning those whose certain small fragment is a scaled copy of the entire object. Therefore,

Correspondence: Katedra Technologii Materiałów i Maszyn, Wydział Nauk Technicznych, Uniwersytet Warmińsko-Mazurski, 10-719 Olsztyn, ul. Oczapowskiego 11, phone: 89 523 38 55, e-mail: miroslaw.bramowicz@uwm.edu.pl

the entire fractal object may be depicted by the subsequent iterations of that small fragment.

Fractal methods are widely used, primarily in computer science, mathematics and in many related fields of science, e.g. materials engineering, geology and even economics. Not only does their use focus on the geometric analysis of physical objects, but it also exceeds beyond it. They are found useful in the description of non-geometric values, e.g. the domain structures of magnetic materials, tendencies in financial markets, etc. (BRAMOWICZ et al. 2014, ANDRONACHE et al. 2016).

Several methods of fractal analyses are in common use, the main ones being: the variance method, box-counting, methods based on the structure function analysis S(l) (SAYLES, THOMAS 1977, MAINSAH et al. 2001) and the spectral density function PSD (KULESZA, BRAMOWICZ 2014, YADAV et al. 2015). As is known from the comparison of the results of a previously conducted experiments and computer simulations, the value of the fractal parameter heavily depends on the used method, which is why in these types of analyses the method should always be specified (KULESZA, BRAMOWICZ 2014).

Fractal methods were also found useful in the description of porous deposits (SUN, KOCH 1998) and turbulent flows (VAN'YAN 1996). The present paper uses the structure function method S(l) for the first time to correlate the characteristics of porous deposits and fractal parameters. When analysing the S(l) dependence plotted on a log-log graph and described by the formula (1), we can determine the above-mentioned fractal parameters: D, Λ, l (Fig. 1).

$$S(l) = \Lambda^{2(D-1)} l^{2(2-D)} \tag{1}$$

where:
l – the distance [m] between two points on the cross-section of the analysed deposit,
D – fractal dimension ($2<D<3$) [–],
Λ – topothesy [–].

The goal of the research described in the paper was to check the dependencies between the porosity of a granular deposit and the fractal parameters, especially in relation to a paper (SUN, KOCH 1998) which stated that for a virtual deposit generated according to the TBM model (Turning Bands Methods) there is no correlation between the fractal dimension and the porosity of the deposit. It should be pointed out that the research conducted in the paper cited herein was related to geological issues, in which the geometric structure of the porous medium was considerably different from the structure of the granular system used in the tests described in this paper.

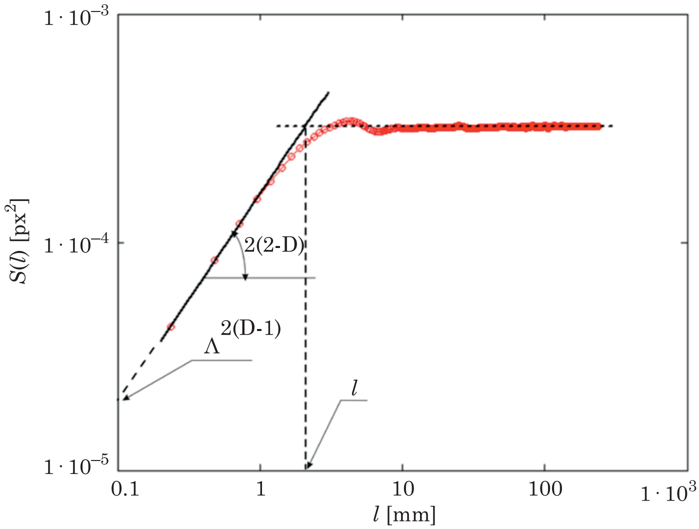

Fig. 1. A sample plot of the structure function

The material present in the article constitutes a fragment of broader research associated, among others, with the methods of the dimensional characterization of the spatial structure of granular deposits (e.g. SOBIESKI, LIPIŃSKI 2016, SOBIESKI et al. 2016). This issue is important not just in the context of the description of the porous material itself, but also in the context of predicting fluid flow resistances in such media. The search for possibilities of introducing fractal parameters into analytical models, such as the Forchheimer equation (SOBIESKI, TRYKOZKO 2011), became the main motivation to commence the research. No attempts to conduct any analyses of similar types had been made in the literature known to the authors.

Virtual deposits

The main parameter that characterizes porous beds is their porosity, that is the ratio of the volume of empty spaces randomly distributed within a solid to its total volume. In order to examine the impact of the degree of porosity of the deposit on the fractal parameters, a set of virtual granular deposits was generated with identical macroscopic dimensions (0.08×0.08×0.16 [m]) and porosities ranging between 0.4 and 0.95 (Fig. 2). The deposits were generated in the Yade numerical code (YADE 2016) using the Discrete Element Method (CUNDALL, STRACK 1979). The number of particles in each deposit was identical and amounted to 5000. The so-called Radius Expansion Method was used in order to obtain various porosities (Yade Documentation 2016, SOBIESKI et al. 2016). This method involves gradually increasing the radius of all particles in the initial cloud (generated randomly), until obtaining the specified porosity. These radii were constant in each particle cloud and depended only on the target porosity (Fig. 3).

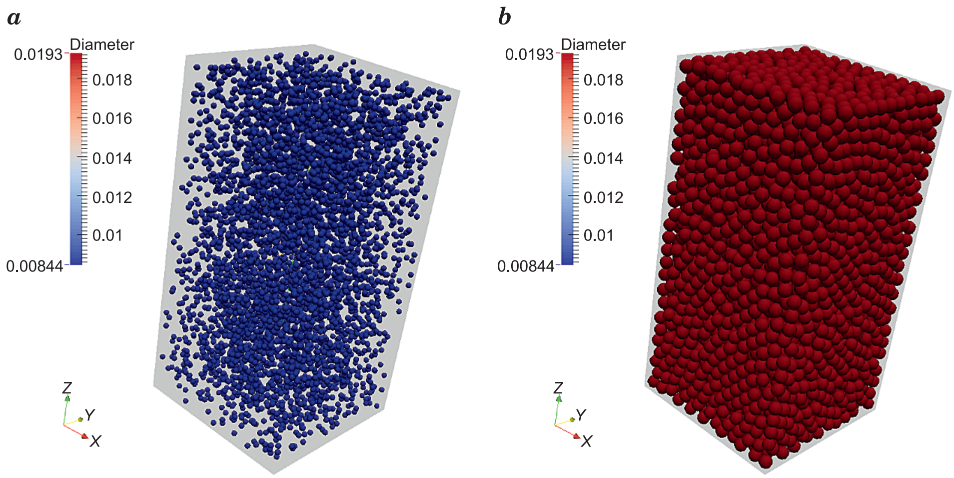

Fig. 2. Visualization of a deposit with porosities of 0.95 (*a*) and 0.4 (*b*)

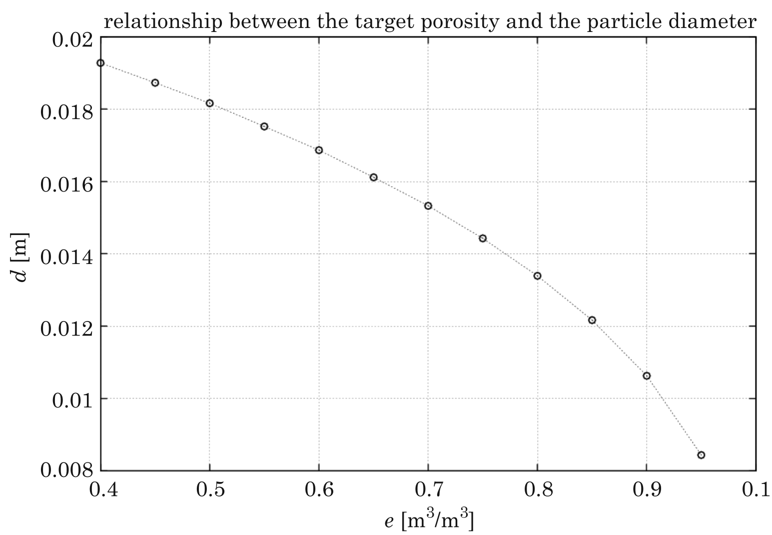

Fig. 3. Plot of particle diameters as a function of porosity

Because fractal analysis is generally performed for shapes in a 2D space, a binarised cross-section in the *YZ* plane was prepared for each virtual deposit for further purposes, passing through the centre of the deposit. Figure 4 presents four selected cross-sections of this type.

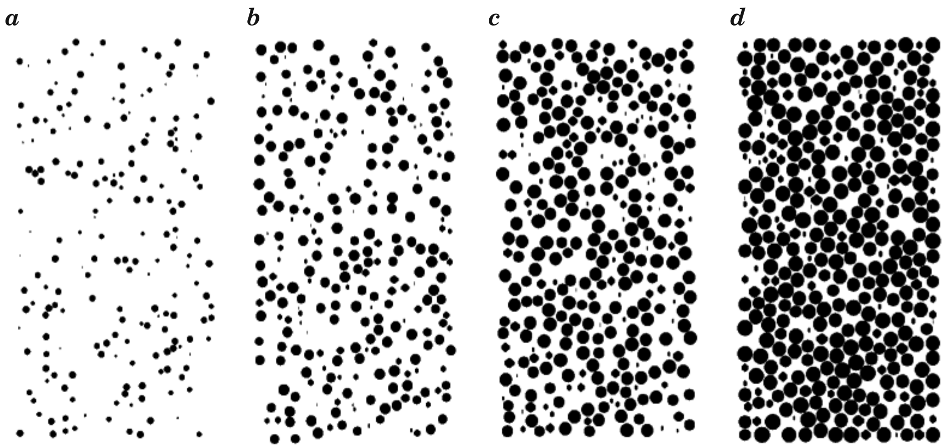

Fig. 4. Visualisation of selected cross-sections of deposits with the following porosities: *a* – 0.95, *b* – 0.8, *c* – 0.6, *d* – 0.4

Fractal analysis of cross-sections of virtual deposits

Subsequently, fractal parameters were determined based on the mean $S(l)$ profiles of the generated deposits, the obtained results being presented in Figure 5.

The conducted simulations indicate that a change in the porosity slightly affects the fractal dimension and other parameters. Observed changes can be fitted numerically with the following function (2–4):

$$D = 0.221P^2 - 0.339P + 2.642 \tag{2}$$

$$l = -16.837P^3 + 28.528P^2 - 14.52P + 4.335 \tag{3}$$

$$\Lambda = \frac{-4.643P + 4.857}{1000} \tag{4}$$

Both fractal dimension and corner frequency depend non-monotonically upon porosity approaching extreme values for P near to 0.8. In contrast, the topothesy is found to decay linearly with the porosity. However, a certain functional dependence is observed with a minimum for $P \approx 0.8$. By determining the average value of the fractal dimension D_{mean} we then get: $D_{mean} = 2.52 \pm 0.01$, while the corner frequency increases to $P \approx 0.8$ and subsequently, after reaching its maximum, it begins to decrease.

The small value of the standard deviation suggests that the fractal dimension can describe granular deposits regardless of their porosity. Similar results, i.e. the lack of a significant impact of the porosity on the fractal dimension, were obtained in a paper (SUN, KOCH 1998). The value of the fractal parameter $D = 2.82 \pm 0.03$ was then obtained for deposits generated using the above-mentioned TBM method. SUN and KOCH (1998) determined the fractal dimension using the autocorrelation function (ACF), which is associated with the structure function according to the formula (5) (MAINSAH et al. 2001):

$$S(l) = 2[ACF(l = 0) - ACF(l)] \tag{5}$$

Therefore, we could say that the results obtained in both experiments are derived using similar method based on the autocorrelation function, which lead to similar conclusions. Detailed analysis of results presented in Figure 4 by SUN and KOCH (1998) exhibits similar trend in the fractal dimension as in our paper providing that the vertical axis is extended.

In the case in question, a decrease in the value of topothesy along with the degree of porosity can also be observed (Fig. 5*b*). In order to examine this trend more precisely, the shape of the autocorrelation function map was analysed (Fig. 6*a*) for the cross-sections of the investigated deposits. Based on the symmetrical shapes of the central peak, it can be unambiguously concluded that in each case under study the deposits are highly isotropic. Using the anisotropy ratio (S_{tr}) of the surface geometrical structure included in the quality standards (PN EN ISO 25178-2:2012, PN EN ISO 25178-3:2012), it can be observed that an increase in porosity is accompanied by a slight increase towards a virtuallỳ ideal' isotropy of the deposits (Fig. 6*b*). Based on the results presented in Figure 4*b* and Figure 5*b*, confirmation of the conclusions presented in papers: THOMAS et al. (1999), BRAMOWICZ (2008) and BRAMOWICZ et al. (2013) was obtained. In these papers it was concluded that slight changes in the anisotropy of the geometric structure are accompanied by changes in topothesy, while the fractal dimension remains unchanged.

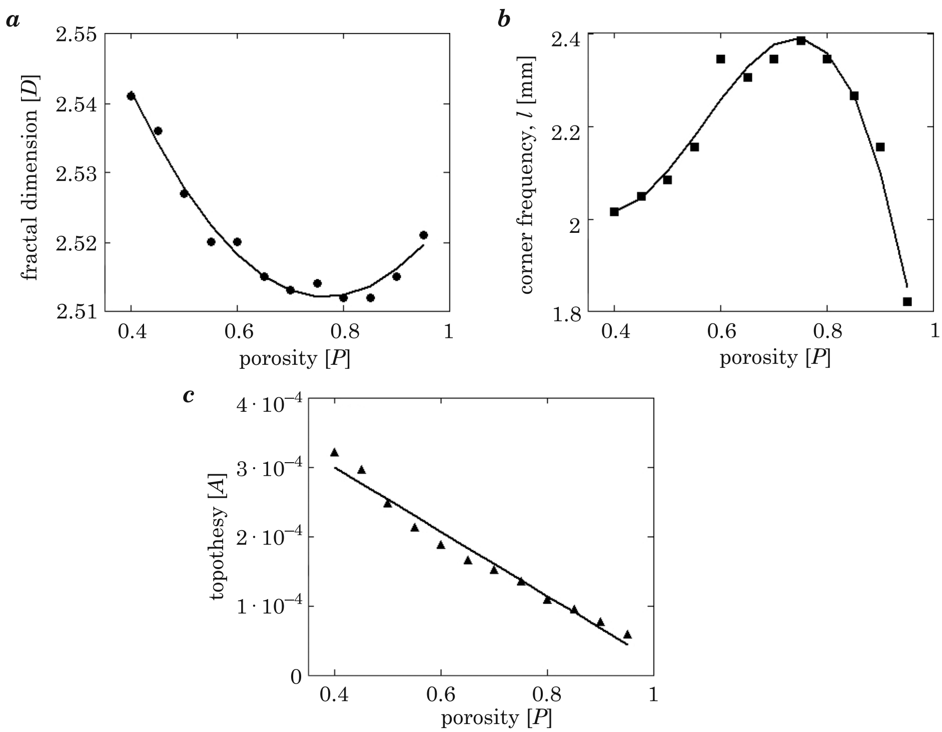

Fig. 5. The influence of the porosity on: *a* – the fractal dimension, *b* – the corner frequency, *c* – the topothesy

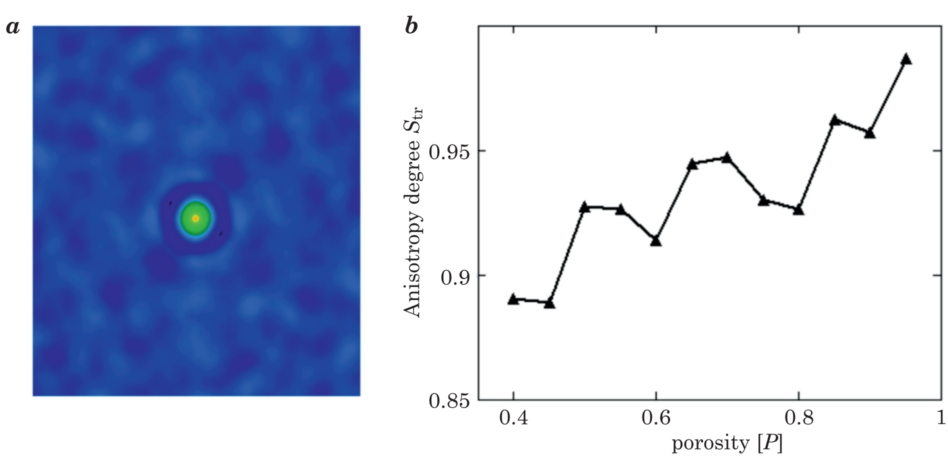

Fig. 6. A sample image of the autocorrelation function of the cross-section of a porous deposit with spherical components (*a*), the plot of porosity vs. anisotropy ratio of the geometric structure of the cross-sections of the analysed deposits (*b*)

Summary

Based on performed simulations the following conclusions can be drawn:

1. Virtual deposits generated by means of the Discrete Element Method may constitute a source of data for fractal analysis.

2. The fractal dimension is most likely dependent on the shape of the deposits' components instead of the degree of their porosity. In this study the components were represented by spherical particles.

3. The fractal dimension determined using the structure function method remains constant at 2.52 ± 0.01.

4. Slight changes in the anisotropy ratio (S_{tr}) for an isotropic cross-sections of deposits composed of spherical particles correspond to changes in the topothesy and the corner frequency.

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