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MONTE CARLO SIMULATION OF CLIMATE-WEATHER CHANGE PROCESS AT MARITIME FERRY OPERATING AREA

Ewa Kuligowska

Department of Mathematics
Gdynia Maritime University

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Abstract

The paper presents a computer simulation technique applied to generating the climate-weather change process at Baltic Sea restricted waters and its characteristics evaluation. The Monte Carlo method is used under the assumption of semi-Markov model of this process. A procedure and an algorithm of climate-weather change process' realizations generating and its characteristics evaluation are proposed to be applied in C# program preparation. Using this program, the climate-weather change process' characteristics are predicted for the maritime ferry operating area. Namely, the mean values and standard deviations of the unconditional sojourn times, the limit values of transient probabilities and the mean values of total sojourn times for the fixed time at the climate-weather states are determined.

Symbols:

$C(t)$ – climate weather change process,
c_b – climate-weather state,
w – number of climate-weather states,
Ξ_{bl} – random conditional sojourn times of a process $C(t)$ at climate-weather states c_b, when its next state is c_l,
$\xi_{bl}^{(k)}$ – realization of the conditional sojourn time Ξ_{bl}, of a process $C(t)$,
ξ – experiment time,
n_{bl} – number of sojourn time realizations during the time ξ,
$[C_{bl}(t)]_{w\times w}$ – matrix of conditional distribution functions of conditional sojourn times Ξ_{bl},
$c_{bl}(t)$ – conditional density function of the distribution function $C_{bl}(t)$,
$C_{bl}^{-1}(h)$ – inverse function of the distribution function $C_{bl}(t)$,

Correspondence: Ewa Kuligowska, Katedra Matematyki, Akademia Morska, ul. Morska 83, 81-225 Gdynia, phone: +48 58 55 86 232, e-mail: e.kuligowska@wn.am.gdynia.pl

g, h, h_1, h_2 – randomly generated numbers from the interval $\langle 0,1)$,
Ξ_b – unconditional sojourn time of a process $C(t)$ at climate-weather state c_b,
$[q_b(0)]_{1\times w}$ – vector of probabilities of a process $C(t)$ at initial states c_b,
$[q_{bl}]_{w\times w}$ – matrix of probabilities of transitions of a process $C(t)$ between climate-weather states c_b and c_l,
$q_b(t)$ – transient probability of a process $C(t)$ at a climate-weather state c_b at the moment t,
q_b – limit value of a transient probability $q_b(t)$,
M_b – mean value of unconditional sojourn time Ξ_b at climate-weather state c_b,
D_b – standard deviation of unconditional sojourn time Ξ_b at climate-weather state c_b,
$\hat{\Xi}_b$ – total sojourn time at climate-weather state c_b, during the fixed time,
$\hat{\xi}_b$ – realisation of the total sojourn time at climate-weather state c_b during the fixed time,
$\hat{M}_b$ – mean value of total sojourn time $\hat{\Xi}_b$ at climate-weather state cb during the fixed time.

Introduction

The general model of the climate-weather change process is proposed in (KOŁOWROCKI, SOSZYŃSKA-BUDNY 2016a). This process is defined by the initial probabilities at its states, the probabilities of transitions between these states and the distributions of the conditional sojourn times at these states. Further, the main characteristics of the considered process, i.e. the mean values and standard deviations of the unconditional sojourn times, the limit values of transient probabilities and the unconditional mean values of total sojourn times at the particular states for the fixed time can be determined. However, very often the analytical approach to the climate-weather change process' characteristics evaluation leads to complicated calculations, obtaining approximate results only (GRABSKI 2014, GRABSKI, JAŹWIŃSKI 2009, KOŁOWROCKI, KULIGOWSKA 2013, LIMNIOS, OPRISAN 2005). This paper proposes another non-analytical approximate approach, i.e. a computer simulation technique based on Monte Carlo method. This method can provide fairly accurate results in a relatively short time spent for calculations (KOŁOWROCKI et al. 2013, KROESE et al. 2011, MARSAGLIA, TSANG 2000, ZIO, MARSEGUERRA 2002). Moreover, the Monte Carlo simulation approach may be successfully applied in joint investigation of the climate-weather change process and its impact on safety of a very wide class of real critical infrastructures (KULIGOWSKA, TORBICKI 2017). To give an example of Monte Carlo simulation application, the climate-weather change process' analysis, identification and prediction at the maritime ferry operating area is performed in this paper.

Materials and methods

Climate-weather change process

We assume that the climate-weather change process for the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, ..., c_w$. Further, we define the climate-weather change process $C(t)$, $t \in \langle 0,\infty)$, with discrete climate-weather states from the set $\{c_1, c_2, ..., c_w\}$. We assume a semi-Markov model (GRABSKI 2014, KOŁOWROCKI 2004, 2014, KOŁOWROCKI et al. 2013, KOŁOWROCKI, KULIGOWSKA 2013, KOŁOWROCKI, SOSZYŃSKA-BUDNY 2011, LIMNIOS, OPRISAN 2005), of the climate-weather change process $C(t)$ and we mark by Ξ_{bl} its conditional sojourn times at the climate-weather states c_b, when its next climate-weather state is c_l, $b, l = 1, 2, ..., w, b \neq l$. Under these assumptions, the climate-weather change process may be described by the following parameters:

– the vector $[q_b(0)]_{1\times w}$ of the initial probabilities $q_b(0) = P(C(0) = c_b)$, $b = 1, 2, ..., w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states at the moment $t = 0$;

– the matrix $[q_{bl}]_{w\times w}$ of the probabilities q_{bl}, $b, l = 1, 2, ..., w, b \neq$ l, of the climate-weather change process $C(t)$ transitions between the climate-weather states c_b and c_l, $b, l = 1, 2, ..., w, b \neq l$, where by a formal agreement $q_{bb} = 0$ for $b = 1, 2, ..., w$;

– the matrix $[C_{bl}(t)]_{w\times w}$ of conditional distribution functions $C_{bl}(t) = P(\Xi_{bl} < t)$, $b, l = 1, 2, ..., w, b \neq l$, of the climate-weather change process $C(t)$ conditional sojourn times Ξ_{bl} at the climate-weather states, where by a formal agreement $\Xi_{bb}(t) = 0$ for $b = 1, 2, ..., w$.

Moreover, we introduce the matrix $[c_{bl}(t)]_{w\times w}$ of the density functions $c_{bl}(t)$, $b, l = 1, 2, ..., w, b \neq l$, of the climate-weather change process $C(t)$ conditional sojourn times Ξ_{bl}, $b, l = 1, 2, ..., w, b \neq$ l, at the climate-weather states, corresponding to the conditional distribution functions $C_{bl}(t)$.

Having in disposal the above parameters, it is possible to obtain the main characteristics of climate weather change process. From the formula for total probability, it follows that the unconditional distribution functions $C_b(t)$ of the climate-weather change process' $C(t)$ sojourn times Ξ_b, $b = 1, 2, ..., w$, at the climate-weather states c_b, $b = 1, 2, ..., w$, are given by $C_b(t) = P(\Xi_b \leq$ t) $= \sum_{l=1}^{w} q_{bl}C_{bl}(t)$, $t \in \langle 0,\infty)$, $b = 1, 2, ..., w$ (KOŁOWROCKI, SOSZYŃSKA-BUDNY 2016a). Hence, the mean values $M_b = E[\Xi_b]$ of the climate-weather change process' $C(t)$ unconditional sojourn times Ξ_b, $b = 1, 2, ..., w$, at the particular climate-weather states can be obtained (KOŁOWROCKI, SOSZYŃSKA-BUDNY 2016b). Further, the limit values of the climate-weather change process' transient probabilities $q_b(t) = P(C(t) = c_b)$, $b = 1, 2, ..., w$, at the particular climate-weather states

$$q_b = \lim_{t \to \infty} q_b(t), \; b = 1, 2, ..., w \tag{1}$$

can be determined (KOŁOWROCKI, SOSZYŃSKA 2011).

Monte Carlo simulation approach to climate-weather change process' modelling

We denote by $c_b = c_b(g)$, $b \in \{1, 2, ..., w\}$, the realization of the climate-weather change process' initial climate-weather state at the moment $t = 0$. Further, we select this initial state by generating realizations from the distribution defined by the vector $[q_b(0)]_{1\times w}$, according to the formula

$$c_b(g) = c_i, \; \sum_{j=1}^{i} q_{j-1}(0) \le \text{g} < \sum_{j=1}^{i} q_j(0), \; i \in \{1, 2, ..., w\} \tag{2}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$ and $q_0(0) = 0$.

After selecting the initial climate-weather state c_b, $b \in \{1, 2, ..., w\}$, we can fix the next climate-weather state of the climate-weather change process. We denote by $c_l = c_l(g)$, $l \in \{1, 2, ..., w\}$, $l \neq b$, the sequence of the realizations of the climate-weather change process' consecutive climate-weather states generated from the distribution defined by the matrix $[q_{bl}]_{w\times w}$. Those realizations are generated for a fixed b, $b \in \{1, 2, ..., w\}$, according to the formula

$$c_l(g) = c_i, \; \sum_{j=1}^{i} q_{bj-1} \le \text{g} < \sum_{j=1}^{i} q_{bj}, \; i \in \{1, 2, ..., w\}, \; i \neq b \tag{3}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$ and $q_b \; 0 = 0$.

We can use several methods generating draws from a given probability distribution. The *inverse transform method* (also known as *inversion sampling method*) is convenient if it is possible to determine the inverse distribution function (GRABSKI, JAŹWIŃSKI 2009, KOŁOWROCKI et al. 2013, KOŁOWROCKI, KULIGOWSKA 2013, KROESE et al. 2011). Unfortunately, this method is not always accurate as not every function is analytically invertible. Thus, the lack of the corresponding quantile of the function's analytical expression means that other methods may be preferred computationally (GRABSKI, JAŹWIŃSKI 2009). One of the proposed methods is a *Box-Muller transform method* that relies on the Central Limit Theorem. It allows generating two standard normally distributed random numbers, generating at first two independent uniformly distributed numbers on the unit interval. Another method is the Marsaglia and Tsang's rejection sampling method, that can be used to generate values from a monotone decreasing probability distributions, e.g. for generating gamma variate realisations (MARSAGLIA, TSANG 2000). The idea is to transform the approximate Gaussian random values to receive gamma distributed realisations.

We denote by $\xi_{bl}^{(k)}$, $b,l \in \{1, 2, ..., w\}$, $b \neq l$, $k = 1, 2, ..., n_{bl}$, the realization of the conditional sojourn times Ξ_{bl}, $b, l \in \{1, 2, ..., w\}$, $b \neq l$, of the climate-weather change process $C(t)$ generated from the distribution function $C_{bl}(t)$, where n_{bl} is the number of those sojourn time realizations during the experiment time ξ. For the particular methods described above, the realization $\xi_{bl}^{(k)}$ is generated according to the appropriate formulae (4)–(6). Thus, for each method we have:

1) the inverse transform method

$$\xi_{bl} = C_{bl}^{-1}(h),\ b,l \in \{1, 2, ..., v\},\ b \neq l \tag{4}$$

where $C_{bl}^{-1}(h)$ is the inverse function of the conditional distribution function $C_{bl}(t)$ and h is a randomly generated number from the interval $\langle 0,1)$;

2) the Box-Muller transform method for generating the realisations from the standard normal distribution

$$\xi_{bl} = \sin(26\pi h_2)\ \sqrt{-2\ ln(1 - h_1)},\ b,\ l \in \{1, 2, ..., v\},\ b \neq l \tag{5}$$

where h_1 and h_2 are two random numbers generated from the uniform distribution on the unit interval.

3) the Marsaglia and Tsang's method for generating Gamma distributed realisations

$$c_{bl}(t) = (t - x_{bl})^{\alpha_{bl}-1} \cdot \beta_{bl}^{\ -\alpha_{bl}} \cdot \Gamma^{-1}(\alpha_{bl}) \cdot \exp[-(t - x_{bl})/\beta_{bl}]\ 1_{\{t\ \in\ <x_{bl},\infty)\}} \tag{6}$$

where $c_{bl}(t)$ is the Gamma density function.

where *alfa* $= \alpha_{bl}$ and *beta* $= \beta_{bl}$, $b,\ l \in \{1, 2, ..., w\}$, $b \neq l$, are the Gamma parameters. The numbers z and u are drawn independently from the normal distribution (using the method presented in the second case) and the uniform distribution on the unit interval (using the command *NextDouble()*), respectively.

Having the realisations $\xi_{bl}^{(k)}$ *of the climate-weather change process* $C(t)$, it is possible to determine approximately the entire sojourn time at the climate-weather state c_b during the experiment time ξ, applying the formula

$$\tilde{\xi}_b = \sum_{\substack{l=1 \\ l \neq b}}^{w} \sum_{k=1}^{n_{bl}} \xi_{bl}^{(k)}, \ b \in \{1, 2, ..., w\} \tag{7}$$

Further, the limit transient probabilities defined by (1) can be approximately obtained using the formula

$$q_b = \frac{\tilde{\xi}_b}{\xi}, \ \xi = \sum_{b=1}^{w} \tilde{\xi}_b, \ b \in \{1, 2, ..., w\} \tag{8}$$

The mean values and standard deviations of the climate-weather change process' unconditional sojourn times at the particular climate-weather states are given respectively by

$$M_b = E[\Xi_b] = \frac{1}{n_b} \tilde{\xi}_b, \ n_b = \sum_{l=1}^{w} n_{bl}, \ b \in \{1, 2, ..., w\} \tag{9}$$

$$D_b = \sqrt{N_b - (M_b)^2}, \ b \in \{1, 2, ..., w\} \tag{10}$$

where

$$N_b = E[(\Xi_b)^2] = \frac{1}{n_b} = \sum_{\substack{l=1 \\ l \neq b}}^{w} \sum_{k=1}^{n_{bl}} (\xi_{bl}^{(k)})^2, \ b \in \{1, 2, ..., w\}$$

Other interesting characteristics of the climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{\Xi}_b$ at the particular climate-weather states cb, during the fixed time $\hat{\xi}$. It is well known (GRABSKI 2014, KOŁOWROCKI, SOSZYŃSKA-BUDNY 2011, LIMNIOS, OPRISAN 2005) that the process' total sojourn time $\hat{\Xi}_b$ at the state c_b, $b \in \{1, 2, ..., w\}$, for sufficiently large time has approximately normal distribution with the expected value given as follows

$$\hat{M}_b = E[\hat{\Xi}_b] = q_b \cdot \hat{\xi}, \ b \in \{1, 2, ..., w\} \tag{11}$$

The above procedures form the following detailed algorithm.

Algorithm 1. Monte Carlo simulation algorithm to estimate climate-weather change process' characteristics.

1. Draw a randomly generated number g from the uniform distribution on the interval $\langle 0, 1)$.
2. Select the initial climate-weather state c_b, $b \in \{1, 2, ..., w\}$, according to (2).
3. Draw another randomly generated number g from the uniform distribution on the interval $\langle 0,1)$.
4. For the fixed b, $b \in \{1, 2, ..., w\}$, select the next climate-weather state c_l, $l \in \{1, 2, ..., w\}$, $l \neq b$, according to (3).
5. Draw a randomly generated number h from the uniform distribution on the interval $\langle 0, 1)$.
6. For the fixed b and l, $b, l \in \{1, 2, ..., w\}$, $b \neq l$, generate a realization ξ_{bl}, of the conditional sojourn time Ξ_{bl}, $b, l \in \{1, 2, ..., w\}$, $b \neq l$, from a given probability distribution, according to (4)–(6).
7. Substitute $b := l$ and repeat 3.–6., until the sum of all generated realisations ξ_{bl} reach a fixed experiment time ξ.
8. Calculate the entire sojourn times at the climate-weather states c_b, $b = 1, 2, ..., w$, according to (7).
9. Calculate limit transient probabilities at the particular climate-weather states c_b, $b = 1, 2, ..., w$, according to (8).
10. Calculate unconditional mean sojourn times at the climate-weather states c_b, $b = 1, 2, ..., w$, according to (9).
11. Calculate standard deviations at the climate-weather states c_b, $b = 1, 2, ..., w$, according to (10).
12. Calculate mean values of the total sojourn times at the climate-weather states c_b, $b = 1, 2, ..., w$, during the fixed time, according to (11).

Results and Discussion

Parameters of climate weather change process for maritime ferry operating area

We consider the maritime ferry operating at the restricted waters of Baltic Sea area. Its climate weather change process $C(t)$, $t \in \langle 0,\infty)$, is taking $w = 6$, different climate-weather states $c_1, c_2, ..., c_6$. We assume a semi-Markov model (GRABSKI 2014, KOŁOWROCKI 2014). On the basis of the statistical data collected in Februaries (the process depends of the season and is a periodic one) during period of years 1988–1993 (KOŁOWROCKI, SOSZYŃSKA-BUDNY 2016b, KULIGOWSKA 2017) and the identification method given in (KOŁOWROCKI,

SOSZYŃSKA-BUDNY 2016a), it is possible to evaluate the unknown parameters of the semi-Markov model of the considered climate-weather change process:
– the vector

$$[q_b(0)] = [0.670,\ 0.271,\ 0.006,\ 0,\ 0.024,\ 0.029] \tag{12}$$

of the initial probabilities $q_b(0)$, $b = \{1, 2, ..., 6\}$ of the climate weather change process staying at the particular states c_b at $t = 0$;
– the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.99 & 0 & 0 & 0.01 & 0 \\ 0.84 & 0 & 0.02 & 0 & 0.14 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 & 0 & 0.64 \\ 0 & 0 & 0.93 & 0 & 0.07 & 0 \end{bmatrix}, \tag{13}$$

of the probabilities q_{bl}, b, $l = 1, 2, ..., 6$, of transitions of the climate-weather change process from the state c_b into the state c_l.

According to (KOŁOWROCKI, SOSZYŃSKA-BUDNY 2011), we may verify the hypotheses on the distributions of the climate-weather change process' conditional sojourn times at the particular climate-weather states. To do this, we, need a sufficient number of realizations of these variables (KOŁOWROCKI 2014), namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. Unfortunately, this condition is not satisfied for all sets of the statistical data we have in disposal.

The sets of the realisations of the conditional sojourn times Ξ_{12} and Ξ_{21} of the climate-weather change process were sufficiently large and we verified that they have Gamma distributions, where the density functions defined by (6) with the following parameters

$$\begin{gathered} x_{12} = 0,\ \alpha_{12} = 0.602,\ \beta_{12} = 169.801, \text{ for } b = 1,\ l = 2 \\ x_{21} = 0,\ \alpha_{21} = 2.059,\ \beta_{21} = 8.671, \text{ for } b = 2,\ l = 1 \end{gathered} \tag{14}$$

The sets of the rest realizations of the climate-weather change process' conditional sojourn times contained less than 30 realizations. Thus, we assumed that the distribution functions of climate-weather change process' conditional sojourn times Ξ_{15}, Ξ_{23}, Ξ_{25}, Ξ_{32}, Ξ_{36}, Ξ_{52}, Ξ_{56}, Ξ_{63}, Ξ_{65} have the empirical distribution functions as follows

$$C_{15}(t) = \begin{cases} 0, & t \le 9 \\ 0.5, & 9 < t \le 18 \\ 1, & t > 18, \end{cases}$$

$$C_{23}(t) = \begin{cases} 0, & t \le 21 \\ 0.5, & 21 < t \le 27 \\ 1, & t > 27, \end{cases}$$

$$C_{25}(t) = \begin{cases} 0, & t \le 3 \\ 0.286, & 3 < t \le 6 \\ 0.5 & 6 < t \le 12 \\ 0.714, & 12 < t \le 18 \\ 0,786, & 18 < t \le 24 \\ 0.857, & 24 < t \le 48 \\ 0.929, & 48 < t \le 63, \\ 1, & t > 63 \end{cases}$$

$$C_{52}(t) = \begin{cases} 0, & t \le 3 \\ 0.8, & 3 < t \le 6 \\ 1, & t > 6, \end{cases}$$

$$C_{56}(t) = \begin{cases} 0, & t \ge 3 \\ 0.444, & 3 < t \le 6 \\ 0.667, & 6 < t \le 9 \\ 1, & t > 9, \end{cases}$$

$$C_{32}(t) = \begin{cases} 0, & t \le 3 \\ 0.5, & 3 < t \le 6 \\ 0.75, & 6 < t \le 9 \\ 0.875, & 9 < t \le 18 \\ 1, & t > 18, \end{cases}$$

$$C_{25}(t) = \begin{cases} 0, & t \le 3 \\ 0.286, & 3 < t \le 6 \\ 0.5 & 6 < t \le 12 \\ 0.714, & 12 < t \le 18 \\ 0,786, & 18 < t \le 24 \\ 0.857, & 24 < t \le 48 \\ 0.929, & 48 < t \le 63, \\ 1, & t > 63 \end{cases}$$

$$C_{36}(t) = \begin{cases} 0, & t \le 3 \\ 0.5, & 3 < t \le 9 \\ 1, & t > 9, \end{cases}$$

$$C_{65}(t) = \begin{cases} 0, & t \le 6 \\ 1, & t > 6. \end{cases}$$

Monte Carlo simulation approach to characteristics evaluation of climate-weather change process for maritime ferry operating area

The simulation is performed according to the data given in the previous section. The first step is to select the initial climate-weather state c_b, $b \in \{1, 2, ..., 6\}$,
at the moment $t = 0$, using formula (2), which is given by

$$c_b(g) = \begin{cases} c_1, & 0 \le g < 0.670 \\ c_2, & 0.670 \le g < 0.941 \\ c_3, & 0.941 \le g < 0.947 \\ c_5, & 0.947 \le g < 0.971 \\ c_6, & 0.971 \le g < 1, \end{cases}$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$. The next climate-weather state $c_l = c_l(g)$, $l \in \{1, 2, ..., 6\}$, $l \ne b$, is generated according to (3), using the procedure defined as follows

$$c_l(g) = \begin{cases} c_2, & 0 \le g < 0.996 \\ c_5 & 0.99 \le g \le 1, \end{cases}$$
if $c_b(g) = c_1$;

$$c_l(g) = \begin{cases} c_1, & 0 \le g < 0.84 \\ c_3 & 0.84 \le g < 0.86 \\ c_5 & 0.86 \le g \le 1, \end{cases}$$
if $c_b(g) = c_2$;

$$c_l(g) = \begin{cases} c_2, & 0 \le g < 0.80 \\ c_6 & 0.80 \le g \le 1, \end{cases}$$
if $c_b(g) = c_3$;

$$c_l(g) = \begin{cases} c_2, & 0 \le g < 0.36 \\ c_6 & 0.36 \le g \le 1, \end{cases}$$
if $c_b(g) = c_5$

$$c_l(g) = \begin{cases} c_3, & 0 \le g < 0.93 \\ c_5 & 0.93 \le g \le 1, \end{cases}$$
if $c_b(g) = c_6$;

Applying (4), the realizations of the empirical conditional sojourn times are generated according to the formulae

$$\xi_{15}(h) = \begin{cases} 9, & 0 \le h \le 0.5 \\ 18, & 0.5 \le h < 1, \end{cases}$$

$$\xi_{23}(h) = \begin{cases} 21, & 0 \le h \le 0.5 \\ 27, & 0.5 \le h < 1, \end{cases}$$

$$\xi_{25}(h) = \begin{cases} 3, & 0 \le h \le 0.286 \\ 6, & 0.286 < h \le 0.500 \\ 12, & 0.500 < h \le 0.714 \\ 18, & 0.714 < h \le 0.786 \\ 24, & 0.786 < h \le 0.857 \\ 48, & 0.857 < h \le 0.929 \\ 63, & 0.929 < h < 1, \end{cases}$$

$$\xi_{32}(h) = \begin{cases} 3, & 0 \le h \le 0.500 \\ 6, & 0.500 < h \le 0.750 \\ 9, & 0.750 < h \le 0.875 \\ 18, & 0.875 < h < 1, \end{cases}$$

$$\xi_{36}(h) = \begin{cases} 3, & 0 \le h \le < 0.5 \\ 9, & 0.5 < h < 1, \end{cases}$$

$$\xi_{52}(h) = \begin{cases} 3, & 0 \le h \le < 0.8 \\ 6, & 0.8 < h < 1, \end{cases}$$

$$\xi_{56}(h) = \begin{cases} 3, & 0 \le h \le 0.8 \\ 6, & 0.444 < h \le 0.667 \\ 9, & 0.6676 < h < 1, \end{cases}$$

$$\xi_{63}(h) = \begin{cases} 3, & 0 \le h \le 0.462 \\ 6, & 0.462 < h \le 0.615 \\ 9, & 0.615 < h \le 0.692 \\ 21, & 0.692 < h \le 0.769 \\ 24, & 0.769 < h \le 0.846 \\ 27, & 0.846 < h \le 0.923 \\ 30, & 0.923 < h < 1, \end{cases}$$

$$\xi_{65}(h) = 6$$

where h is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$.

The climate-weather change process characteristics, for Februaries of the years 1988–1993, are calculated using the Monte Carlo simulation method with time of the experiment fixed as

$$\xi = 6 \text{ years} \cong 52\,595 \text{ hours}.$$

Applying (8) the limit values of the climate-weather change process' transient probabilities at the particular climate-weather states are as follows:

$$q_1 = 0.807,\ q_2 = 0.162,\ q_3 = 0.009,\ q_4 = 0,\ q_5 = 0.007,\ q_6 = 0.015 \quad (15)$$

Based on the formula (9), the climate-weather change process' unconditional mean sojourn times measured in hours at the particular climate-weather states are given by

$$M_1 = 101.79,\ M_2 = 17.23,\ M_3 = 6.85,\ M_4 = 0,\ M_5 = 4.96,\ M_6 = 11.15 \quad (16)$$

whereas applying (10), the standard deviations of the climate-weather change process' unconditional sojourn times, are as follows

$$D_1 = 126.05,\ D_2 = 13.71,\ D_3 = 5.38,\ D_4 = 0,\ D_5 = 2.61,\ D_6 = 10.55 \quad (17)$$

Hence, applying (11) and according to (15), the climate-weather change process' expected values $\hat{M}_b$ measured in days of the total sojourn times $\hat{\Xi}_b$ at the particular climate-weather states and during the fixed time $\hat{\xi} = 10 \cdot 28$ February days = 280 days, are given by

$$\hat{M}_1 \cong 226,\ \hat{M}_2 \cong 45,\ \hat{M}_3 \cong 3,\ \hat{M}_4 \cong 0,\ \hat{M}_5 \cong 2,\ \hat{M}_6 \cong 4 \quad (18)$$

Comments on the climate-weather change process characteristics evaluation

The experiment was performed basing on the statistical data sets collected in Februaries during a 6-year period of time. It can be expected that for other months, the result will be different. Thus, before the climate-weather change process identification, the investigation of these empirical data uniformity is necessary. The data sets collected per each month of the year during the experiment time should be uniformly tested, and if it is reasonable, the data from selected month sets can be joined into season sets. This way, the sets of the analyzed data will be larger and processes created on them will be better reflected to the considered real climate-weather change process. These improvements of the accuracy of the climate-weather change processes identification and prediction are the future steps in the research.

Conclusions

The Monte Carlo simulation method was applied to the approximate evaluation of the climate-weather change process' main characteristics at the maritime ferry operating area for a fixed month February. The obtained results may be considered as an illustration of the possibilities of the proposed Monte Carlo simulation method application to the climate-weather change process' analysis and prediction. Moreover, the results justify practical sensibility and very high importance of considering the climate-weather change process at critical infrastructure different operating areas. Especially, this considering is important in the investigation of the climate weather change process influence on the critical infrastructure safety as it could be different at various operating areas and various months of the year (KULIGOWSKA, TORBICKI 2017).

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