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THE SINGULARITIES AT THE VICINITY OF THE TRIPLE POINT OF CONTACT OF THREE ORTHOTROPIC WEDGES IN PLANE ELASTICITY

Aleksandra Wieromiej-Ostrowska

Department of Technological and Computer Science Education
University of Warmia and Mazury in Olsztyn

Key words: anisotropic elasticity, stress singularities.

Abstract

The particular problem of the order of singularity at the vicinity of the triple point of contact of three wedges made of the same (rotated) materials is considered.

The order of singularity λ changes with the change of the opening angle φ and the wedge rotation angle ψ of at least one material. Relations λ–φ for different sets of elastic constants corresponding to the composites of the epoxy resin and kevlar fiber, epoxy resin and boron fiber and the real metallic cubic crystal (aluminium and tungsten) have been studied. The real solutions were taken into considerations only.

General cases of the lack of symmetry were considered. Modes of stress distribution for different values of λ were found. It is not difficult to notice that neither of them describes symmetric nor skew-symmetric stress field.

RZĄD OSOBLIWOŚCI W OTOCZENIU WIERZCHOŁKA POTRÓJNEGO PUNKTU KONTAKTU TRZECH KLINÓW W PŁASKIM ZAGADNIENIU TEORII SPRĘŻYSTOŚCI

Aleksandra Wieromiej-Ostrowska

Zakład Edukacji Technicznej i Informatycznej
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: anizotropia sprężysta, osobliwości pól naprężeń.

Streszczenie

W pracy rozpatrzono problem rzędu osobliwości λ w otoczeniu wierzchołka potrójnego punktu kontaktu trzech klinów wykonanych z tych samych (obróconych) lub różnych ortotropowych materiałów linowo sprężystych. Zbadano przebiegi zmienności rzędu osobliwości λ ze zmianą kąta rozwarcia klina φ przy różnych kątach obrotu ψ przynajmniej jednego z materiałów i dla różnych kombinacji stałych sprężystych odpowiadających takim materiałom, jak: kompozyt żywicy epoksydowej i włókna kewlarowego, kompozyt żywicy epoksydowej i włókna borowego, rzeczywisty kryształ metaliczny w układzie regularnym (aluminium i wolfram). Poszukiwano rozwiązań o rzeczywistych wartościach λ.

Dla wybranego przypadku niesymetrycznego znaleziono rozkłady naprężeń odpowiadających wartościom λ. Rozkłady te nie wykazywały symetrii ani antysymetrii.

Introduction

Strength of singularity at the tip of the orthotropic wedge embedded into infinite two-dimensional elastic orthotropic body was considered in the Part One of the previous paper (Blinowski and Wieromiej-Ostrowska (2005)). Considerations were restricted to the wedges symmetrically oriented with respect to the axes of orthotropy. They concerned mainly, the relations between the order of singularity and the wedge opening angle and/or elastic properties of materials. Mixed boundary value conditions were assumed: continuity of both, tractions and displacements at the interfaces was demanded.

Two modes of the stress distribution corresponding to different values of λ: symmetric and skew-symmetric were found. For the case of nearly isotropic materials the quantitive results roughly repeated these obtained by the authors for isotropic materials reported in the paper by Blinowski and Wieromiej (2004), where the symmetries of solutions were assumed in advance.

In the Part Two of the mentioned paper the considerations were generalized on the wedges arbitrarily oriented with respect to the axes of orthotropy and on the complex solutions $\lambda = a + ib$ as well.

It was found that the real solutions for λ are solitary in the complex plane, no complex solutions corresponding to the finite elastic energy in the vicinity of the wedge tip ($0 \leqslant \mathrm{Re}\lambda \leqslant 2$) were found.

The particular problem of the order of singularity at the vicinity of the triple point of contact of three wedges made of the same (rotated) materials is considered in the present paper. Solution of such a problem may bring a valuable contribution into understanding of material fatigue process. Mathematical treatment of such a problem does not differ from the used in the previous papers by Blinowski and Wieromiej-Ostrowska (2005) and (2007). The only difference consists in the use of larger 12 x 12 matrices.

Formulation of the boundary value problem

As it has been already mentioned, we shall focus our attention in the present paper on the mixed boundary value problem assuming continuity of displacement fields as well as the normal and tangential stresses at the interfaces in the vicinity of the point of contact of three wedges.

We shall consider the general non-symmetric problems: compare Figure 1. All quantities in the domain of the first material will be noted using the

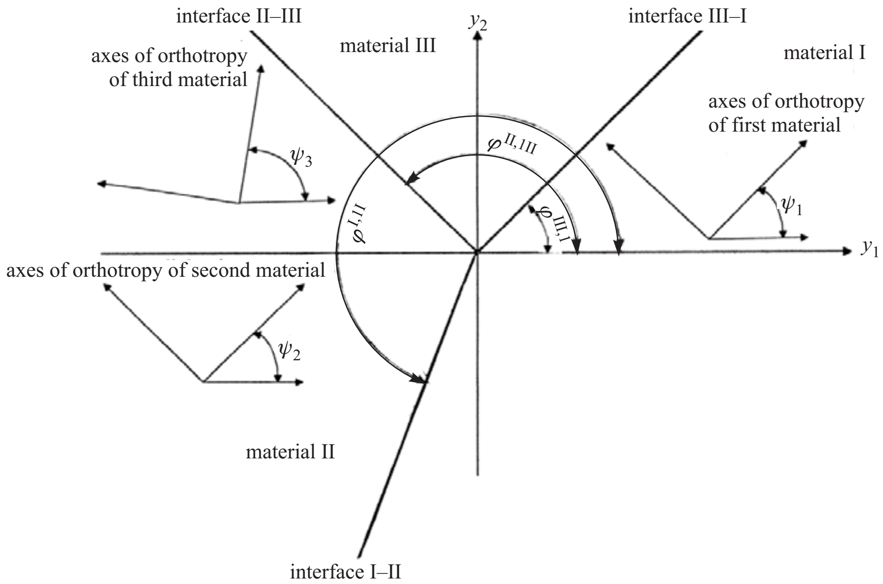

Fig. 1. Orientation of the triple point of contact of three wedges made of the same (rotated) or different materials with respect to the observer frame axes $\{y_1, y_2\}$
– $(\varphi^{\mathrm{I,II}} - \psi^{\mathrm{I}})$ the angle between the first material axis of ortothropy and I–II interface
– $(\varphi^{\mathrm{I,II}} - \psi^{\mathrm{II}})$ the angle between the second material axis of ortothropy and I–II interface
– $(\varphi^{\mathrm{II,III}} - \psi^{\mathrm{II}})$ the angle between the second material axis of ortothropy and II–III interface
– $(\varphi^{\mathrm{II,III}} - \psi^{\mathrm{III}})$ the angle between the third material axis of ortothropy and II–III interface
– $(\varphi^{\mathrm{III,I}} - \psi^{\mathrm{III}})$ the angle between the third material axis of ortothropy and III–I interface
– $(\varphi^{\mathrm{III,I}} - \psi^{\mathrm{I}})$ the angle between the first material axis of ortothropy and III–I interface.

Roman number one (I), while for the second and third material indices II and III will be used.

The complete set of the boundary value conditions must be taken as follows:

$$\left.\begin{array}{l} u^{I}_{rI,II} - u^{II}_{rI,II} = 0 \\ u^{I}_{\varphi I,II} - u^{II}_{\varphi I,II} = 0 \end{array}\right\} \text{– displacement continuity for I–II interface}$$

$$\left.\begin{array}{l} \sigma^{I}_{\varphi\varphi I,II} - \sigma^{II}_{\varphi\varphi I,II} = 0 \\ \sigma^{I}_{r\varphi I,II} - \sigma^{II}_{r\varphi I,II} = 0 \end{array}\right\} \text{– traction continuity for II–I interface}$$

$$\left.\begin{array}{l} u^{II}_{rII,III} - u^{III}_{rII,III} = 0 \\ u^{II}_{\varphi II,III} - u^{III}_{\varphi II,III} = 0 \end{array}\right\} \text{– displacement continuity for II–III interface} \quad (1)$$

$$\left.\begin{array}{l}\sigma^{II}_{\varphi\varphi II,III}-\sigma^{III}_{\varphi\varphi II,III}=0\\ \sigma^{II}_{r\varphi II,III}-\sigma^{III}_{r\varphi II,III}=0\end{array}\right\}\text{– traction continuity for II–III interface}$$

$$\left.\begin{array}{l}u^{III}_{rIII,I}-u^{I}_{rIII,I}=0\\ u^{III}_{\varphi III,I}-u^{I}_{\varphi III,I}=0\end{array}\right\}\text{– displacement continuity for III–I interface}$$

$$\left.\begin{array}{l}\sigma^{III}_{\varphi\varphi III,I}-\sigma^{I}_{\varphi\varphi III,I}=0\\ \sigma^{III}_{r\varphi III,I}-\sigma^{I}_{r\varphi III,I}=0\end{array}\right\}\text{– traction continuity for III–I interface}$$

where $\sigma_{\varphi\varphi}$ and $\sigma_{r\varphi}$ are the transversal and tangential stresses in polar coordinates, while u_r and u_φ denote the radial and transversal components of the displacement field.

Conditions (1) and relations (29) from the paper of BLINOWSKI and WIEROMIEJ-OSTROWSKA (2005) generate the following homogenous system of twelve equations for the determination of the unknown multipliers in the linear combinations of four particular solutions (A_1, A_2, B_1, B_2) for the all three materials I, II, III.

$$\underline{\mathbf{A}}\ \underline{\mathbf{x}}=0 \qquad (2)$$

where $$\underline{\mathbf{x}}=\begin{bmatrix}A_1^I\\ B_1^I\\ A_2^I\\ B_2^I\\ A_1^{II}\\ B_1^{II}\\ A_2^{II}\\ B_2^{II}\\ A_1^{III}\\ B_1^{III}\\ A_2^{III}\\ B_2^{III}\end{bmatrix}$$

Regarding the terms of the matrix **A** – see Appendix.

Results and conclusions

As one can expect the order of singularity λ changes with the change of the wedge opening angle φ and of the wedge rotation angle ψ of at least one material. Calculations have been done for different values of the elastic constants corresponding to some composites: epoxy resin and kevlar fiber, epoxy resin and boron fiber and for the real metallic cubic crystal (aluminium and tungsten). The real solutions only were taken into considerations only.

Some particular problems were solved. For the first of them the elastic properties of material corresponded to the composite of the epoxy resin and kevlar fiber. The wedge opening angle φ_1 of the first material varied within the interval $\varphi_1 \in \left(0, \frac{\pi}{2}\right)$. For the second material the opening angle was taken $\varphi_2 = \pi$, for the third material the opening angle varied corresponding to change of the first one $(\varphi_1 + \varphi_3 = \pi)$[1]. The first material was symmetrically oriented with respect to the symmetry plane of the body, the second material of the wedge was rotated by the angle $\psi_2 = \frac{2}{3}\pi$, while the third material was rotated by the angle $\psi_3 = -\frac{2}{3}\pi$. For given angles of rotation a single solutions only for the wedge opening angles φ_1 within the interval $\left(\frac{\pi}{12}, \frac{\pi}{2}\right)$ was found, while no solutions for φ_1 within interval $\left(0, -\frac{\pi}{12}\right)$ was observed, see Figure 2.

Similar problem for the same material and different angle $\psi_1 = \frac{\pi}{2}$ was studied. No solution was observed in this case.

The same problem was considered for composite of epoxy resin and boron fiber and no results were obtained.

For the case of the boron-epoxy composite and the geometry corresponding to the first problem double solution for the wedge opening angles $\varphi_1 \in (0.18, 0.7)$ were found. For $\varphi_1 \in (0.04, 0.18)$ and $\varphi_1 \in (0.18, 0.7)$ single solu-

[1] We adopt the following notation:
$\varphi_1 = \varphi^{I,II} - \varphi^{I,III}$, $\varphi_2 = \varphi^{I,II} - \varphi^{II,III}$, $\varphi_3 = \varphi^{II,III} - \varphi^{I,III}$.

a

b

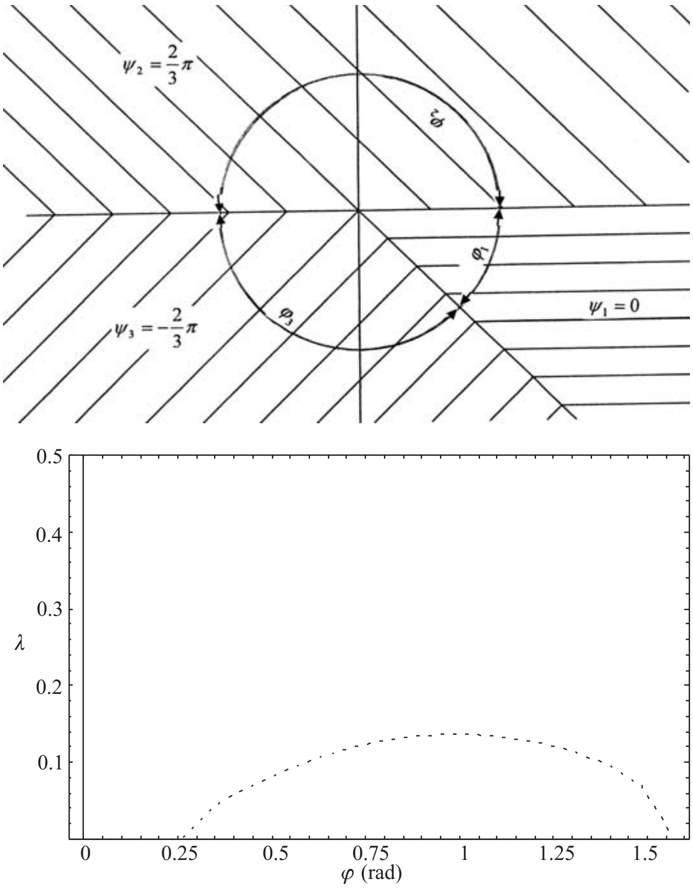

Fig. 2. The point of contact of three wedges made of the same (rotated) materials corresponding to the composite of the epoxy resin and kevlar fiber, compare Table 1.
The order of singularity λ versus opening angles φ, and given wedge rotation angles ψ.

Table 1

Elastic constans	E_1	E_2	γ_1	γ_2	γ_3
Material I	91.1	15.87	0.59	4.01	2.92
Elastic constans	E_1	E_2	γ_1	γ_2	γ_3
Material II	91.1	15.87	0.59	4.01	2.92

tions were found while no solutions were observed at the remaining part of the interval $\left(0, \frac{\pi}{2}\right)$, see Figure 3.

For the same case with the first material rotated by $\frac{\pi}{2}$, the results were qualitatively similar: compare Figure 4. This makes a difference with

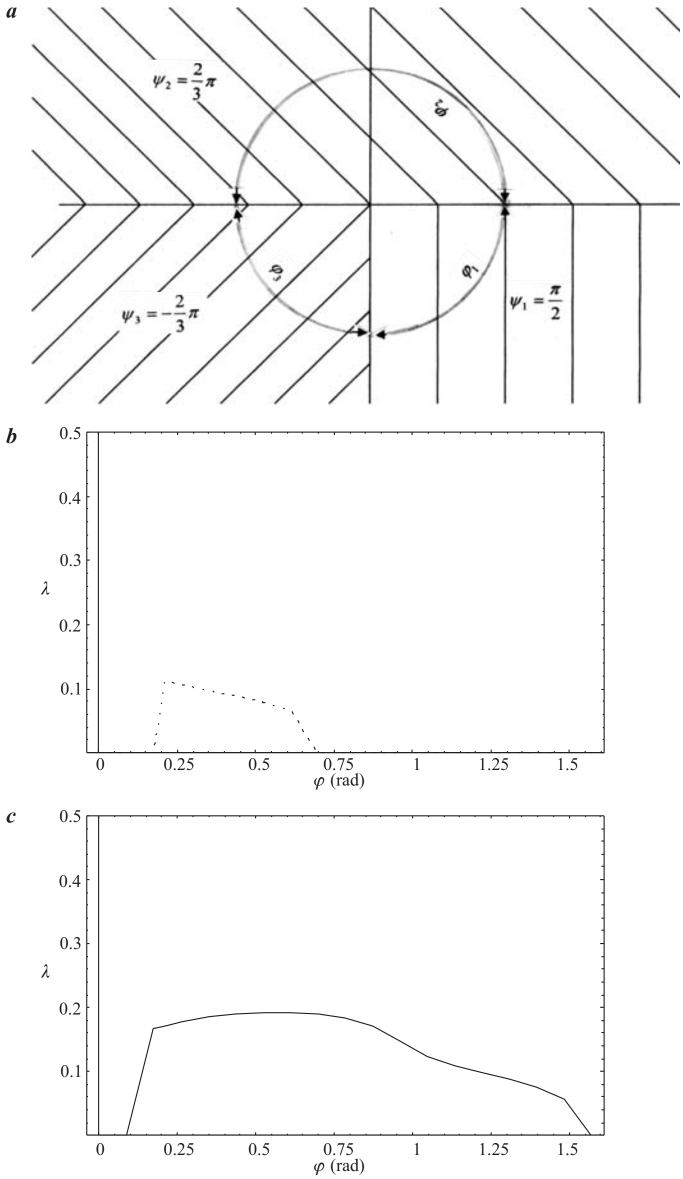

Fig. 3. The point of contact of three wedges made of the same (rotated) materials corresponding to the composite of the epoxy resin and boron fiber, compare Table 2.
The order of singularity λ versus opening angles φ, and given wedge rotation angles ψ.

Table 2

Elastic constans	E_1	E_2	γ_1	γ_2	γ_3
Material I	266.9	16.6	1.05	3.79	2.82
Elastic constans	E_1	E_2	γ_1	γ_2	γ_3
Material II	66.5	9	0.6	4.51	3.27

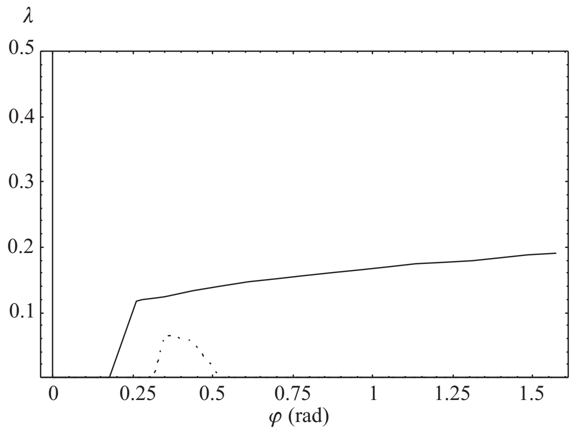

Fig. 4. The point of contact of three wedges made of the same (rotated) materials corresponding to the composite of the epoxy resin and boron fiber, compare Table 2.
The order of singularity λ versus opening angles φ, and given wedge rotation angles ψ.

respect to the case of kevlar-epoxy composite, where solutions disappeared under such a rotation.

For the case of the real metallic cubic crystal (aluminium and tungsten), where the point of contact of three wedges made of the same (rotated) materials was considered, under the same geometry as in the problem depicted in the Figure 2. No real solution for λ from the interval (0,1) corresponding to finite elastic energy was observed.

For the symmetric problems considered in paper BLINOWSKI and WIEROMIEJ-OSTROWSKA (2005) the symmetric and skew-symmetric distributions of stress have been found, while for the non- symmetric problems (considered in paper BLINOWSKI and WIEROMIEJ-OSTROWSKA (2007)) two non-symmetric distributions of stress have been found.

In the present paper the lack of symmetry of the boundary value problem yields two non-symmetric solutions as well.

For the case of the point of contact of three wedges made of the same materials corresponding to epoxy-boron composite for the first wedge opening angle $\varphi_1 = \frac{5}{36}\pi$, the second wedge opening angle $\varphi_2 = \pi$ and the first material of the wedge rotated by the angle $\psi_1 = \frac{\pi}{2}$, the second material of the wedge rotated by the angle $\psi_2 = \frac{2}{3}\pi$, while the third material of the wedge was rotated by the angle $\psi_3 = -\frac{2}{3}\pi$ one obtains: $\lambda_1 = 0.088$, while the corresponding eigenvector is the following:

$[0.10;-0.58;0.04;-0.006;0.21;-0.17;0.05;0.001;-0.08;-0.74;0.009;-0.002]$[2], for the second value of $\lambda_2 = 0.189$ the corresponding eigenvector looks as follows: $[-0.30;-0.47;0.01;0.02;-0.56;-0.36;0.03,0.012;0.28;-0.37;-0.02;0.019]$, compare Figure 4. It is not difficult to notice that neither of them describes symmetric nor skew-symmetric stress field.

From the results mentioned above no firm conclusions concerning the existence and the number of solutions as well as for the strength of the singularities can be formulated at the present stage of study of that subject. For particular combinations of the materials employed in engineering constructions the problems must be separately solved for every case under consideration.

References

BLINOWSKI A., OSTROWSKA-MACIEJEWSKA J. 1995. *On the stress distribution in bending of strongly anisotropic beams,* Engng.Trans., 43: 1–2, 83–89.

BLINOWSKI A., WIEROMIEJ A. 2004. *Singular stress in the vicinity of a sharp inclusion in an elastic matrix,* Techn. Sc., 7: 159–174.

BLINOWSKI A., WIEROMIEJ-OSTROWSKA *A.* 2005. *On the singularities at the tips of the orthotropic wedges in plane elasticity.* Part One, Techn. Sc., 8: 107–124.

BLINOWSKI A., WIEROMIEJ-OSTROWSKA *A.* 2006. *On the singularities at the tips of the orthotropic wedges in plane elasticity.* Part Two, Techn. Sc., 9:65–72.

ŚLEZIONA J. 1998. *Podstawy technologii kompozytów.* Wydawnictwo Politechniki Śląskiej, Gliwice.

NYE J.F. 1962. *Własności fizyczne kryształów.* PWN, Warszawa.

SEWERYN A. 1997. *Kumulacja uszkodzeń i pękanie elementów konstrukcyjnych w złożonych stanach obciążeń.* Białystok.

SEWERYN A., MOLSKI K. 1996. *Elastic stress singularities and corresponding generalized stress intensity factors for angular corners under various boundary conditions.* Engineering Fracture Mechanics, 55: 529–556.

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Appendix

For reference, we shall expose here explicit expressions for the components of the matrix **A.** For the sake of brevity we shall subdivide our matrix into nine square submatrices as follows:

$$\underline{\mathbf{A}} = \begin{bmatrix} \mathbf{A}^1 & \mathbf{A}^2 & 0 \\ 0 & \mathbf{A}^3 & \mathbf{A}^4 \\ \mathbf{A}^5 & 0 & \mathbf{A}^6 \end{bmatrix},$$

For submatrix $\mathbf{A}^1$ one can write:

[2] Zeros at the even positions in eigenvectors of the matrix **A** describe the symmetric stress of distribution, while zeros of the odd positions corresponds to skew-symmetric of the stress field.

$$A_{11}=-\left[\begin{array}{l}\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)+\\ \gamma_1^I\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{12}=-\left[\begin{array}{l}\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)-\\ \gamma_1^I\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{13}=-\left[\begin{array}{l}\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)+\\ \gamma_2^I\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{14}=-\left[\begin{array}{l}\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)\\ -\gamma_2^I\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{21}=\left[\begin{array}{l}\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)-\\ \gamma_1^I\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{22}=\left[\begin{array}{l}\dfrac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)+\\ \gamma_1^I\dfrac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_2^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)\end{array}\right]/E^I,$$

$$A_{23}=\begin{bmatrix}\frac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)-\\ \gamma_2^I\frac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)\end{bmatrix}/E^I,$$

$$A_{24}=\begin{bmatrix}\frac{\left(\gamma_1^I\right)^2-\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)+\\ \gamma_2^I\frac{-\left(\gamma_1^I\right)^2+\left(\gamma_2^I\right)^2-2\left(\gamma_3^I\right)^2}{\gamma_1^I}\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)\end{bmatrix}/E^I,$$

$$A_{31}=-\gamma_1^I\operatorname{Im}\left(\varphi^{I,II}-\psi^I,2-\lambda,\gamma_1^I\right),$$
$$A_{32}=-\gamma_1^I\operatorname{Re}\left(\varphi^{I,II}-\psi^I,2-\lambda,\gamma_1^I\right),$$
$$A_{33}=-\gamma_2^I\operatorname{Im}\left(\varphi^{I,II}-\psi^I,2-\lambda,\gamma_2^I\right),$$
$$A_{34}=-\gamma_2^I\operatorname{Re}\left(\varphi^{I,II}-\psi^I,2-\lambda,\gamma_2^I\right),$$
$$A_{41}=\left[\left(\gamma_1^I\right)^2\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)-\gamma_1^I\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\right],$$
$$A_{42}=-\left[\left(\gamma_1^I\right)^2\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)+\gamma_1^I\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_1^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\right],$$
$$A_{43}=\left[\left(\gamma_2^I\right)^2\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)-\gamma_2^I\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\right],$$
$$A_{44}=-\left[\left(\gamma_2^I\right)^2\operatorname{Im}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\cos\left(\varphi^{I,II}-\psi^I\right)+\gamma_2^I\operatorname{Re}\left(\varphi^{I,II}-\psi^I,1-\lambda,\gamma_2^I\right)\sin\left(\varphi^{I,II}-\psi^I\right)\right],$$

The elements of remaining submatrices can be obtained easily by the following modifications of matrix $\mathbf{A^1}$:

- for matrix $\mathbf{A^2}$ variables $E^I\ \psi^I,\ \gamma_1^I,\gamma_2^I$ should be replaced with E^{II} $\psi^{II},\ \gamma_1^{II},\gamma_2^{II}$ and signs of all terms should be changed;
- matrix $\mathbf{A^3}$ can be obtained from $\mathbf{A^1}$ by replacing variable $\varphi^{I,II}$ with $\varphi^{II,III}$ and variables $E^I\ \psi^I,\ \gamma_1^I,\gamma_2^I$ should be replaced with $E^{II}\ \psi^{II},\ \gamma_1^{II},\gamma_2^{II}$;

- for $\mathbf{A^4}$ one should replace in $\mathbf{A^2}$ variable $\varphi^{I,II}$ with $\varphi^{II,III}+2\pi$ and variables E^{II} $\psi^{II}, \gamma_1^{II}, \gamma_2^{II}$ should be replaced with E^{III} $\psi^{III}, \gamma_1^{III}, \gamma_2^{III}$;
- matrix $\mathbf{A^5}$ can be obtained from $\mathbf{A^1}$ by replacing variable $\varphi^{I,II}$ with $\varphi^{III,I}$;
- matrix $\mathbf{A^6}$ can be obtained from $\mathbf{A^2}$ by replacing variable $\varphi^{I,II}$ with $\varphi^{III,I}$ and variables E^{II} $\psi^{II}, \gamma_1^{II}, \gamma_2^{II}$ should be replaced with E^{III} $\psi^{III}, \gamma_1^{III}, \gamma_2^{III}$.