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POSSIBILITIES OF APPLICATION OF STOCHASTIC PROCESS THEORY IN EVALUATION OF TRIBOLOGICAL EXPERIMENT

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Key words: statistical dynamics, stochastic process, power spectral density, autocorrelation function, tribologic experiment.

Abstract

Mathcad 11, a software product of Mathsoft, was used for random numbers generation. The program enables generating sets of random numbers of various statistical properties. We used random distribution for our application. The set of random numbers had undergone statistical analysis using statistical dynamics methods. We applied low-permeable numeric filter. After filtration, the statistical analysis results have shown, that the power spectrum is relatively low. This has been confirmed also by the Fisher test. There are 15 important frequencies total, ranging in interval of 0.5 Hz – 6.2 Hz. Statistically, the most signifficant frequency is 0.5 Hz. Evaluated and described courses with their frequency spectrums can be used as input parameters of rationalizing and optimizing methods of tools�' and machinery's design modification. They can be also used as input data of load in adhesive wear testing within tribologic experiments.

MOŻLIWOŚCI WYKORZYSTANIA TEORII PROCESÓW STOCHASTYCZNYCH DO OCENY EKSPERYMENTU TRIBOLOGICZNEGO

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Słowa kluczowe: dynamika statystyczna, proces stochastyczny, gęstość spektralna, autokorealcja, eksperyment trybologiczny.

Streszczenie

Do wygenerowania zbioru liczb przypadkowych zastosowano produkt Mathcada 11 firmy Mathsoft. Program pozwala na generowanie liczb przypadkowych o różnych właściwościach statystycznych. Na potrzeby eksperymentu przyjęto podział przypadkowy. Zbiór liczb przypadkowych poddano analizie statystycznej z zastosowaniem metod dynamiki statystycznej. W analizie zastosowano dolnoprzepuszczalny filtr liczbowy. Po filtracji analiza statystyczna wykazała, że realizacyjne spektrum częstotliwości jest relatywnie niskie, co potwierdzono również w teście Fishera. Istotnych jest tylko 15 częstotliwości w zakresie 0,5÷6,2 Hz. Statystycznie najbardziej istotna jest częstotliwość 0,5 Hz.

Opisywane funkcje i ich spektra częstotliwościowe mogą być wykorzystane jako wstępne parametry metod optymalizowania konstrukcyjnego narzędzi i maszyn, a także jako wstępne parametry obciążeń w badaniach trybologicznych związanych z adhezyjnym zużyciem materiałów.

Introduction

In practice, it is normally continuous random process that occur. Random process analysis by digital computer assumes numeric formulation of the examined problem. Several operations go before the analysis.

First of all it is scanning the examined process. Concerning non-electrical process, it is tranformed into electrical signal. Normally, direct real-time analysis of the process is not executed, but it is recorded on an appropriate data carrier. It is possible to replay the record anytime. Using A/D converter, data are regularly scanned from the stochastic process application which is analysed. These data are expressed numerically. This is what we call sampling of the stochastic process realization. Throughout the whole process several errors may occur. Errors may affect correctness of the analysis results. These errors may be caused by inertia, non-linearity of the elements used in measuring sequence, additive noise etc. For the process analysis to be successful, all the errors must be negligible. We pay great attention to errors that occur during sampling.

Checking the pattern of the process is a part of the whole scanned process analysis. We mean by this the stacionarity test, detection of periodical components and normality test. Single steps of the analysis give us answers to these questions.

Ergodicity of the process is advantageous for its analysis. Single realization of the examined process is sufficient as a source of information about it. We may eliminate errors caused by sampling the continuous realization of the random process into numeric data by convenient choice of interval between moments of sampling to negligible value. Sampling period Δt is chosen so that the limiting frequency f_m will be bigger than the maximum value of the frequency $f_{\max}$ contained in sampling signal spectrum (of the realization). Usually, $f_m = (1.5 \div 2) f_{\max}$ is chosen with reserve.

Random process can be reached by experimental measuring of the particular device (certain nodes) in real working conditions, or by generating a group of random numbers with defined statistical features. For our application we used standard distribution with following defined arguments: number of figures with corresponding distribution, mean value - median, standard deviation.

We adjusted the generated group of random numbers using digit filter. We filtered off frequencies over 10 Hz. Adjusted group was analysed within the correlation theory.

Specific working conditions of agricultural machinery interact its working life, which is often relatively short, as a result of heterogenous modes of loading and subsequent deterioration of its components and their surface. In agriculture, from 70% to 90% of all damaged components are inactivated because of an abnormal wear. Tribologic node:

– pivot or shaft
– slide bearing, shaft seal etc.
– grease cap
– environment

ranks among frequent nodes in agricultural machinery and plays an important role in transmitting the power from driving unit to functional parts of adapters. The wear and loss of functional capacity of node‘s parts is a result of relative motion and load.

Real tribologic node of an agricultural device in work is a subject to complex dynamic stress of working load. Generators of the work load are variable values of power effects, deformations and motion acceleration of functional parts of the device, which determine its operation reliability. Therefore it is necessary to pay attention to such random processes. Using statistical dynamics in evaluation of values measured in tribologic experiment conditions it is possible to gain information revealing hidden relations between single values and between various statuses of the examined system.

Material and methods

Weight-loss of the samples within tribologic experiment were simulated as a demonstration of exploitation facilities of the random process theory. It was executed using random number generator with defined statistical parameters. Simulated procedures of realization after the transformation of data to PC were adjusted by specific program technique (filtration, basic deterministic parameters of random realization, stacionarity test, trend test for mean value and dispersion, histogram, distribution function).

After the accumulation of the set of digitalized parameters in the PC memory the following values have been defined:

Mean value dependent on time:

$$m(t_i) = \frac{1}{N_i} \cdot \sum_{i=1}^{N_i} p(i) \qquad (1)$$

where:

$N_i = \frac{N}{I}$ is longitude of the chosen interval

N is number of p values in realization
I is number of values in the chosen interval

Centred values of the process

$$p^{\circ}(i) = p(i) - m(t_i) \tag{2}$$

Dispersion of the random value

$$\mu_2 = \int_{-\infty}^{\infty} (x - m_x)^2 f(x) dx = D(x) \tag{3}$$

Standard deviation of the random value

$$\sigma_x = \left| \sqrt{D(x)} \right| \tag{4}$$

After the calculation of the basic statistical characters it is necessary to determine the stationarity or non-stationarity of the process applying an appropriate test. For the advisement on the stationarity of the process within the correlation theory it is sufficient to deal with attributes of the mean value, dispersion and autocorrelation function only. If we prove that mean value and dispersion are constant along the whole evaluation stage of the process and that the autocorrelation function depends only on the difference of two periods $\tau = t_2 - t_1$, the process is stationary.

Autocorrelation function for discrete data will be:

$$R_p(r) = \frac{1}{N - r} \sum_{n=1}^{N-r} p^{\circ}(n) \cdot p^{\circ}(n + r) \tag{5}$$

where: $r = 0,1,2, \ldots, m$

Spectral power density is defined by the basic relation

$$S_p(\omega) = \frac{2}{\pi} \int_0^{\infty} R_p(\tau) \cos \omega \tau d\tau \tag{6}$$

After adjusting we get equation, that can be used in computer algoritm

$$S_p\left(\frac{\pi \cdot l}{m \cdot \Delta t}\right) = \frac{\Delta t}{\pi}\left[R_p(0) + 2\sum_{r=1}^{m-1} R_p(\tau) \cos \frac{\pi \cdot l}{m} \cdot r + R_p(m) \cos(\pi \cdot l)\right] \tag{7}$$

where: $l = 0,1,2, \ldots, m$.

Having been prepared, these functions were graphically visualized and evalueated in terms of stationarity, ergodicity and significance of the power spectrum in relation to relevant frequencies.

The calculation of spectral density function is needed to the determination of the spectrum of the stochastic process. The significance of detected frequencies was confirmed by Fisher test in the process.

Function $I(\lambda)$ is determineed for the sequence of random values by the following formula:

$$I(\lambda) = \frac{1}{2\pi N}\left|\sum_{T=1}^{N} x_t e^{-it\lambda}\right|^2 , \quad -\pi \leqslant \lambda \leqslant \pi \tag{8}$$

Function $I(\lambda)$ stands for periodicity chart of sequence $x_1, ..., x_n$.

For the statistical attributes of the periodicity chart to be examined in an easier way, another Figures of the formula (8) have been derived:

$$C_k = \frac{1}{N}\sum_{T=1}^{N-k} x_t \quad x_{t+k}, \quad k = 0,1,...,N-1 \tag{9}$$

then

$$I(\lambda) = \frac{1}{2\pi}(C_0 + 2 \quad \sum_{k=1}^{N-1} C_k \cos k) \tag{10}$$

The periodicity chart shows high values in points corresponding frequencies $\lambda_1, ..., \lambda_{p.}$ R. A. Fisher derived a test that can be used to discover the statistical significance of high values of the periodicity chart.

This way we determine significant frequencies $\lambda_1, ..., \lambda_p$ of the periodicity chart.

Design of the conditions of the experiment

Tribologic qualities of the friction couples are often checked using different devices repecting methods of the specific test room. Based on the experiment we would like to point out possibility and also necessity of using statistical dynamics in this field.

The wear test of selected types of material can be executed on one of devices of TE 97/A type – Figure 1, which ranks among “pivot-disk” test devices with area contact of the frictional-node elements. The test device is suitable for comparative tests of selected materials. The core of the test is imprinting the samples of the forms of pivot to front area of the spinning disk by constant power using hydraulic cylinder. The result of the test is a comparison of wear resistance of single types of material in conditions of the given experiment.

Parameters of the test were derived from the parameters of the adhesive wear test without lubrication used in tribologic laboratory VÚZ Bratislava. They were chosen in order to enable the best possible comparison of tribologic qualities of provided samples. Parameters of the test are following:

- pressure in hydraulic circuit (simulated) 1.47 MPa

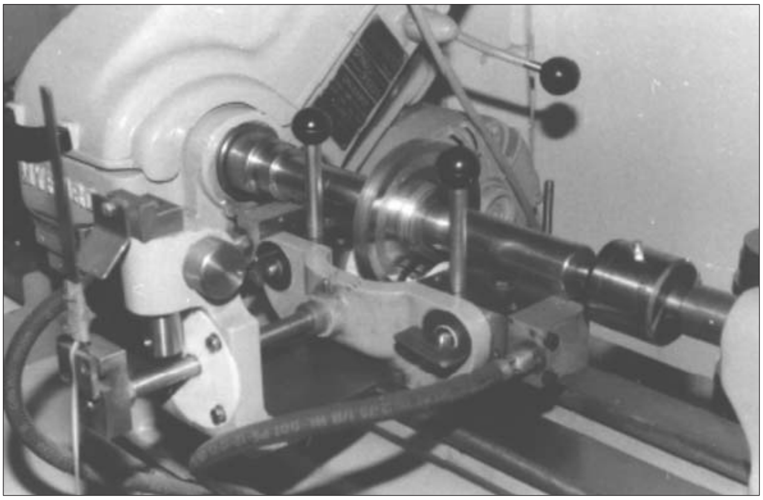


Fig. 1. The view of active part of equipment TE 97/A

• adherence pressure on pivot (simulated)	74.3 N
• circumferential velocity at the test radius	3.2 m.s^{-1}
• exposure time	15, 30, 45, 75s
• material of the companion part	steel 12 020
• dimension of the sample	10 x 50 mm

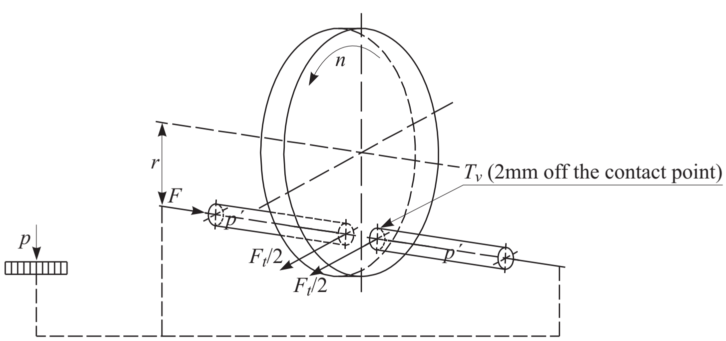


Fig. 2. TE 97/A appliance diagram

Table 1

Fisher test

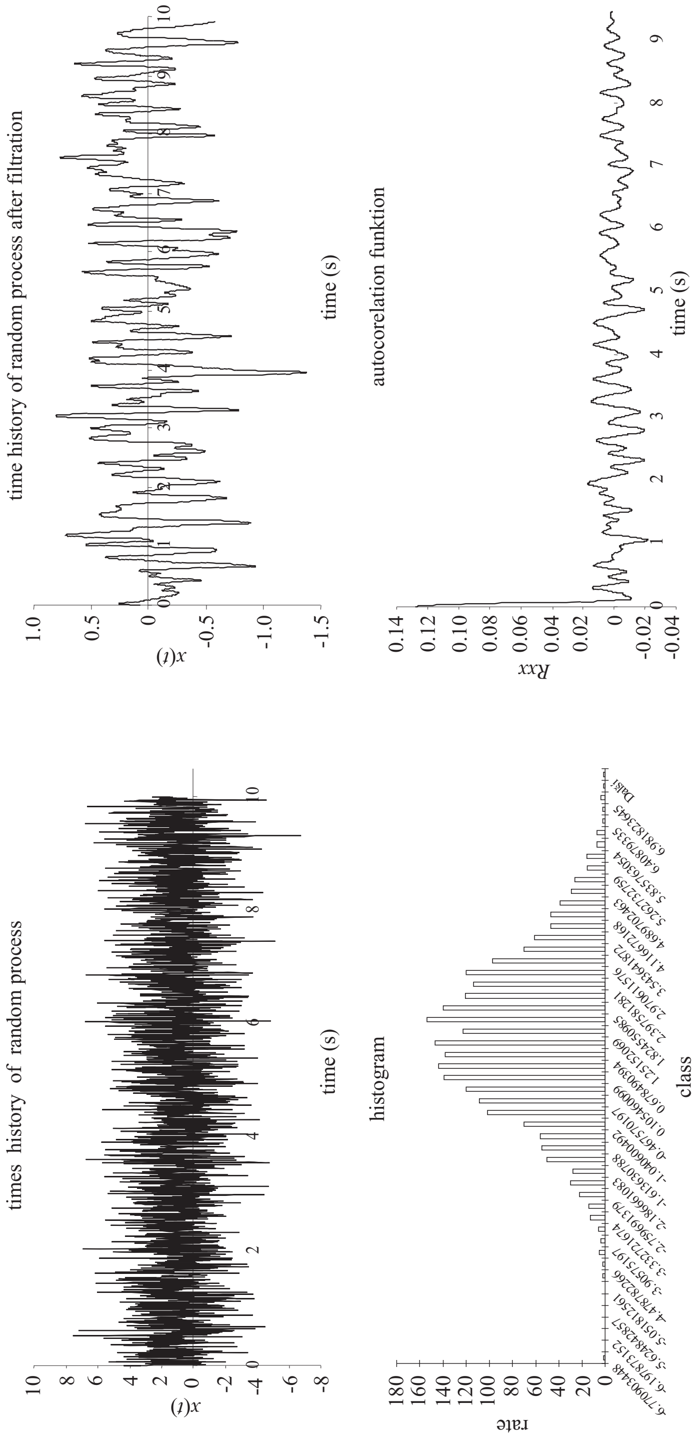
j	$\lambda(j)$	$T(j)$	$I(\lambda)\cdot 10^6$	W	W_{crit}	Frequency Hz	Number
5	0.01267	495.800	1440487.63	0.0189	0.0095	0.5042	1
41	0.10392	60.463	1411544.50	0.0189	0.0095	4.1347	2
16	0.04055	154.938	1227834.13	0.0168	0.0095	1.6136	3
24	0.06083	103.292	1209192.63	0.0168	0.0095	2.4203	4
33	0.08364	75.121	1082818.63	0.0153	0.0095	3.3280	5
2	0.00507	1239.500	992838.13	0.0142	0.0095	0.2017	6
30	0.07604	82.633	951267.13	0.0138	0.0095	3.0254	7
27	0.06843	91.815	921187.69	0.0136	0.0095	2.7229	8
45	0.11406	55.089	893927.88	0.0134	0.0095	4.5381	9
15	0.03802	165.267	849326.75	0.0129	0.0095	1.5127	10
11	0.02788	225.364	826285.63	0.0127	0.0095	1.1093	11
31	0.07857	79.968	732318.69	0.0114	0.0095	3.1263	12
22	0.05576	112.682	688170.50	0.0108	0.0095	2.2186	13
62	0.15714	39.984	667185.81	0.0106	0.0096	6.2525	14
37	0.09378	67.000	633632.13	0.0102	0.0096	3.7313	15

Results and discussion

Figure 3 shows the course of the generated set with determined statistical characteristics. This set was filtered out by means of low-permeable numeric filter and it was centered to a zero mean value. Adjusted course has undergone the stationarity test. Iteration test and of the trend test for mean value and dispersion were executed. Iteration tests have shown, that this process is stationary in mean value and dispersion, which is proven by histogram showing the distribution of frequency of the centered realization. Its course is similar to standard frequency distribution.

Figure 3 also shows autocorrelation function describing the form of the spectrum of generated numbers in the course of time. After evaluation we detect its standard course, drop from high value and gradual damping.

Frequency analysis of this process is introduced by spectral power density. From the course of the spectral power density it is obvious, that there are several significant frequencies in the generated set. Fisher test proves the above. The most significant frequency of all is the frequency of 0.5 Hz, which is number 5. The corresponding periodicity chart is illustrated in Figure 3.


time history of random process after filtration
x(t)
time (s)
autocorelation funktion
Rxx
time (s)
times history of random process
x(t)
time (s)
histogram
rate
class

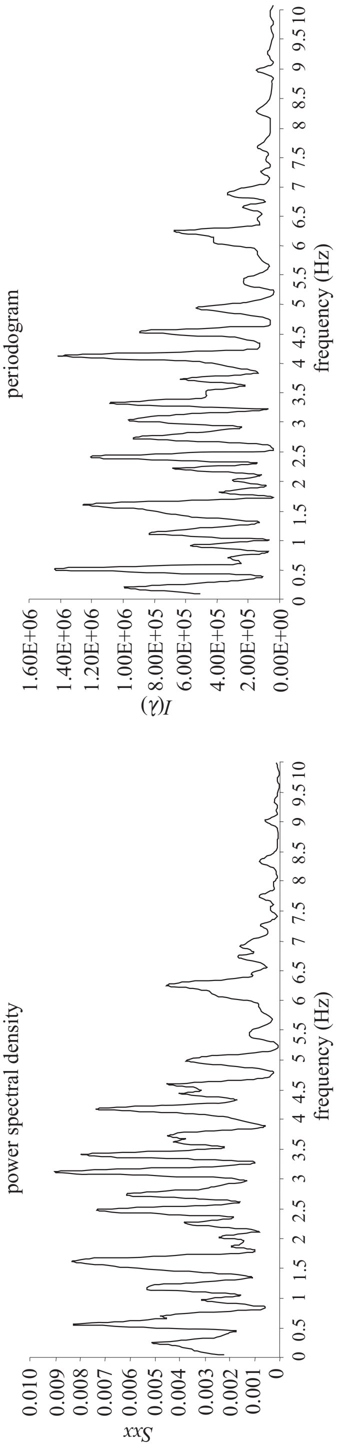


Fig. 3. Random data with defined statistical properties

Conclusion

We applied statistical dynamics methods for evaluation of random realization of the generated set of random numbers.

Evaluated and described courses with their frequency spectrums may be used as input parameters of rationalizing and optimizing methods of tools‘ and machinery‘s design modification. They may also be used as input data of load in adhesive wear testing of various types of material.

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