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SINGULAR STRESS FIELDS IN THE VICINITY OF A SHARP INCLUSION IN AN ELASTIC MATRIX

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Abstract

An analytical description of the order of singularity around angular corners of isotropic elastic wedges embedded in an isotropic elastic medium has been considered in the paper. Blinowski and Ostrowska-Maciejewska (3) have used a displacement function to solve this kind of problems. This method has been also applied by Blinowski and Rogaczewski (4) to study stress singularities at sharp notches in anisotropic materials.

In the present paper this method has been used for solving a mixed boundary value problem, assuming continuity of displacement fields and normal and tangential stresses at interfaces. The choice of the solution in polar coordinates reduces the problem to a transcendent equation describing the stress singularity λ. The numerical solution for λ makes it possible to determine effectively the unknown multipliers in the singular part of the solution for a stress field in the following form:

$$\sigma_{ij}(r,\varphi) = r^{-\lambda}\Phi_{ij}(\varphi),$$

where functions $\Phi_{ij}(\varphi)$ are linear combinations of trigonometric functions.

OSOBLIWE POLA NAPRĘŻEŃ W OTOCZENIU WIERZCHOŁKA OSTREGO WTRĄCENIA W SPRĘŻYSTEJ OSNOWIE

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Słowa kluczowe: klin sprężysty, osobliwość pola naprężenia.

Streszczenie

Do badania rzędu osobliwości na ostrych wtrąceniach (klinach) w liniowo-sprężystych materiałach izotropowych zastosowano funkcję przemieszczeń oraz funkcję naprężeń Airy.

Rozwiązania osobliwe w materiale izotropowym przedstawiono w postaci superpozycji pola symetrycznego (rozrywanie-ściskanie) i antysymetrycznego (ścinanie).

Zakładano ciągłość pól przemieszczeń oraz zgodność wektorów naprężeń na powierzchniach podziału. Poszukiwano rozwiązań w rozdzielonych zmiennych w układzie biegunowym, sprowadzając zagadnienie do równania przestępnego na nieznany rząd osobliwości λ. Numeryczne rozwiązanie tego równania pozwala na analityczne wyznaczenie stałych całkowania i skonstruowanie osobliwej części rozwiązań dla pól naprężeń w postaci

$$\sigma_{ij}(r,\varphi) = r^{-\lambda}\Phi_{ij}(\varphi),$$

gdzie funkcje $\Phi_{ij}(\varphi)$ są kombinacjami liniowymi funkcji trygonometrycznych o znanych współczynnikach.

Introduction

In the theory of elasticity an important role is played by plane problems concerning wedges, notches and cracks. A review of the professional literature on the topic can be found in papers by SEWERYN (1997), SEWERYN and MOLSKI (1996).

The estimation of the order of singularity in an isotropic linear-elastic half-space on wedge-shaped inclusions of another isotropic linear-elastic material is not only of a high cognitive and practical value (triangular singular elements in the finite element method), but also of primary importance for the evaluation of limiting cases in problems formulated for anisotropic materials.

SEWERYN and MOLSKI (1997), REEDY Jr. and GUESS (2001) analyzed special cases of rigid inclusions with sharp corners. BOGY (1968) considered a case of two rectangular elastic wedges. BLINOWSKI and ROGACZEWSKI (2000) focused their attention on a special case of a triangular notch in orthotropic material, where the symmetry of the problem was consistent with the symmetry of the material (symmetry axes coincided with orthotropic axes). Solution of such a problem can be represented as a superposition of two separate cases: symmetric and antisymmetric. Such a representation is always possible for isotropic materials, both with notches and wedges.

The standard approach to plane problems, based on the Airy function, can be inadequate in the case of mixed boundary conditions. BLINOWSKI and OSTROWSKA-MACIEJEWSKA (1995) proposed the application, in the orthotropic case, of a displacement function fulfilling a homogenous quartic differential equation in the form:

$$\frac{\partial^4 F}{\partial x_1^4} + \left(\gamma_1^2 + \gamma_2^2\right)\frac{\partial^4 F(x_1, x_2)}{\partial x_1^2 \partial x_2^2} + \gamma_1^2 \gamma_2^2 \frac{\partial^4 F(x_1, x_2)}{\partial x_2^4} = 0 \tag{1}$$

where γ_i i γ_2 are dimensionless material constants. The Airy stress function must also fulfill this equation.

In the case of isotropy, equation (1) can be reduced to a biharmonic form for both functions, which allows to apply standard solutions known from the theory of plane elastic states (NOWACKI 1970). It should be noted that passing to the isotropic case changes the type of equation (e.g. for solutions with separated variables the multiplicity of characteristic equation roots changes). Thus, the solutions describing the behavior of systems composed of isotropic materials cannot be described as a special case of anisotropic solutions by a simple substitution of relevant elastic constants.

Problem formulation

The objective of the present study was to determine, the order of singularity in the vicinity of a sharp inclusion (wedge) of an isotropic elastic material in an elastic matrix. Therefore, two cases: symmetric and antisymmetric are to be considered as plane problems of the theory of elasticity.

The quantities concerning the right domain are indexed by 1, and those concerning the left domain – by 2 (Fig. 1).

Let us assume that in the unloaded state there are no internal stresses in the system, and demand continuity of stress and displacement on the intefaces at loading. Let us focus on the singular part of the solution, especially on the order of singularity. Thus, for the value $\varphi = \alpha$ (as well as for the value $\varphi = -\alpha$, due to symmetry), the following relations must obey:

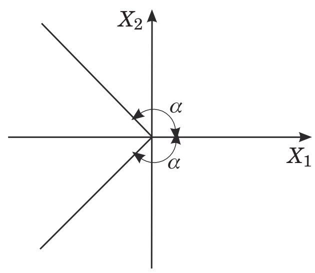

Fig. 1. Plane problem with a wedge, opening angle 2α, $0 < \alpha < \pi$

$$\begin{aligned} \sigma_{\varphi\varphi 1} - \sigma_{\varphi\varphi 2} &= 0 \\ \sigma_{r\varphi 1} - \sigma_{r\varphi 2} &= 0 \\ U_{r1} - U_{r2} &= 0 \\ U_{\varphi 1} - U_{\varphi 2} &= 0 \end{aligned} \tag{2}$$

where $\sigma_{\varphi\varphi}$ i $\sigma_{r\phi}$ denote circumferential and tangential stresses, respectively, in the polar coordinate system, and U_r i U_φ denote the radial and transversal components of the displacement vector, respectively.

Problem solution

The problem can be solved using the displacement function $G(r, \varphi)$ (BLINOWSKI, OSTROWSKA-MACIEJEWSKA 1995), reduced to a biharmonic form for isotropy. Thus, one can seek for the solution of the form $G(r,\varphi)=r^{4-\lambda}F(\varphi)$.

Applying the well-known Laplace formula in polar coordinates (cf. NOWACKI 1970):

$$\Delta G = \frac{\partial^2 G}{\partial r^2} + \frac{1}{r}\frac{\partial G}{\partial r} + \frac{1}{r^2}\frac{\partial^2 G}{\partial \varphi^2}, \tag{3}$$

it is easy to reduce the problem to an ordinary equation for $F(\varphi)$, whose solutions can be represented as linear combinations of the following functions:

$$e^{i(4-\lambda)\varphi},\ e^{-i(4-\lambda)\varphi},\ e^{i(2-\lambda)\varphi},\ e^{-i(2-\lambda)\varphi},$$

thus for the case of symmetric load the function $G(r, \varphi)$ takes the following forms in domains 1 and 2 respectively:

$$G(r,\varphi) = r^{4-\lambda}\left[P_1 \sin[(4-\lambda)\varphi] + Q_1 \sin[(2-\lambda)\varphi]\right] \tag{4a}$$

$$G(r,\varphi) = r^{4-\lambda}\left[P_2 \sin[(4-\lambda)(\pi-\varphi)] + Q_2 \sin[(2-\lambda)(\pi-\varphi)]\right] \tag{4b}$$

where P_1, Q_1, P_2, Q_2 – integration constants for domains 1 and 2 respectively, whereas for the case of antisymmetric load, the above function assumes the following forms:

$$G(r,\varphi) = r^{4-\lambda}\left[P_1 \cos[(4-\lambda)\varphi] + Q_1 \cos[(2-\lambda)\varphi]\right] \tag{4c}$$

$$G(r,\varphi) = r^{4-\lambda}\left[P_2 \cos[(4-\lambda)(\pi-\varphi)] + Q_2 \cos[(2-\lambda)(\pi-\varphi)]\right] \tag{4d}$$

The power $4-\lambda$ in the functions $G(r, \varphi)$ generates singularity of the order $r^{-\lambda}$ in stresses. Generally, the relations describing the displacement fields U_r, U_φ in polar coordinates by derivatives of displacement functions $G(r, \varphi)$ are extremely complicated, so the formulas expressing displacement components in the Cartesian system can be applied (2.3).

$$\begin{aligned} u_1 &= \frac{1}{2E}\left(G_{,22} - \nu G_{,11}\right)_{,2} \\ u_2 &= \frac{1}{2E}\left(G_{,11} - \nu G_{,22}\right)_{,2} \end{aligned} \tag{5}$$

Commas stand for partial derivatives with respect to Cartesian coordinates x_1, x_2

Displacement components in the polar coordinate system can be presented as Cartesian:

$$U_r = u_1 \cos\varphi + u_2 \sin\varphi \qquad U_\varphi = -u_1 \sin\varphi + u_2 \cos\varphi \tag{6}$$

Substituting the dependencies (5) for (6) and presenting the results in polar coordinates, we obtain, for the symmetric problem:
For domain 1,

$$U_r = -\frac{r^{1-\lambda}}{2E_1}(2-\lambda)(3-\lambda)\{P_1 \cos[(2-\lambda)\varphi](4-\lambda)(1+v_1) - Q_1 \cos(\lambda\varphi)[2+\lambda-(2-\lambda)v_1]\}$$

$$U_\varphi = \frac{r^{1-\lambda}}{2E_1}(2-\lambda)(3-\lambda)\{P_1 \sin[(2-\lambda)\varphi](4-\lambda)(1+v_1) + Q_1 \sin(\lambda\varphi)[-4+\lambda+\lambda v_1]\} \tag{7a}$$

For domain 2,

$$U_r = \frac{r^{1-\lambda}}{2E_2}(2-\lambda)(3-\lambda)\cdot \cdot\{P_2 \cos[\lambda\pi + (2-\lambda)\varphi](4-\lambda)(1+v_2) - Q_2 \cos[\lambda(\pi-\varphi)][2+\lambda-(2-\lambda)v_2]\} \tag{7b}$$

$$U_\varphi = -\frac{r^{1-\lambda}}{2E_2}(2-\lambda)(3-\lambda)\cdot \cdot\{P_2 \sin[\lambda\pi + (2-\lambda)\varphi](4-\lambda)(1+v_2) - Q_2 \sin[\lambda(\pi-\varphi)][-4+\lambda+\lambda v_2]\}$$

For domain 1 the Young's modulus and Poisson's ratios are denoted by E_1 i v_1, for domain 2 – by E_2 i v_2.

For the antisymmetric problem we obtain,

For domain 1,

$$U_r = \frac{r^{1-\lambda}}{2E_1}(2-\lambda)(3-\lambda)\cdot \cdot\{P_1 \sin[(2-\lambda)\varphi](4-\lambda)(1+v_1) + Q_1 \sin(\lambda\varphi)[2+\lambda-(2-\lambda)v_1]\} \tag{8a}$$

$$U_\varphi = \frac{r^{1-\lambda}}{2E_1}(2-\lambda)(3-\lambda)\{P_1 \cos[(2-\lambda)\varphi](4-\lambda)(1+v_1) - Q_1 \cos(\lambda\varphi)[-4+\lambda+\lambda v_1]\}$$

For domain 2,

$$U_r = \frac{r^{1-\lambda}}{2E_2}(2-\lambda)(3-\lambda)\cdot$$
$$\cdot\{P_2 \sin[\lambda\pi + (2-\lambda)\varphi](4-\lambda)(1+\nu_2) - Q_2 \sin[\lambda(\pi-\varphi)][2+\lambda-(2-\lambda)\nu_2]\} \tag{8b}$$

$$U_\varphi = \frac{r^{1-\lambda}}{2E_2}(2-\lambda)(3-\lambda)\cdot$$
$$\cdot\{P_2 \cos[\lambda\pi + (2-\lambda)\varphi](4-\lambda)(1+\nu_2) - Q_2 \cos[\lambda(\pi-\varphi)][-4+\lambda+\lambda\nu_2]\}$$

The displacement function is related to the Airy stress function by the dependence:

$$\Phi(r,\varphi) = G(x_1, x_2)_{,12} \tag{9}$$

(cf. (2.8) BLINOWSKI, OSTROWSKA-MACIEJEWSKA 1995)

Thus, in the Cartesian system the stress function is a mixed derivative of the displacement function. Both these functions fulfill the biharmonicity condition. Substituting expressions (3a and 3b) into dependence (9) for the symmetric problem in domains 1 and 2, after transformations, we obtain:

$$\Phi(r,\varphi) = r^{2-\lambda}(3-\lambda)[P_1(4-\lambda)\cos[(2-\lambda)\varphi] + Q_1(2-\lambda)\cos(\lambda\varphi)],$$
$$\Phi(r,\varphi) = -r^{2-\lambda}(3-\lambda)[P_2(4-\lambda)\cos[\lambda\pi + (2-\lambda)\varphi] + Q_2(2-\lambda)\cos[\lambda(\pi-\varphi)]] \tag{10}$$

and for the antisymmetric problem we obtain:

$$\Phi(r,\varphi) = -r^{2-\lambda}(3-\lambda)[P_1(4-\lambda)\sin[(2-\lambda)\varphi] - Q_1(2-\lambda)\sin(\lambda\varphi)],$$
$$\Phi(r,\varphi) = -r^{2-\lambda}(3-\lambda)[P_2(4-\lambda)\sin[\lambda\pi + (2-\lambda)\varphi] + Q_2(2-\lambda)\sin[\lambda(\pi-\varphi)]] \tag{11}$$

Applying known formulas for describing stresses with derivatives of the Airy functions in polar coordinates (NOWACKI 1970):

$$\sigma_{rr} = \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}$$
$$\sigma_{\varphi\varphi} = \frac{\partial^2\Phi}{\partial r^2} \tag{12}$$
$$\sigma_{r\varphi} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\right)$$

it is possible to calculate the values of stresses for both symmetric and antisymmetric problems:
in domain 1 for the symmetric problem:

$$\begin{aligned}
&\text{a) } \sigma_{rr} = -r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\cos[(2-\lambda)\varphi] + Q_1(2+\lambda)\cos(\lambda\varphi)],\\
&\text{b) } \sigma_{\varphi\varphi} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\cos[(2-\lambda)\varphi] + Q_1(2-\lambda)\cos(\lambda\varphi)],\\
&\text{c) } \sigma_{r\varphi} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\sin[(2-\lambda)\varphi] - Q_1\lambda\sin(\lambda\varphi)],
\end{aligned} \qquad (13)$$

in domain 2 for the symmetric problem:

$$\begin{aligned}
&\text{d) } \sigma_{rr} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\cos[\lambda\pi + (2-\lambda)\varphi] - Q_2(2+\lambda)\cos[\lambda(\pi-\varphi)]],\\
&\text{e) } \sigma_{\varphi\varphi} = -r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\cos[\lambda\pi + (2-\lambda)\varphi] + Q_2(2-\lambda)\cos[\lambda(\pi-\varphi)]],\\
&\text{f) } \sigma_{r\varphi} = -r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\sin[\lambda\pi + (2-\lambda)\varphi] - Q_2\lambda\sin[\lambda(\pi-\varphi)]].
\end{aligned}$$

in domain 1 for the antisymmetric problem:

$$\begin{aligned}
&\text{g) } \sigma_{rr} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\sin[(2-\lambda)\varphi] - Q_1(2+\lambda)\sin(\lambda\varphi)],\\
&\text{h) } \sigma_{\varphi\varphi} = -r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\sin[(2-\lambda)\varphi] - Q_1(2-\lambda)\sin(\lambda\varphi)],\\
&\text{j) } \sigma_{r\varphi} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_1(4-\lambda)\cos[(2-\lambda)\varphi] - Q_1\lambda\cos(\lambda\varphi)],
\end{aligned}$$

in domain 2 for the antisymmetric problem:

$$\begin{aligned}
&\text{k) } \sigma_{rr} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\sin[\lambda\pi + (2-\lambda)\varphi] - Q_2(2+\lambda)\sin[\lambda(\pi-\varphi)]],\\
&\text{l) } \sigma_{\varphi\varphi} = -r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\sin[\lambda\pi + (2-\lambda)\varphi] + Q_2(2-\lambda)\sin[\lambda(\pi-\varphi)]],\\
&\text{m) } \sigma_{r\varphi} = r^{-\lambda}(1-\lambda)(2-\lambda)(3-\lambda)[P_2(4-\lambda)\cos[\lambda\pi + (2-\lambda)\varphi] - Q_2\lambda\cos[\lambda(\pi-\varphi)]],
\end{aligned}$$

Conditions (2) and relations (7a), (7b), (8a), (8b) and (13) generate the following system of homogenous equations, allowing to determine the unknowns P_1, Q_1, P_2, Q_2..

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} P_1 \\ Q_1 \\ P_2 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (14)$$

where the components of the matrix **A** assume the following values for the symmetric problem:

$$
\begin{aligned}
A_{11} &= (4-\lambda)\cos[(2-\lambda)\alpha], & A_{12} &= (2-\lambda)\cos(\lambda\alpha),\\
A_{13} &= (4-\lambda)\cos[\lambda\pi+(2-\lambda)\alpha], & A_{14} &= (2-\lambda)\cos[\lambda(\pi-\alpha)],\\
A_{21} &= (4-\lambda)\sin[(2-\lambda)\alpha], & A_{22} &= -\lambda\sin(\lambda\alpha),\\
A_{23} &= (4-\lambda)\sin[\lambda\pi+(2-\lambda)\alpha], & A_{24} &= -\lambda\sin[\lambda(\pi-\alpha)],\\
A_{31} &= -\frac{1}{E_1}(4-\lambda)(1+\nu_1)\cos[(2-\lambda)\alpha], & A_{32} &= \frac{1}{E_1}[2+\lambda-(2-\lambda)\nu_1]\cos(\lambda\alpha),\\
A_{33} &= -\frac{1}{E_2}(4-\lambda)(1+\nu_2)\cos[\lambda\pi+(2-\lambda)\alpha], & A_{34} &= \frac{1}{E_2}[2+\lambda-(2-\lambda)\nu_2]\cos[\lambda(\pi-\alpha)],\\
A_{41} &= \frac{1}{E_1}(4-\lambda)(1+\nu_1)\sin[(2-\lambda)\alpha], & A_{42} &= \frac{1}{E_1}[-4+\lambda+\lambda\nu_1]\sin(\lambda\alpha),\\
A_{43} &= \frac{1}{E_2}(4-\lambda)(1+\nu_2)\sin[\lambda\pi+(2-\lambda)\alpha], & A_{44} &= -\frac{1}{E_2}[-4+\lambda+\lambda\nu_2]\sin[\lambda(\pi-\alpha)]
\end{aligned}
\tag{15}
$$

and for the antisymmetric problem:

$$
\begin{aligned}
A_{11} &= -(4-\lambda)\sin[(2-\lambda)\alpha], & A_{12} &= (2-\lambda)\sin(\lambda\alpha),\\
A_{13} &= (4-\lambda)\sin[\lambda\pi+(2-\lambda)\alpha], & A_{14} &= (2-\lambda)\sin[\lambda(\pi-\alpha)],\\
A_{21} &= (4-\lambda)\cos[(2-\lambda)\alpha], & A_{22} &= -\lambda\cos(\lambda\alpha),\\
A_{23} &= -(4-\lambda)\cos[\lambda\pi+(2-\lambda)\alpha], & A_{24} &= \lambda\cos[\lambda(\pi-\alpha)],\\
A_{31} &= \frac{1}{E_1}(4-\lambda)(1+\nu_1)\sin[(2-\lambda)\alpha], & A_{32} &= \frac{1}{E_1}[2+\lambda-(2-\lambda)\nu_1]\sin(\lambda\alpha),\\
A_{33} &= -\frac{1}{E_2}(4-\lambda)(1+\nu_2)\sin[\lambda\pi+(2-\lambda)\alpha], & A_{34} &= \frac{1}{E_2}[2+\lambda-(2-\lambda)\nu_2]\sin[\lambda(\pi-\alpha)],\\
A_{41} &= \frac{1}{E_1}(4-\lambda)(1+\nu_1)\cos[(2-\lambda)\alpha], & A_{42} &= -\frac{1}{E_1}[-4+\lambda+\lambda\nu_1]\cos(\lambda\alpha),\\
A_{43} &= -\frac{1}{E_2}(4-\lambda)(1+\nu_2)\cos[\lambda\pi+(2-\lambda)\alpha], & A_{44} &= \frac{1}{E_2}[-4+\lambda+\lambda\nu_2]\cos[\lambda(\pi-\alpha)]
\end{aligned}
\tag{16}
$$

The system of homogenous equations has a nontrivial solution when the determinat is equal to zero.

In the case considered the equation

$$\det \mathbf{A} = 0 \tag{17}$$

is a complex transcendental equation for determining the unknown order of singularity λ. Such an equation requires a numerical solution.

If the rank of the matrix **A** is equal to three $R[A_{ij}] = 3$, the values of integration constants are proportional to a single parameter, let us denote it by σ. Let a_{ij} be a cofactor of the element A_{ij}, then:

$$
\begin{aligned}
P_1 &= a_{i1}\sigma \\
Q_1 &= a_{i2}\sigma \\
P_2 &= a_{i3}\sigma \\
Q_2 &= a_{i4}\sigma
\end{aligned}
\tag{18}
$$

where i is any number from the set {1,2,3,4}; the values of cofactors a_{ij} are easy to determine provided that the value λ had been found. The parameter σ performs a similar role as the stress intensity factor in problems concerning stress fields surrounding the crack tip. However, it should be noted that its dimension is not fixed and depends on the order of singularity, i.e. on the opening angle and material constants of the system, so the parameter σ most probably cannot play the role of a measure of stress energy, and its possible critical value cannot be interpreted as a physical constant.

Results and Discussion

Figures 2 and 3 illustrate the dependence between the order of singularity and wedge opening angles for various loads (symmetric and antisymmetric cases) and values of elastic constants. In Figures 2 the values of the parameter α (wedge half-angle) vary within $0 \le \alpha \le \pi$, so two cases have to be considered: a wedge of material (1) in material (2) for $0 \le \alpha \le \pi/2$, and a wedge of material (2) in material (1) for $\pi/2 \le \alpha \le \pi$. Figure 2 shows the symmetric case. Materials (1) and (2) differ in the values of the Young's modulus: ten-fold (solid line) and hundred-fold (broken line), at identical of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$. The singularity whose value changes con-

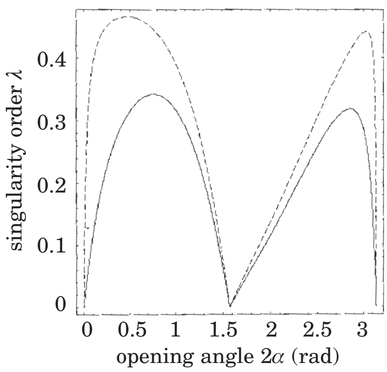

Fig. 2. Relation between the order of singularity λ and the wedge opening angle 2α for symmetric load (diagram I). For the values of the Young's modulus $E_1 = 1$, $E_2 = 10$ (solid line). For $E_1 = 1$, $E_2 = 100$ (broken line). (Material of domain 2, on the right side is more rigid than on the left side), identical values of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$

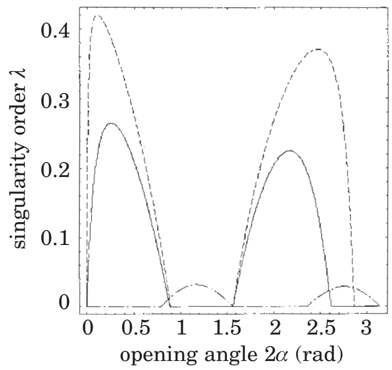

Fig. 3. Dependence between the order of singularity λ and the wedge half-angle 2α for antisymmetric load (diagram II). For the values of the Young's modulus $E_1 = 1$, $E_2 = 10$ (solid line). For $E_1 = 1$, $E_2 = 100$ (broken line). (Material of domain 2, on the right side is more rigid than on the left side), identical values of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$. For identical values of the Young's modulus $E_1 = E_2 = 1$ (dot-and-dash line), for different values of the Poisson's ratios $\nu_1 = 1/10$ i $\nu_2 = 1/2$

stantly reaching the maximum lower than $1/2$ within the above ranges can be observed for all values of the parameter α, except the ends of the range and the value $\pi/2$ corresponding to the contact of the two half-spaces. It should be emphasized that at the ends of the range $\lambda = 0$, and not $\lambda = 1/2$, as in the case of a notch. This is a natural consequence of the condition of transversal displacement continuity on the contact surface.

Figure 3 illustrates a case of antisymmetric load for the same combinations of material constants as above. Singularity is not present for all values of the parameter , similarly as with triangular notches (BLINOWSKI, ROGACZEWSKI 2000, SEWERYN, MOLSKI 1966) (broken line); the dot-and-dash line describes the antisymmetric problem for identical Young's moduli and different values of the Poisson's ratios $\nu_1 = 1/10$ and $\nu_2 = 1/2$. The value of singularity is small and cannot be observed for all values α. It is quite surprising that with the same material relations, but at symmetric load, there were no singular solutions at all, despite the fact that the symmetric system – in light of the results presented in Figure 2 – seemed more sensitive to the difference between material constants. The behavior of such systems at identical or very similar values of the Young's modulus requires further, more detailed, studies.

The results of computations concerning stress distribution for a specified value of the parameter $\alpha = 1/4$ (wedge half-angle) are given in Table 1:

Substituting the calculated values into stress formulas[1], we obtain stress distributions shown in Figures 4–7. Figure 4 presents the symmetric prob-

[1] In stress formulas in domain 2, at negative angles φ, the value $\pi - \varphi$ was substituted by $-\pi - \varphi$, to maintain symmetry

Table 1

Specification	Symmetry		Antisymmetry	
	$E_1 = 1 \quad E_2 = 10$	$E_1 = 10 \quad E_2 = 1$	$E_1 = 1 \quad E_2 = 10$	$E_1 = 10 \quad E_2 = 1$
λ	0.341365	0.217427	0.0624755	0.194658
P_1	–0.113522	0.249407	0.0151697	–0.144458
Q_1	–0.187108	–0.795303	–0.100713	–0.978319
P_2	0.255741	–0.25726	0.090059	–0.0474593
Q_2	0.941647	0.488985	–0.990715	–0.140614

lem – materials (1) and (2) differ ten-fold in the values of the Young's modulus, at identical values of Poisson's ratios $\nu_1 = \nu_2 = 1/4$. Radial stress is discontinuous on the contact surface, whereas circumferential and tangential stresses are continuous, according to the boundary conditions assumed. Figure 5 describes the antisymmetric load for the same combinations of material constants. Stress distribution on the contact surface of the wedge is similar as in the case of symmetric load. Figures 6 and 7 show the same situations as Figures 4 and 5, for a different combination of materials (1) and (2), i.e. $E_1 = 10$, $E_2 = 1$.

As shown by the courses of the diagrams of the order of singularity, versus opening angle 2α, its values do not differ qualitatively from the results obtained for triangular notches (BLINOWSKI, ROGACZEWSKI 2000, SEWERYN, MOLSKI 1966). However, the behavior of the system at small differences in material constants requires further, thorough investigations and a detailed interpretation.

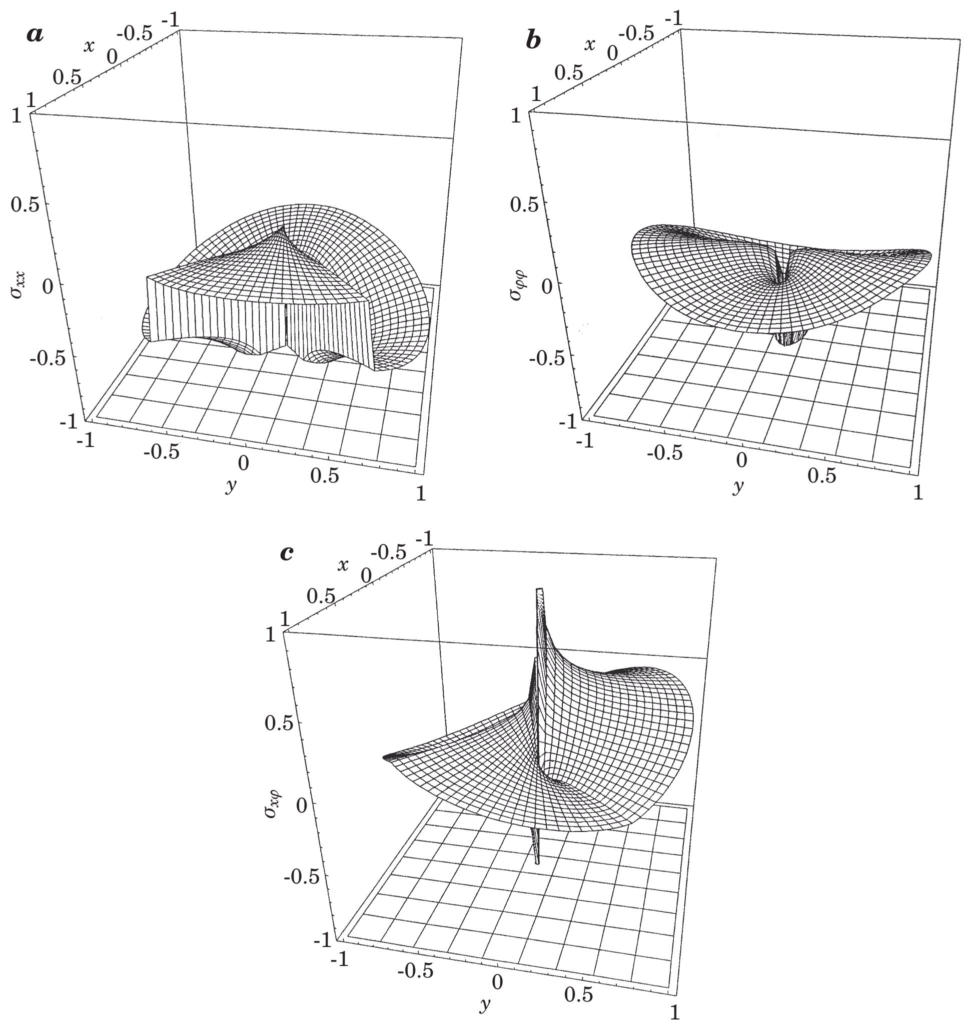

Fig. 4. Radial s_{rr} (a), circumferential $\sigma_{\varphi\varphi}$ (b) and tangential $\sigma_{r\varphi}$ (c) stresses for symmetric load, at a specified value of the wedge opening angle $2\alpha = {}^{\pi}/_{2}$, at different values of the Young's modulus $E_1 = 1$ i $E_2 = 10$ and identical values of the Poisson's ratios $\nu_1 = \nu_2 = {}^{1}/_{4}$

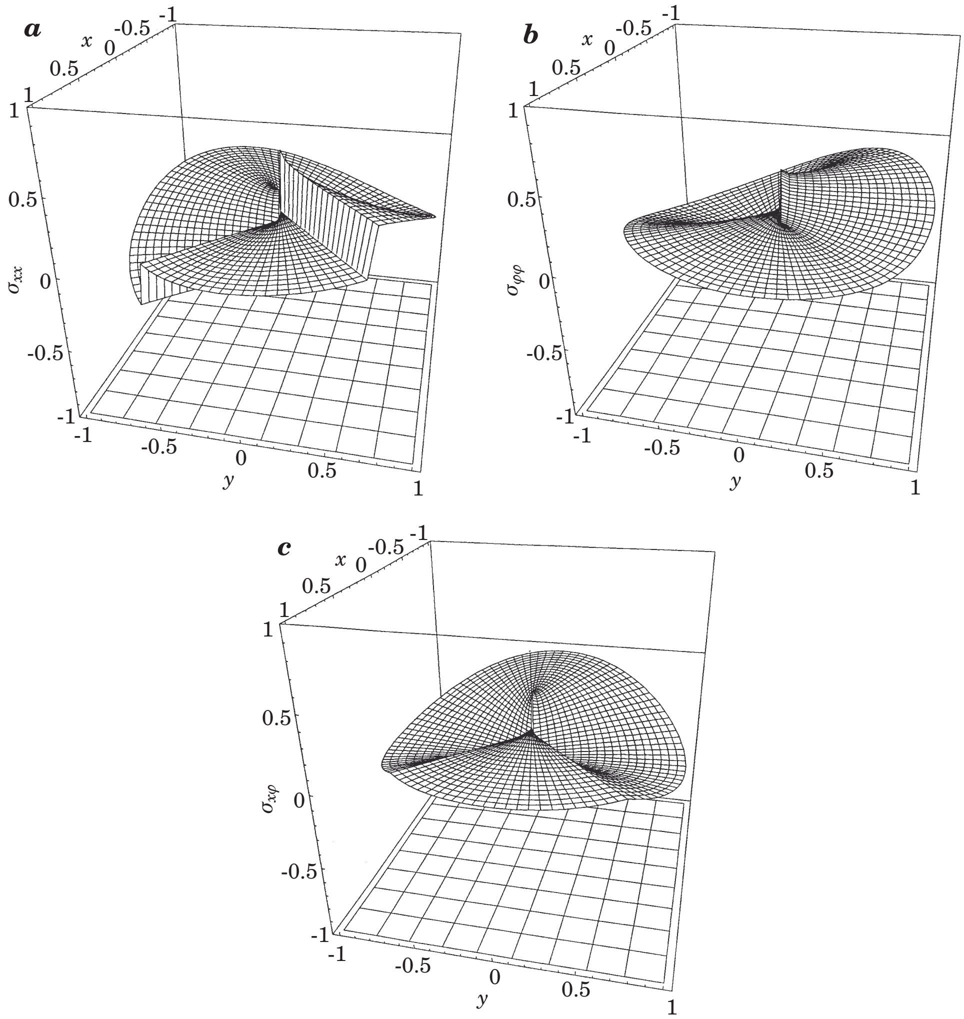

Fig. 5. Radial σ_{rr} (a), circumferential $\sigma_{\varphi\varphi}$ (b) and tangential $\sigma_{r\varphi}$ (c) stresses for antisymmetric load, at a specified value of the wedge opening angle $2\alpha = \pi/2$, at different values of the Young's modulus $E_1 = 1$ i $E_2 = 10$ and identical values of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$

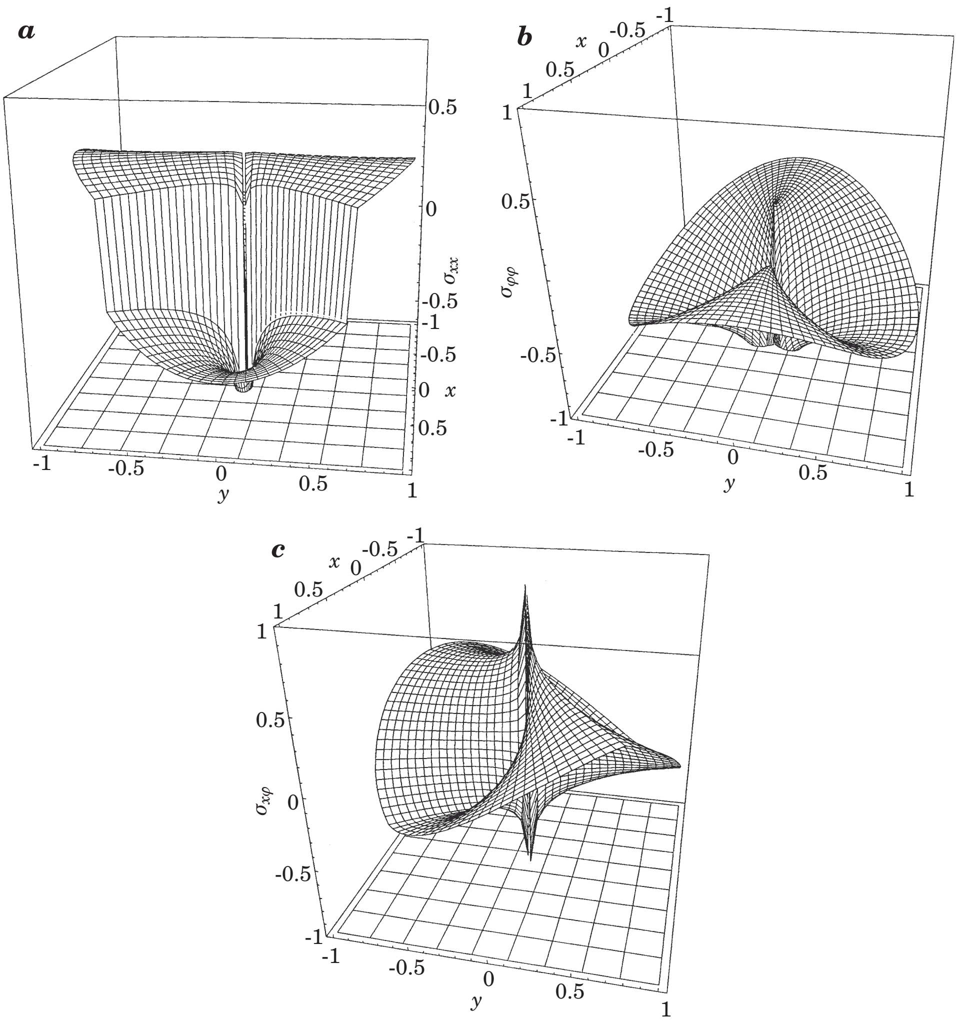

Fig. 6. Radial σ_{rr} (a), circumferential $\sigma_{\varphi\varphi}$ (b) and tangential $\sigma_{r\varphi}$ (c) stresses for symmetric load, at specified value of the wedge opening angle $2\alpha = \pi/2$, at different values of the Young's modulus $E_1 = 10$ i $E_2 = 1$ and identical values of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$

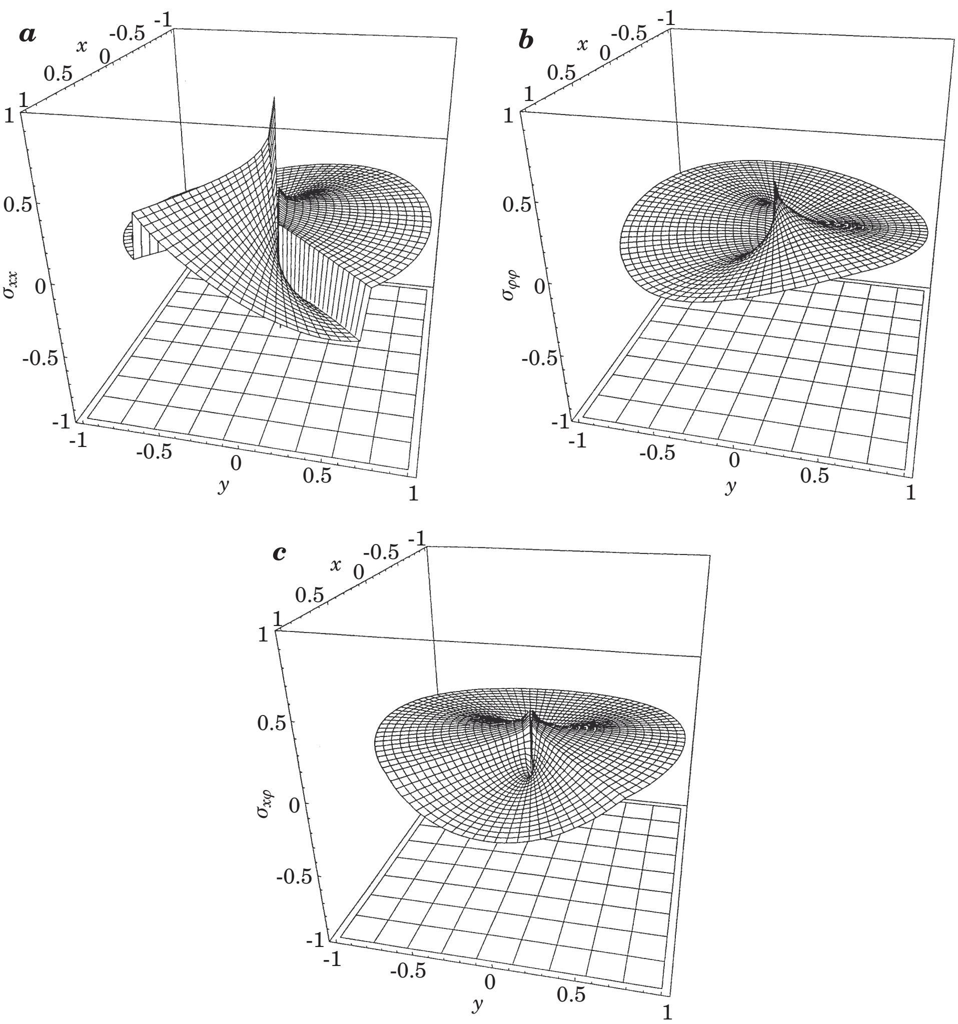

Fig. 7. Radial σ_{rr} (a), circumferential $\sigma_{\varphi\varphi}$ (b) and tangential $\sigma_{r\varphi}$ (c) stresses for antisymmetric load, at specified value of the wedge opening angle $2\alpha = \pi/2$, at different values of the Young's modulus $E_1 = 100$ i $E_2 = 1$ and identical values of the Poisson's ratios $\nu_1 = \nu_2 = 1/4$

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