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MATHEMATICAL MODEL FOR EVALUATING THE OPERATION QUALITY OF TRANSPORT SYSTEMS

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Abstract

The paper discusses some problems related to the evaluation of the operation quality of transport systems. An algorithm enabling to develop a mathematical model and a general model for evaluating the operation quality of transport systems are presented. Certificates that allow to compute the value of the operation quality of the above systems have been worked out. Relationships of the partial and correct order of systems as regards the quality of their operation have been applied. Elementary concepts of metric space were necessary to compute the operation quality of transport systems. The presented certificates are very useful since they permit the evaluation of the operation quality of a single system at different moments in time, as well as the assessment of the operation quality of two distinct systems at the same moment, and of distinct systems in different moments in time.

MODEL MATEMATYCZNY OCENY JAKOŚCI DZIAŁANIA SYSTEMÓW TRANSPORTOWYCH

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Słowa kluczowe: jakość działania systemu, metoda oceny, matematyczny model oceny, przestrzeń metryczna.

Streszczenie

Praca dotyczy oceny jakości działania systemów transportowych. Przedstawiono algorytm wyznaczania modelu matematycznego oraz zbudowano ogólny model oceny jakości ich działania. Opracowano metryki służące do wyznaczania jakości działania rozpatrywanych systemów. Zastosowano relację częściowego porządku i dobrego uporządkowania w celu jakościowej klasyfikacji systemów oraz zastosowano podstawowe pojęcia przestrzeni metrycznej do wyznaczenia poziomu jakości ich działania.

1. Introduction

The essential operational aim of the transport systems is to provide passenger transportation over a specific territory, within specified density and time. The evaluation and assurance of the required operation quality of these systems both in terms of efficiency, reliability, safety and in terms of economic aspects are key factors in the process of their operation and maintenance.

An analysis of the relevant literature shows that the problems of sociotechnical system operation quality of the *<H–M–E>* type (human – machine – environment), including the discussed transport systems, have not been studied in detail so far. Therefore, on the basis of the results of our own operation and maintenance investigations (WOROPAY, MUŚLEWSKI 2002) and identification of the processes performed within the investigation object (WOROPAY, KNOPIK, LANDOWSKI 2001), this paper presents a method to build a model for evaluating the operation quality of transport systems. This method makes it possible to evaluate and compare the operation quality of various transport systems of the same (*H–M–E*) type and contributes to rational control over the processes performed within the discussed systems.

For the purposes of this paper it has been assumed that "the operation quality of the system is a set of the system features expressed by means of their numeral values at a given moment t, determining the level of accomplishing the required conditions" (WOROPAY, MUŚLEWSKI 2000).

2. Concept of building a model for evaluating the operation quality of transport systems

This chapter presents a description of the rules that provided a basis for evaluating the operation quality of transport systems. A general scheme of system evaluation is shown in Fig. 1.

As shown in this diagram, an external observer – *EO*, based on the determined quality criteria K, performs identification of the feature set – *X*, describing the transport system from the point of view of the quality of its

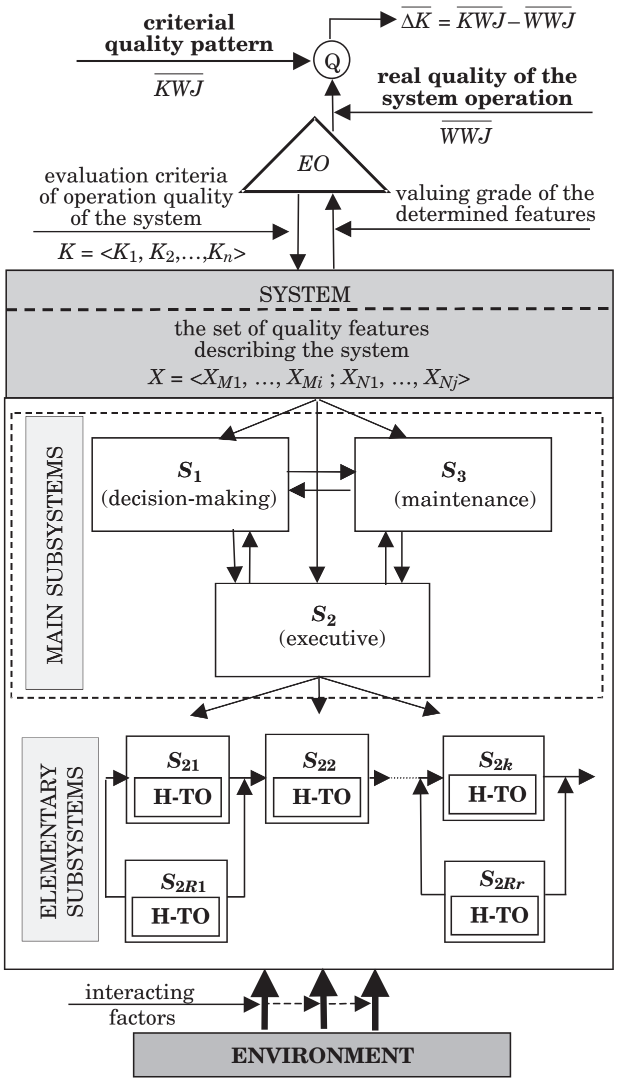

Fig. 1. Scheme of evaluating the operation quality of transport systems

operation. In this paper the term criterion is defined as one of the significant conditions, imposed on a feature, describing the quality of the analysis subject at the given moment t, whereas the feature is a property or quality of the analysis subject. It must be taken into account that the set of features adopted to describe the quality of the system under investigation consists of two subsets: measurable features and non-measurable features. The measurable features are those which are beyond "the reach" of the possibilities to measure them because of the technical nature problems or because of the lack of knowledge of the investigator. For each measurable feature describing the system under investigation X_{Mi} (i = 1,2,...,n)., the permissible limits of their changeability are to be set $X_{M,i}^{\min}$, $X_{M,i}^{\max}$, which correspond to the correct (required) operation quality of the system. Likewise, for each feature which is agreed to be non-measurable one, X_{Nj} (j = 1,2,...,m), it is needed to determine the conditions for the correct quality in a way enabling unambiguous statement whether or not a specific feature meets them. For that reason different values from 0 to m are assigned to the non-measurable features. Then the condition of the correct quality of the system operation at the given moment t, $t \in [t_0, t_k]$ is presented by the formula stated below:

$$J_S = \begin{cases} X_{M,1}^{\min} < X_{M,1,t} < X_{M,1}^{\max} \ldots X_{M,n}^{\min} < X_{M,n,t} < X_{M,n,}^{\max} \\ X_{N,1}^{\min} < X_{N,1,t} < X_{N,1}^{\max} \ldots X_{N,m}^{\min} < X_{N,m,t} < X_{N,m}^{\max} \end{cases} \tag{1}$$

This formula means that at the given moment t, the system operates with the required quality only when the values of its measurable features remain within the determined limits and when the non-measurable features meet the determined conditions of the correct (required) quality of its operation (WOROPAY, MUŚLEWSKI 2001).

The following symbols have been adopted for the diagram shown above:
- EO – external observer,
- S – transport system,
- S_1, $S2$, ..., S_n – transport subsystems,
- S_{21}, S_{22}, ..., S_{2k} – elementary subsystems – essential ones,
- S_{2R1}, ..., S_{2Rr} – elementary subsystems – reserve ones:
- H– human (operator),
- TO – technical object,
- $\overline{\Delta K}$ – vector of the system operation quality grade,
- $\overline{WWJ}$ – multidimensional vector of the system operation at the moment t,
- $\overline{KWJ}$ – criterial quality pattern Q.

The direct impact on the operation quality level of the transport system is exerted by its subsystems and by interaction of the environment with each of those subsystems. In the decomposition process, shown in Fig. 1, the main subsystems of the transport system have been set at the first level as follows:
- executive subsystem,
- maintenance subsystem,
- decision-making subsystem.

An analysis of the problems of complex maintenance and operation systems, and especially of transport systems (NIZIŃSKI 2002), shows that these subsystems are sets based on the identification of the system under investigation and their number depends on its complexity, kind and intended purpose.

At the second decomposition level, keeping in mind the clarity of representation, only subsystems of the executive system have been presented. These are essential and reserve elementary subsystems of the (*H–TO*) type, where a human is interlinked with a technical structure by means of a serial structure, as presented in Fig. 2.

Fig. 2. Structure of the elementary subsystem of the executive system:
H – operator (driver), *TO* – technical object (bus)

They carry out the tasks resulting from the necessity to satisfy their users' needs, who are treated in the model description as external observers. It is obvious that the operation quality of the transport system is also influenced on by its other subsystems. However, it must be taken into account that the person who evaluates the operation quality of the transport system is the so called external observer – a user of the transport system, within whose observation range, most of all, is the executive subsystem. For this reason the external observer evaluates the operation quality of the transport system based on the example of this subsystem.

The operation quality of the transport system determined at the moment t, $t \in < t_0, t_k >$ is described by means of the so called Multidimensional Quality Vector. The set of features adopted to describe the operation quality of the system $(X_1, X_2, \dots, X_p)$ determines p – dimensional space of the quality evaluation. The values of the investigated features at a given moment t are projected on individual coordinate axes. These values enable to set point M', with the coordinates $[k'_{x_1(t)}, k'_{x_2(t)}, \dots, k'_{x_p(t)}]$. Point M' in the multidimensional space is the vector end, the beginning of which is the beginning of the coordinate system. This vector describes the operation quality at the moment t, and it has been denoted with the symbol $\overline{WWJ}$. Afterwards, in the same investigated p – dimensional space, the model (required) feature values are projected on each of the adopted coordinate axes, based on which point M is determined. Point M with the coordinates $[k_{x_1}, k_{x_2}, \dots, k_{x_p}]$ is the end of the model vector representing a model system state, which has been called the Criterial Quality Pattern and has been denoted with the symbol $\overline{KWJ}$. The distance between the ends of the vectors $\overline{KWJ}$ and $\overline{WWJ}$, in the adopted p – dimensional space, determines the system operation quality grade $\overline{\Delta K}$. It may be described as follows:

$$\overline{\Delta K} = \overline{KWJ} - \overline{WWJ} \tag{2}$$

Whereas point M', being the end of the vector $\overline{WWJ}$, within the time interval with the length Δt, draws a trajectory representing changes in individual feature values of the investigated system, in the considered p – dimensional space. It means that the operation quality of the system is changeable in time, because the vector component values are changed on each axis, in the investigated p – dimensional space in time $(t + \Delta t)$.

A simplified geometrical interpretation of the vector $\overline{\Delta K}$, determining the quality grade of the system operation in space R_3, is shown in Fig. 3.

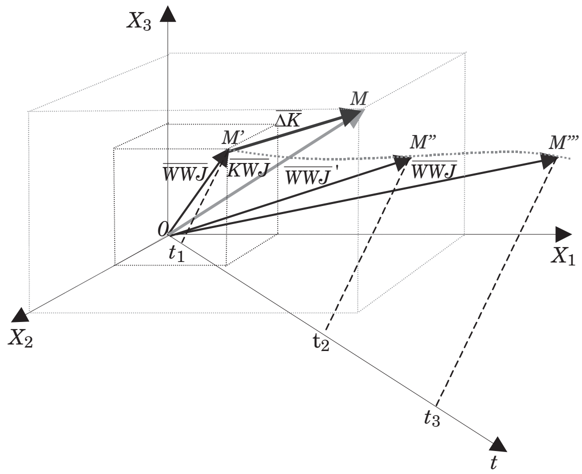

Fig. 3. Geometrical interpretation of vector $\overline{\Delta K}$, determining the quality grade of the system operation in space R_3

3. Mathematical model for evaluating the operation quality of transport systems

3.1. Assumptions to build the model

Let $X_i(t)$, i = 1, 2, …, p, denote a feature being a random variable which depends on time, whose realisation at a given moment t describes the operation quality of the system. In this paper the quality feature vector is considered as follows:

$$X(t) = <X_1(t), X_2(t), ..., Xp(t)> \tag{3}$$

The component $X_i(t)$, $i = 1, 2, \ldots, p$, of the vector $X(t)$, is a one-dimensional random process in space R, describing the *i-th* feature of the operation quality of the system. However, the vector X(t) is p – a dimensional random process describing comprehensively the operation quality of the system in space R^p, at a given moment t. Then the formula:

$$X : T \times \Omega \rightarrow R^p \tag{4}$$

means that for each pair (t,ω), where $t \in T, \omega \in \Omega$, $X(t,\omega)$ is p – a dimensional vector whose components are real numbers expressing the values of the quality features of the investigated system at a given moment t. Where:

X – p – a dimensional random process (in the geometrical interpretation representing the vector $\overline{WWJ}$),
T = <0, +∞) – a time moment set,
Ω – a set of elementary events,
ω – an elementary event,
R^p – p – dimensional space composed of vectors in the form: $(x_1, x_2, \ldots, x_p)$,
x_i – p – element number sequences
$x_i \in R$, i = 1, 2, …, p.

3.2. General model for evaluating the operation quality of transport systems

In order to evaluate the operation quality of transport systems a set of quality features Z is determined, which is divided into n – disjoint subsets $Z_1, Z_2,\ldots,Z_n$ such that:

$$Z_i \cap Z_j = \emptyset \text{ for } \neq j$$

$$Z = Z_1 \cup Z_2 \cup \ldots \cup Z_n = \{X_1(t), X_2(t), \ldots, X_p(t)\} = X(t) \tag{5}$$

$$n, p \in N,\ n \leq p$$

Each of the n-th subsets of set Z is a set of features describing the operation quality of individual system elements. In this case, when discussing the sociotechnical systems of the <*H–M–E*> type, the elements of the system are: human – operator, machine – technical object and environment. The number of elements and features is determined based on the identification of the system being investigated, and depends on its type, complexity and characteristics.

An evaluation of the operation quality of the system is performed based on the determined, significant for the investigation performance purpose, quality features $X_i(t)$ i=1,2,…,p. The values of these features determine the components of the Multidimensional Quality Vector ($\overline{WWJ}$). This vector represents the operation quality of the system at the moment t. The above considerations may be formulated as:

$$\begin{aligned} &Z_1(t) = \{X_1(t),\ldots,X_{k_1}(t)\} \\ &Z_2(t) = \{X_{k_1+1}(t),\ldots,X_{k_2}(t)\} \\ &Z_3(t) = \{X_{k_2+1}(t),\ldots,X_{k_3}(t)\} \\ &Z_n(t) = \{X_{k_n-r}(\ddot{t}),\ldots,X_{k_n-1}(t),X_{k_n}(t)\} \end{aligned} \tag{6}$$

where: $k_n = p$.

In this paper it has been assumed that the operation quality of transport systems is a representation of the formula:

$$Y : T \times \Omega \rightarrow R \tag{7}$$

which means that $Y(t,\omega)$ $t \in T, \omega \in \Omega$, is a measure of the operation quality of the system at the moment t, and it depends on the elementary event ω, where:

Y – a measure of evaluation of the operation quality of the system, being the function of the random variable vector $X(t)$, (representing the length of the vector $\overline{\Delta K}$),
T = <0, +∞) – a set of time moments,
Ω – a set of elementary events,
R – a set of real numbers,
ω – an elementary event.

3.3. Measures of evaluating the operation quality of transport systems

3.3.1. Partial order relation

Let $t_1 < t_2 < \ldots < t_n$ be the moments at which the feature values of the operation quality of the investigated system S were measured.

In the set of the vectors $X(t_1)$, $X(t_2)$, …, $X(t_n)$ it is possible to formulate the partial order relation as stated below:

Definition 1

The vector $X(t_k)$ is in relation with the vector $X(t_r)$, if for each i ∈ {1,2,…,p} there is:

$$X_i\ (t_k) \leq X_i\ (t_r) \tag{8}$$

The above formula means that the system S at the moment $t_r \in T$ has a higher operation quality grade than at the moment t_k.

In this paper it has been assumed that that equation (3) determines a set of features describing the operation quality vector of the system at the moment t.

Whereas $X_i\ (t_k)$, $X_i\ (t_r)$ stand for the same sets of quality features describing the investigated system at the time moments t_r and t_k.

Definition 2

The system S at the moment tk has a higher operation quality grade than at the moment t_r, if the following inequalities are true:

$$X_i\ (t_r) \leq X_i\ (t_k) \tag{9}$$

for $i = 1, 2, \ldots, p$.

It can been seen that the partial order relation may be used in order to describe changes in the operation quality of the investigated systems at the different time moments t_k and t_r.

3.3.2. Correct order relation

The partial order relation, introduced in point 3.3.1, allows to determine if the investigated system at the moment t_r has a higher operation quality grade than at the moment t_k, only in specific cases.

In order to describe the order relation for any systems at set time moments, for the quality vector described by dependence (3), a function described on this vector is introduced, which takes the values from the set of real numbers. The values of this function create an ordered set as presented below:

$$q(X(t)) = q(X_1(t), X_2(t), \ldots, Xp(t)) \tag{10}$$

where q is a function of p – variables such that $q(X(t))$ is a stochastic process. This function is a measure of the operation quality of the system.

In the considerations regarding the operation quality of the system it has been assumed that each of the coordinates of the vector $X(t)$ is smaller than or equal to a certain limiting value of the pattern for individual quality features:

$$X_i(t) \leq q_i \tag{11}$$

for: $t \in T$, $i = 1, 2, \ldots, p$.

A set of the quality criterial features fulfilling the above inequality is represented by the model state of the operation quality of the system (in the geometrical interpretation $\overline{KWJ}$).

The application of (10) makes it possible to introduce the correct order relation of systems in terms of their operation quality (RASIOWA 1999).

Definition 3

The investigated system at the moment $t_k \in T$ has a higher operation quality grade than at the moment $t_r \in T$, *if*:

$$q(X(t_r)) < q(X(t_k)) \tag{12}$$

For the investigated system a random process is defined, representing the operation quality of the system, and is formulated as:

$$Z_X(T) = \sum_{i=1}^{p} \alpha_i X_i(t); \quad \alpha_i \geq 0, \quad \sum_{i=1}^{p} \alpha_i = 1 \tag{13}$$

where α_i, i = 1, 2, ..., p stand for the values of the quality weights of individual features, determining the operation quality of the investigated system.

$Z_X(t)$ – is a random process, being a finite combination of the processes $X_i(t)$, i=1,2,...,p For the process $Z_X(t)$ the below inequality is obvious:

$$Z_X(t) \leq \sum_{i=1}^{p} \alpha_i q_i, \quad t \in T \tag{14}$$

The above mentioned inequality indicates that the process $Z_X(t)$ determined by means of equation (13) is limited, thus the feature values determining the operation quality of the system shall not go beyond the preset threshold, that means the right side of the inequality (14).

For the average value it can be noted that:

$$EZ_X(t) = \sum_{i=1}^{p} \alpha_i \, EX_i(t) \tag{15}$$

The average value $EZ_X(t)$ is a linear combination of the average values $EX_i(t)$, i=1,2,...,n. The formula (15) is applicable irrespectively of the fact whether the processes $X_i(t)$, i=1,2,...,n are dependent.

For the process variation $Zx(t) = \sum_{i=1}^{p} \alpha_i X_i(t)$ there is:

$$D^2 Z_x(t) = \sum_{i=1}^{p} \alpha_i^2 \, D^2 X_i(t) + 2 \sum_{i>j} \alpha_i \alpha_j \operatorname{cov}(X_i(t), X_j(t)) \tag{16}$$

where cov $(X_i(t), X_j(t))$ means covariation between the random variables $X_i(t)$ and $X_j(t)$. In the case when the random processes $X_i(t)$, i=1,2,...,p are independent, all the covariations $\operatorname{cov}(X_i(t), X_j(t))$ are equal to zero. In this case the process variation $Z_X(t)$ is a sum of the variations.

In real cases the processes $X_i(t)$, i=1,2,…,p are dependent and it is to be expected that the covariations cov($X_i(t)$, $Xj(t)$) will be positive. This fact may be expressed in such a way that the processes $X_i(t)$ are positively correlated by pairs. That means that the coefficient of the correlation between the random variables $X_i(t)$ and $X_j(t)$, i,j=1,2,…n, is positive.

For the investigated system S at any moment t it is possible to determine the length between the point describing the operation quality of this system at the moment t (in the geometrical interpretation this point is the end of $\overline{WWJ}$) from the model system (the point determining $\overline{KWJ}$) by means of the following formula:

$$d(X(t),q) = (\sum_{i=1}^{p}(X_i(t) - q_i)^2)^{\frac{1}{2}} \tag{17}$$

Formula (17) may be applied as a tool to classify systems in terms of their operation quality.

Functions (13) and (17) are particular cases of additive functionals set on p – dimensional stochastic process $X(t)$.

3.3.3. Application of essential concepts of metric space to evaluate the operation quality of the system

Formula (17) is one of the examples to describe the operation quality of the system at the moment t, in the space R^p. The space R^p is composed of p – element number sequences:

$$R^p = \{(x_1, x_2, \ldots, x_p)\ x_i \in R,\ \ \mathrm{i} = 1, 2, \ldots, p\}$$

In p – element space of the sequences consisting of real numbers, it is possible to determine the length (metric) – the operation quality grade of the system ($\overline{\Delta K}$) in many different ways (KUDREWICZ 1976). The metric described with the formula (17) is called Euclidean metric. If:

$$\bar{x} = (x_1, x_2, \ldots, x_p) \in R^p,$$

$$\bar{y} = (y_1, y_2, \ldots, y_p) \in R^p,$$

then the Euclidean metric is determined as follows:

$$\rho_2(\bar{x}, \bar{y}) = (\sum_{i=1}^{p}(x_i - y_i)^2)^{\frac{1}{2}} \tag{18}$$

The metric may also be determined by means of the following formula:

$$\rho_1(\bar{x}, \bar{y}) = \sum_{i=1}^{p}\left|x_i - y_i\right| \tag{19}$$

The above metric is referred to in professional literature as the urban metric.

The next metric is determined by means of the following formula:

$$\rho_{\infty}(\bar{x},\bar{y}) = \max_{1\le i\le p}\left|x_i - y_i\right| \tag{20}$$

This metric is referred to as the Chebyshev metric.

The set R^p along with metric (18) or (19) or (20) create a metric space. The metric space (R^p, ρ_2) is most commonly used in practice.

In general the pair (R^p, ρ), where $\rho\ (\bar{x},\bar{y})$ is one of the metrics, is called a metric space, if a non-negative number $\bar{x},\bar{y} \in R^p$ is assigned to each pair of the elements in such as way that the following metric conditions are satisfied:

– $\rho(\bar{x},\bar{y}) = \rho(\bar{y},\bar{x})$ – (symmetry).

The first axiom of the metric means that the distance from the point $\bar{x}$ to $\bar{y}$ is the same as the distance from $\bar{y}$ to $\bar{x}$:

– $\rho(\bar{x},\bar{y}) =$ 0 only then, if $\bar{x} = \bar{y}$.

The next axiom of the metric means that the distance between two points $\bar{x} = \bar{y}$ is greater than 0 then and only then, when these points do not coincide.

– $\rho(\bar{x},\bar{y}) + \rho(\bar{x},\bar{y}) \ge \rho(\bar{x},\bar{y})$ – (triangle inequality).

The third axiom of the metric means that the points $\bar{x},\bar{y},\bar{z}$ create a triangle or they lie on a single straight line. It is known that the sum of the lengths of two sides is always greater than or equal to the length of the third side in this triangle.

$\rho(\bar{x},\bar{y})$ is called a metric and defines the length between the points $\bar{x}$ and $\bar{y}$.

When evaluating the operation quality of the system the concept of the distance from the set $E \subset R^p$ to the point is also applicable (SMALKO 1972). The distance from the set E to the point $\bar{x}$ is as follows:

$$\rho(\bar{x},E) = \inf_{y\in E}\ \rho(\bar{x},\bar{y}) \tag{21}$$

where inf stands for the infimum of the numeral set.

If E stands for the model set of the values of system operation quality features, then the distance

$$\rho(X)(t),E = \inf_{y\in E}\ \rho(X(t),Y) \tag{22}$$

represents the state of the operation quality of the system at the moment t. In the geometrical interpretation the operation quality of the system $\overline{\Delta K}$ is the distance between the point being the end of the vector $\overline{WWJ}$ and the infimum of the set E.

Another case considered in this paper is the use of the distance between the sets W, E when evaluating the operation quality of the investigated system. If the set E means the model value set of the system quality features and the set W means the value set of the system quality features at the moment t, then the distance:

$$\rho(W,E) = \inf_{y \in W, y \in E} \rho(\bar{x}, \bar{y}) \tag{23}$$

determines the operation quality of the system at the moment t. In the geometrical interpretation, the operation quality of the system $\overline{\Delta K}$ is the distance between the sets W and E.

The distances $\rho(X(t),E)$ and $\rho(W,E)$ may be determined by means of one of the metrics described by formulas (18, 19, 20).

Summary

The method presented in this paper and the model built to evaluate the operation quality of transport systems are tools to support rational control over the processes carried out within the discussed systems, depending on changes of the changeable values describing the actions of the operators, technical objects controlled by them and impact of the environment.

The metrics presented in the paper are applicable for evaluating the operation quality of the same system at different time moments, as well as for assessing the operation quality of two different systems at the same time moment, and of different systems at different time moments.

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