VARIABILITY OF SOLAR IRRADIANCE

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Abstract

The variation of solar irradiance resulting in additional difficulties for the utilization of the solar energy is assessed by changes in the transparency of atmosphere mainly due to the impact of stochastic clouds. The variation of solar irradiance can be described in the best way by the "alternating irradiance", which we define as a difference between the measured and calculated beam and diffuse irradiance for the clear-sky conditions which both are periodical variables, but determined for each instant. The alternating irradiance is investigated by correlation analysis methods, which allow to identify the significantly diurnal periodical character of diffuse irradiance. During the years 1955–2000 all components of solar irradiance decreased at a rate of 0.1% per year, showing a noticeable linear trend, but the time function may have a periodical component for long periods of time, exceeding 30 years. However, the ratio of diffuse and global irradiance has remained a constant value.

ZMIENNOŚĆ PROMIENIOWANIA SŁONECZNEGO

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Słowa kluczowe: promieniowanie bezpośrednie, promieniowanie rozproszone, promieniowanie całkowite, promieniowanie zmienne, analiza korelacyjna.
Streszczenie

Zmienność promieniowania słonecznego, wywołując trudności wykorzystania energii słonecznej, wynika ze zmian przejrzystości atmosfery spowodowanej głównie przez wpływ losowego rozkładu zachmurzenia. Ta zmienność promieniowania słonecznego może być najlepšiej opisana przez "promieniowanie zmienne", które może być zdefiniowane jako różnica między pomierzonym i obliczonym promieniowaniem bezpośrednim i rozproszonym dla warunków bezchmurnego nieba, które są okresowo zmienne, ale ustalone dla każdej chwili. To promieniowanie zmienne jest badane metodami analizy korelacyjnej, które pozwalają zidentyfikować istotną cechę okresowej zmienności dzienną promieniowania rozproszonego.

W okresie 1955–2000 wszystkie elementy składowe promieniowania słonecznego uległy zmniejszeniu z prędkością 0.1% w ciągu roku, z zauważalną tendencją liniową, lecz funkcja czasu może mieć składniki okresowe dla długości okresu przekraczającego 30 lat. Stosunek promieniowania rozproszonego do promieniowania całkowitego pozostaje wartością stałą.

Nomenclature of symbols

\( A \) – amplitude of the periodical component, \( \text{W} \cdot \text{m}^{-2} \),
\( B \) – auxiliary variable,
\( a \) – relative amplitude of the periodical component,
\( D \) – diffuse irradiance by the measurements, \( \text{W} \cdot \text{m}^{-2} \),
\( E \) – correction, minutes (time equation),
\( G_{0n} \) – (normal) solar irradiance outside the atmosphere, \( \text{W} \cdot \text{m}^{-2} \),
\( G_b \) – beam irradiance for the clear-sky conditions, \( \text{W} \cdot \text{m}^{-2} \),
\( G_{bL} \) – alternating beam irradiance, \( \text{W} \cdot \text{m}^{-2} \),
\( G_D \) – diffuse irradiance for the clear-sky conditions, \( \text{W} \cdot \text{m}^{-2} \),
\( G_{DL} \) – alternating diffuse irradiance, \( \text{W} \cdot \text{m}^{-2} \),
\( L \) – longitude, degrees,
\( k \) – constant, matched empirically,
\( n \) – sequence number of the hour (day, week, year),
\( Q \) – global irradiance by the measurements, \( \text{W} \cdot \text{m}^{-2} \),
\( s \) – beam irradiance by the measurements, \( \text{W} \cdot \text{m}^{-2} \),
\( T \) – period, (day, year),
\( \beta \) – tilt angle, degrees,
\( \Phi \) – latitude, degrees,
\( \theta_z \) – zenith angle, degrees,
\( \theta_{b,T} \) – incidence angle for the beam radiation on the tilted surface, degrees,
\( \delta \) – declination (of the sun), degrees,
\( \mu \) – symbol of any average value,
\( \rho \) – symbol of any autocorrelation function,
\( \sigma \) – standard deviation,
\( \sigma^2 \) – variance,
\( \tau \) – correlation interval, (hour, week, year),
\( \tau a \) – transparence-absorbance product,
\( \tau_b \) – transparence of atmosphere for the clear-sky conditions,
\( \omega \) – hour angle, degrees.
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Introduction

The resource of solar energy in North-Europe is modest: the actinometric average resource of the global irradiance has the value of 1028 kWh/m\(^2\)y\(^{-1}\) in Copenhagen, 976 in Stockholm, 978 in Tartu and 977 in Warsaw correspondingly. High variability of the solar radiation in the Baltic area results in additional difficulties as the required storages have to be expanded and dynamical losses are growing. Considering co-operation of PV electrical plants with the grid (Jenkins 2004), the dynamical behavior of solar irradiance will be of importance.

The solar irradiance both on the ground and tilted technological devices includes both periodical and random components. The periodical component is a determined variable and depends on the normal solar irradiance outside the atmosphere \(G_0\), and the zenith angle \(\Theta\). The periodical component has been well investigated and summarized in (\(\text{ASHRAE handbook}\) 1982, Duffie and Beckman 1991, \textit{Solar Energy, the state of the art} 2001). Stochastic effects of solar irradiance due to clouds are investigated in (\textit{Stochastic structure of clouds and radiation}, 1972, Brinkworth 1977) and considered for the solar irradiance in the simulation models (Moref 1998, Boland and Ridley 2002).

Since the value of the stochastic component is of the same order as the amplitude of (the main) periodical component, its behavior and characteristics have to be specified. Both of them define the required characteristics of technological devices, first of all the volume of (daily, seasonal) storage, etc.

Data

We shall investigate the variability (instability) of beam and diffuse radiation in Estonia, based on continuous measurements in the Tartu-Tõravere Meteorological Station (T-TMS) which is located 58°15’N, 26°27’ (26.5° in the text below), 76 m above sea level. T-TMS is described in detail in (Tooming 2003). Main data used in the work have been recorded as one-minute averaged values of irradiance (1999–2001) in the computer memory. All other data used in the study are calculated as averaged values over the corresponding time intervals. In the work we shall look for the stochastic variables which can be considered stationary random processes (“signals”). The steady-state condition is valid for the averaged values like hourly, (daily), weekly and yearly mean values of solar irradiance. These variables are beneficial for engineering and can be used for designing the volumes of daily or seasonal storages, for agro-meteorological forecasting, etc. The minute-long mean values of solar irradiance, which define transient losses in solar collectors and co-operation of PV-devices with the grid, do not correspond to the steady-state condition and have to be investigated in a different way. Also, in this work the stochastic characteristics of the measured beam \(s\) and diffuse component \(D\) of the solar irradiance are calculated to evaluate their instability (variability).
Evaluation of instability of solar irradiance

The theoretical beam $G_b$ and the diffuse irradiance $G_D$ can be well calculated for the ideally transparent atmosphere (clear-sky conditions) for each instant of time. In these conditions the diffuse component has a circumsolar character. The said $G_b$ and $G_D$ change always in time, being determined variables and not instable. The measured beam irradiance $s < G_b$ is always less than the calculated variable, because the real atmosphere is only partly and randomly transparent. The lost fraction of the solar beam irradiance $G_{bL}$, which does not reach the ground, is hereby named “alternating beam irradiance” and in fact it defines the instability of solar beam irradiance.

$$G_{bL} = G_b - s > 0$$ (1)

In a similar way, the instability of diffuse irradiance is defined by the alternating diffuse irradiance

$$G_{DL} = D - G_D$$ (2)

which has no certain sign. For very thick clouds $G_{DL} < 0$, but mostly $G_{DL} > 0$. Both alternating components include pure variability of solar irradiance. Due to time limits described below, $G_D(\omega)$ is a process which includes a periodical component (consisting of a big number of harmonics) with a relatively small amplitude (<20%) and a high average value. It means it is close to the constant value, and its impact on random characteristic of $s$ is irrelevant, too.

Solar irradiance for the clear-sky conditions

For the clear-sky conditions the following calculation rules are valid.

$$G_b = \tau_b(\omega) \cdot G_{0n} \ [2.8.2]$$ (3)

where $G_{0n}$ is the normal irradiance outside the atmosphere, depending on the sequence number $n$ of the instant day.

$$G_{0n} = 1353 \cdot (1 + 0.033 \cos(n \cdot 360/365)) \ [1.4.1]$$ (4)

and $\tau_b(\omega)$ is the transparency of the ideal atmosphere depending on the zenith angle $\Theta_z(\omega)$

$$\tau_b(\omega) = a_0 + a_1 \cdot \exp (-k / \cos \Theta_z) \ [2.8.1a]$$ (5)

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1Hereby we refer to (DUFFIE and BECKMAN 1991), the formula numbers in the quadratic brackets corresponds the said reference.
For the location of T-TMS the following values by (Duffie and Beckman 1991) are recommended

\[ a_0 = 0.1399 \text{ and } a_1 = 0.738, \text{ however} \]

\[ k = 0.2 \text{ is an empirically matched constant, which provides the best coincidence of the calculated irradiance with the envelope of real radiation. The zenith angle is a function of the hour (clock) angle } \omega. \text{ For the angle in degrees} \]

\[ \omega = 0.25 \cdot (\text{UTC+MTZ+DLM+E}) \]  

(6)

Here: UTC – Universal Coordinated Time (GMT), min.,
MTZ – Time Zone Meridian, min. For T-TMS, MTZ = 120,
DLM – Deviation for the Local Meridian, min. For T-TMS.

\[ \text{DLM}=4 \cdot (L_{TO}-L_{MTZ})=4 \cdot (26.5-30)=-14 \]  

(7)

The correction

\[ E = 229.2 \cdot (0.000075 + 0.001868 \cos(B) – 0.032077 \sin(B) – 0.014615 \cos(2B) \cos(2B) – 0.0408 \sin(2B)) \]

and the auxiliary variable

\[ B = (n-1) \cdot 360/365 \]  

(8)

The meaning of the symbols is described in the nomenclature. The reference (Duffie and Beckman 1991) shows an empirical formula of diffuse irradiance for the clear-sky conditions

\[ G_d = G_{0n} \cdot \cos \Theta \cdot (0.271 – 0.294 \cdot \tau_d / \omega) \]  

(9)

Using formulas (1) – (9), both theoretically determined components of solar irradiance for each instant of time can be calculated, and based on these values – also the instable variables \( G_{bL} \) and \( G_{DL} \). The diagram of calculation procedures is presented in Fig. 1.

As an output of digital analysis, the average value, variance, standard deviation, correlation interval and amplitude of possible periodical components are calculated. These parameters can be determined by the autocorrelation function (ACF) of the input variables (\( n, \text{ UTC, } \Phi, s, D \)), which is shown as an output of the graph in Fig. 1.
Limitations used in the work and selected time basis

We expect the results of the work to be used in technical projects. Therefore only the solar irradiance suitable for technological targets is analyzed here. In the conditions for Estonia, this condition is valid for the daytime between 06:00 and 18:00. In this time interval the incidence angle of the beam radiation \( \Theta_{b,T} < 70^\circ \) is provided for a tilted \( \beta = 45^\circ \) surface. For the big incident angle is \( \Theta_{b,T} > 70^\circ \), \( t_a \) of any solar collector decreases fast to zero. In wintertime there is no solar radiation available for technological use and therefore we limit the data between the spring and autumn equinoxes with a one hour time step basis. We shall use the hourly mean values of both measured irradiance data \( s \) and \( D \) (2244 samplings) between 21.03.02 until 23.09.02, to find the diurnal variability of solar irradiance. We shall use a weekly time step basis to assess the seasonal variability with a one week mean value for the summer seasons 1999–2002 (108 samplings). We shall use a yearly time step basis for defining long-time variability with annual total values for the years 1955–2000 (46 samplings). The history of the measurements and description of the used apparatus can be found in (Tooming 2003).

Diurnal instability of solar irradiance

Fig. 2 shows an example of analyzed variables for mostly sunny days (14.05.02–16.05.2002) and in Fig. 3 such an example is given for the days with variable radiation (14.06.02–16.06.2002). In Table 1 summarized data of diurnal variability are presented.

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![Flow chart of the simulation program](image-url)
### Table 1

Calculated and measured values of irradiances according to investigation

<table>
<thead>
<tr>
<th>Specification</th>
<th>Symbol</th>
<th>Beam r.</th>
<th>Symbol</th>
<th>Diffuse r.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The calculated clear-sky average value, W·m⁻²</td>
<td>( \mu G_b )</td>
<td>817</td>
<td>( \mu G_D )</td>
<td>56</td>
</tr>
<tr>
<td>The measured average value, W·m⁻²</td>
<td>( \mu s )</td>
<td>409</td>
<td>( \mu D )</td>
<td>159</td>
</tr>
<tr>
<td>The average value of the alternating irradiance, W·m⁻²</td>
<td>( \mu G_{bL} )</td>
<td>411</td>
<td>( \mu G_{DL} )</td>
<td>102</td>
</tr>
<tr>
<td>The variance of the alternating irradiance, W²·m⁻⁴</td>
<td>( \sigma^2 G_{bL} )</td>
<td>90681</td>
<td>( \sigma^2 G_{DL} )</td>
<td>6971</td>
</tr>
<tr>
<td>The quadric deviation of the alternating irradiance, W·m⁻²</td>
<td>( \sigma G_{bL} )</td>
<td>301</td>
<td>( \sigma G_{DL} )</td>
<td>83.5</td>
</tr>
<tr>
<td>The amplitude of periodical component of diffuse irradiance, W·m⁻²</td>
<td>( A \rightarrow 0 )</td>
<td></td>
<td>( A )</td>
<td>64</td>
</tr>
</tbody>
</table>

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**Fig. 2. Example of some mostly sunny days 14 V 02–16 V 02**

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**Fig. 3. Example of some days 14 VI 02–16 VI 02 with variable radiation**
It can be concluded from the table that practically half of the solar radiation falling into the atmosphere is reflected, absorbed or transposed to diffuse radiation. The last one in turn is nearly half of the alternating beam radiation.

The estimated steady-state of hourly mean values is proven in Fig. 4, where the slipping average values of a mean hourly alternating beam and diffuse irradiance are shown. The averaging window is a month (30 days) long, and is moving over the season. The presented interval between hour numbers 409 and 2009 belongs to the middle of season to avoid the “border effects”. The numeration of the performance-hours starts with “1” at the spring equinox. The condition of stationarity is proven with the zero tilt-angle of the trend lines for both the slipping average and slipping variance.

Due to the said steady-state condition, we can calculate their autocorrelation functions (Fig. 5). There we can find a correlation interval $r(G_{bl})$ 5 h of the alternating beam irradiance and $r(G_{dl})$ 3 h for the alternating diffuse irradiance. Both of them are significantly shorter than a performance-day (12 h) and present an evidence of instability of solar radiation in this region. The maximum of variance in the middle of the season (Fig. 4), which coincides with the maximum of alternating irradiance, shows that the time in the middle of the season is also the most instable period. The alternating diffuse radiation consists of a large periodical component (with the period of selected performance time 12 h) and we have to clean $\rho(G_{dl})$ from that. We do it with the help of an artificial periodical signal “per” by subtracting it from $\rho(G_{dl})$. The relative amplitude of the said periodical signal $\alpha$ is set in such a way that the rest $\rho(G_{dl})$ should be possibly close to an exponent function. The amplitude of a real periodical component can be found from ACF as $A = (\alpha^2 \sigma^2 G_{dl})^{0.5}$. The phase of the periodical component cannot be detected by ACF, for this purpose we have to build a graph of hourly mean values over the season (Fig. 6). In this graph we can see that $G_{bl}$ is nearly constant during the day (an exception is the last working hour 17–18), but
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$G_{DL}$ has a character of a cosine function with the maximum at the noon. We can conclude that in our region cloudiness is more stable in the afternoon and cloud cover is the most variable in the sky at noon.

Since by measurements of the beam radiance $s$ the sensor follows the (virtual) position of the sun in the sky, the measured and calculated data are close $s = G_b$ at most clear-sky conditions (Fig. 2). According to these measurements, the average value of beam radiation is much higher than its periodical component, and we almost cannot see any periodical effects on $\rho(G_{bL})$ (Fig. 5). When measuring the diffuse radiation, the sensor is fixed horizontally and any radiation parallel to the beam component has a moving incidence angle during the day. This moving incidence angle involves a large periodical component of alternating diffuse radiance (dependence of the diffuse irradiance on the zenith angle). We can see significant periodical effects on $\rho(G_{DL})$ (Fig. 5) and find the amplitude of the periodical component nearly equal to the standard deviation (Table 1).

![Fig. 5. Autocorrelation function of the alternating beam and diffuse irradiance](image1)

![Fig. 6. Average daily value of the alternating beam and diffuse irradiance](image2)
Seasonal instability of solar irradiance

The diagrams built on the weekly mean values of $G_b$, $s$, $G_D$ and $D$ are presented in Fig. 7. Since the diagrams are easy to survey, there is no need for additional control of the steady state.

![Fig. 7. Diagrams of the irradiance for summer seasons 1999–2002](image)

The diagrams of alternating beam irradiance and diffuse irradiance are shown in Fig. 8, where we can also find a periodical component for the alternating diffuse irradiance. This phenomenon can be observed especially in the ACF diagram (Fig. 9). By the way, this result repeats and confirms the quality found for the season 2002 only (Fig. 4). It can be well seen in Fig. 9, where the lines of ACF for $G_{bL}$ and $s$ (also for $G_{DL}$ and $D$) are very close.

![Fig. 8. Diagrams of the alternating irradiance for summer seasons 1999–2002](image)
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When the averaging interval exceeds the period of the periodical component significantly, there is no need to use alternating irradiances, since the measured natural radiation guarantees the right result. The correlation interval for the beam irradiance in the “weekly range” is $t(G_{BL}) 1.5 w$ and that for the diffuse irradiance $t(G_{DL}) 3w$. The latter is evaluated without filtering out its periodical component and therefore the correlation interval of purely random diffuse irradiance is in fact less $t(G_{DL}) < 3w$. Using the seasonal data of two seasons 1999–2000 and known yearly sum of $s$ and $D$ for the same years, we can evaluate that the seasonal resource is close to the level of 0.76 from the yearly total. This ratio is valid both for the beam and diffuse radiation.

Long-time instability of solar irradiation

Fig. 10 shows the diagram of the yearly sum of the measured beam $s$, diffuse $D$ and global irradiance $Q$ from 1955 to 2000 (Tooming 2003). A negative linear trend of 0.1% per year can be observed for each value, which confirms previous findings (Russak, 1998). It results probably from the atmospheric pollution and warming up of the climate. The ratio $D/Q = 0.51$ is constant without showing any trend of change.

In Fig. 11 the autocorrelation function of the annual totals is presented. We can see that most probably the solar radiation has a long-time periodical component with a period $T \approx 30 \text{ y}$, which is equal both for the beam and diffuse components, and this phenomenon is evidently shown first. Due to a comparatively short control interval (46 years), this hypothesis has not
been correctly proven yet. Like in the case of seasonal instability investigation, there is no difference whether we look for the instability of radiation by the calculated alternating irradiance or natural radiation.
Conclusions

1. Principally, instability (or variability) of solar radiation can be best found by formal alternating irradiance defined as a difference between the radiations calculated for the clear sky conditions and the actual, measured values.

2. The measured data of beam irradiance, which are picked up by the sensor following the sun’s position, describe the random component of radiation quite well.

3. An approximation effect can be seen when averaging the data: if the averaging window or sampling interval are longer than the period of the periodical component, it will be sufficient to analyze the natural measured data.

4. The alternating diffuse irradiance preserves diurnal and seasonal periodicity, whose quality is lost for the alternating beam irradiance.

5. The instability of diffuse irradiance has its maximum at noon and in midsummer.

6. Yearly insolation seems to have a periodical component, which up to now has been considered a hypothesis.

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