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INTEGRATION OF GPS AND PSEUDOLITES – EFFECT ON THE POSITIONING ACCURACY

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K e y w o r d s: pseudolites, DOP factors, positioning accuracy, satellite configuration.

S u m m a r y

It is well known that for monitoring of engineering structures it is desirable and recommended, that the measuring system used could provide equal precision in all 3D coordinates, all the time. In many cases the spaceborn position should be augmented by other additional means, especially when the configuration of satellites is not very good (signal shadowing, small number of satellites). There are many possibilities, one of them is to use additional ranging signals transmitted from ground-based devices simulating satellites. The devices are called pseudosatellites or, more often in abbreviated form, pseudolites (PLs). Most often, they are used to strengthen geometry of positioning.

In this paper background theory of pseudolites as well as Dilution of Precision (DOP) is presented. Results of accuracy pre-analysis performed are also given.

INTEGRACJA GPS I PSEUDOLITÓW – WPŁYW NA DOKŁADNOŚĆ POZYCJONOWANIA

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S ł o w a k l u c z o w e: pseudolity, współczynniki DOP, dokładność pozycjonowania, konfiguracja satelitów.

S t r e s z c z e n i e

Wiadomo, że do monitorowania obiektów inżynierskich jest wymagane, aby wykorzystywany system pomiarowy mógł zapewnić równą dokładność wszystkim trzem współrzędnym przez cały czas trwania pomiaru. Przy wykorzystaniu pozycjonowania satelitarnego GPS często się zdarza, szczególnie w warunkach występujących zasłon sfery niebieskiej, że warunek ten nie jest spełniony. Wtedy system satelitarny powinien być zintegrowany z jakimś innym, niezależnym systemem pomiarowym. Jedną z możliwości jest zastosowanie urządzeń naziemnych, symulujących satelity GPS. Urządzenia takie są zwane pseudosatelitami lub w skrócie pseudolitami (PL). Najczęściej wykorzystuje się je do wzmocnienia geometrii pozycjonowania.

W prezentowanej pracy podano podstawy teorii pseudolitów oraz wykonywania analizy geometrii pozycjonowania (Dilution of Precision). Podano także wyniki przeprowadzonej wstępnej analizy dokładności pozycjonowania przy użyciu systemu GPS wspomaganego pseudolitami.

Introduction

All man-made constructions, in this number also bridges, are subject to many factors like strong winds, temperature influence, pressure of drifting ice float, floods, etc. Especially the floods force searching for such measuring techniques which enable quick ascertainment whether the bridge is safe and can be open for exploitation after a temporary shutdown, for example during or after the flood.

That is why in many scientific and technological centers all over the world research works are conducted aimed at elaboration of new techniques and methods of bridge deformation monitoring, which would be cheap, fast, reliable and which would not reqiure full approach to investigated object. It seems that amongst many other modern techniques, especially promising is taking advantage of satellite postioning, like GPS or GPS integrated with GLONASS. But, on the other hand, there are many such applications or circumstances, where accuracy and reliability of such spaceborn positions is too small, or its quality falls down temporarily, together with poorer satellite configuration. In such cases additional augmentation of the satellite systems is needed

Institute of Geodesy, Olsztyn University of Warmia and Mazury, has widely used satellite positioning (GPS and GLONASS) in surveying and geodesy. On the basis of our earlier experience as well as GPS literature it is known that positioning accuracy in GPS RTK mode is of the order of 1–2 cm for horizontal coordinates and about 3 cm for the vertical one. In the case of post-processing a millimeter accuracy for horizontal determinations and 1 cm for heights is obtainable. Thus the spaceborn position should be augmented by other additional means.

There are many possibilities, one of them is to use additional ranging signals transmitted from ground-based devices simulating satellites. The devices are called pseudosatellites or, more often in abbreviated form, pseudolites (PLs). Pseudolites can in fact play three different roles depending on operational conditions (LEE et al., 2004):

a) strengthening geometry of the ranging intersection - when a very high accuracy is required, like in deformation monitoring,
b) improving reliability of satellite solution - when there are very strict demands concerning the system used, like in aircraft landing,
c) providing possibility of satellite-like positioning indoors, like in positioning in mines.

Our general scientific and practical goal is application of integrated position determinations GPS and PLs to monitoring of bridges. Therefore it is seen that the purpose falls into the first point of above listed. It is foreseen that application of pseudolites will improve determinations of height and that it will provide continuous sub-centimeter accuracy.

In the last decade pseudolite equipment has been available and been applied to a range of applications (WANG 2002), such as aircraft landing (HOLDEN & MORLEY 1997, HEIN et al. 1997), deformation monitoring (BARNES et al., 2002, BARNES et al. 2003), Mars exploration (LEMASTER & ROCK 1999), precision approach applications, and others (BARLTROP et al. 1996; DAI et al., 2001b, WEISER 1998, CHOI et al. 2000, WANG et al. 2000, STONE & POWELL 1999, O'KEEFE et al. 1999).

Compared with satellites in space, pseudolites can be optimally located, which can significantly improve the geometric strength of positioning solutions, particularly for the height component but here it is obvious that installation of pseudolites in locations that ensure good geometry is the key to good positional precision. Thus geometrcal analyses is needed in preliminary stage of pseudolite-base experiments or applications. Such analyses are usually performed on the basis of the so called DOP (Dilution of Precision) factors (or their relative counterparts called RDOPs (eg. in HOFMANN-WELENHOF 1997, ERICKSON 1992).

In this paper the results of such preliminary analyses are given, but they are preceded by short and general description of pseudolites and some basic notes on the DOP factors.

The introduction about pseudolites is needed here since Polish literature devoted to this subject is very poor. On the other hand, the remarks given on the basis of DOP is given for completeness of the paper.

Pseudolites

In many papers devoted to pseudolites such a definition can be found: "Pseudolites are ground-based GPS-like signal transmitters, which can improve the 'open air' signal availability, or even replace the GPS satellites constellation for some indoor applications" (eg. LEE 2002). And actually, this describes the basic feature of the device. On this basis one can deduce how the device works and what is the reason of introducing the devices into scientific as well as practical applications. Also it is well known and repeated in numerous publications on pseudolites, that the use of pseudolites dates back as early as the 1970's. Even before the launch of the GPS satellites, pseudolites had been used to test the initial GPS user equipment (HARRINGTON & DOLLOFF 1976). KLEIN and PARKINSON (1984) analysed the geometric advantages of integrating GPS and pseudo-satellites.

It is well known that for spaceborne satellite positioning systems (like GPS or GLONASS) the accuracy, availability and reliability of the positioning results is very dependent on both the number and geometric distribution of satellites being tracked. Pseudolites may be used to increase the number and optimise the geometry of the determination. They also may be used without satellites at all, like in indoors applications (eg. WANG 2002).

They typically transmit signals at the GPS frequency L1 or L2 or both. They were proposed to transmit up to five frequencies: two in the 900 MHz ISM band, two in the 2.4 GHz ISM band, and the GPS L1 frequency (ZIMMERMAN et al. 2000). An advantage of such multi-frequency pseudolite systems is that the integer carrier phase ambiguities can be resolved instantaneously, due to redundant measurements and the extra wide-lane observables that can be constructed from the different frequencies. Currently the majority of the pseudolites transmit GPS-like signals at the frequencies of L1 (1575.42 MHz) and possibly on L2 (1227.6 MHz). Both pseudo-range and carrier phase measurements can be made on the pseudolite signals.

The mathematical models for the pseudolite pseudo-ranges and phases are very similar to those for GPS receiver, and they read (WANG et al. 2001a, 2001b):

$$R_A^A = \rho_A^p + c(dt^p - dt_A) + T_A^p + dr_A^p + dm_A^p + \varepsilon_A^p$$

$$\phi_A^p = \frac{1}{\lambda_p}\rho_A^p + \frac{c}{\lambda_p}(dt^p - dt_A) + N_A^p + \frac{1}{\lambda_p}T_A^p + \frac{1}{\lambda_p}dr_A^p + \delta m_A^p + e_A^p \qquad (1)$$

where the lower index A means the station, upper index p means the pseudolite, R, ϕ, ρ are pseudorange measurement, carrier phase measurement, topocentric distance between the station and the pseudolite respectively, c is the speed of light, λ_p is the wavelength of the carrier frequency for pseudolite dt^p, dt_p are the pseudolite and the receiver clock errors, N_A^p integer carrier phase ambiguity, T_A^p is the tropospheric delay on the path from the pseudolite to the receiver, dr_A^p is the pseudolite location error, $dm_A^p, \delta m_A^p$ are multipath errors in the pseudo-range and carrire phase, and ε_A^p, e_A^p are pseudo-range and carrier phase measurement errors espectively.

Some remarks are needed here. First of all, it should be noted that there is no ionosphere in the equations (1), the reason is obvious: the signal does not pass ionosphere between the pseudolite and the receiver. But when forming double differences with GPS satellite and the pseudolite involved, it should be remembered that the influence of the ionosphere on the satellite signal will not be reduced.

In both the equations (1) the geometric distance term occurs, in case of carrier phases it is expressed in units of carrier wavelengths. It is the main term, in the sense that it contains coordinates of the station, which, in most cases, are the unknowns we are looking for. Of course, the distance is a nonlinear function in which the coordinates are involved. The nonlinearity may be significant in case of using near pseudolites, eg. (WANG 2001b; TSUJI et al., 2001) report that an error of 15 m in one of approximate coordinate components causes an error of nonlinearity of 0.6 m (estimated for separation between the pseudolite and the receiver equal to 200 m). At the same time, in case of satellite positioning, an error of even 200 m in approximate coordinates cause an error of nonlinearity of the order of 1 mm. It is seen then, that in case of pseudolite positioning one should use the approximate coordinates as accurate as possible, and the processing should be performed in iterative mode.

It can be seen that the clock terms in (1) look just like in observation equations for GPS satellites, but it should be emphasized here that pseudolites have TXCO (Temperature Compensated Cristall Oscillators), which are not so accurate. So, the receivers tracking the pseudolite signals cannot synchronize their sampling time exactly and get data at different times. Even though the receiver clock error (dt_A) and pseudolite clock error (dt^p) terms are removed through double difference, the different sampling times cause range error from Doppler (time-tag).

The next factor to be considered is the troposphere: one should use a different model for tropospheric delay of the signal coming from the pseudolite, since it propagates only through the lower part of the troposphere, not crossing different layers, like the signal from satellite does. Several models for refractivity at the surface of the Earth have been derived (HARTMANN and LEITINGER 1984). Typical tropospheric corrections between two on-ground stations are of the order of 35 cm/km (GREJNER-BRZEZINSKA and YI 2002).

The next term to look at is the location error of the pseudolite dr_A^p. It is obvious that for satellite positioning, users should know the accurate position of the phase center of the transmission antennas. In case of GPS satellite, the positions of the satellites can be computed using ephemeris data, by the receiver or by a computer programme, in case of post-processing mode. But for an integrated GPS/PL system the accurate positions of the pseudolite transmission antennas must be determined and provided to users. The location error of pseudolite is much more serious than that of real satellite; a small error of transmission position creates relatively big line of sight (LOS) vector error because of very short distances between the user and the pseudolite. The accurate position of the phase center of the transmitting antennas of the PL should be measured, but it can be very difficult for some types of antennas used in PLs, like a helical one (KEE et al., 2000). The so called inverse method (KEE et al., 2000) may be used here for determination of PL antenna phase center, where the receivers of GPS are placed at known locations, they receive signals from the PL, which is regarded as unknown.

According to the equations (1) also the multipath errors, for pseudo-range as well as for phase measurements should also be considered. What is more, due to the low elevation angle, multipath is of much greater concern with a pseudolite signal than with a GPS satellite signal. It should be noted that for static sessions, this error theoretically is a constant, thus it would give constant biases which could be removed from the observations (BARNES et al., 2003). But in the experiment cited there, the need of more sophisicated procedures for multipath reduction are reported, like choke-ring antennas, developing a multipath "signature" of the bridge measured or removing the low frequency trends from the position time series. In precise kinematic cases this problem is much more difficult. One possible approach is to use proper transmitting antennas in the pseudolites, like helical ones (KEE et al. 2000).

The terms ε_A^p, e_A^p , reflecting pseudo-range and carrier phase measurement errors respectively, may be greater in case of pseudolite than spaceborn satellite (WANG et al., 2000). It should be mentioned here that the received pseudolite signal enters the recever's antenna through a low gain portion of it. This may result in a low signal-to-noise ratio for the received pseudolite transmission, yielding noisier code and carrier phase measurements.

Besides the factors listed, there are also several technical issues that need to be addressed, like the 'near-far' problem and signal design (eg. Wang, 2002). GPS can be regarded as CDMA (Code Division Multiple Access) system (PARKINSON and SPILKER 1996, WANG, 2002). It means that each satellite uses its own code. When the codes chosen have suitable properties (in case of GPS the so called "golden codes") it prevents interference between signals coming from different satellites. But in such systems the power of all signals reaching the antenna should be of similar level. If the power of one signal is much higher than the others, a receiver tracks only this one signal because the high-powered signal acts as noise to the channels tracking the others (near problem). And if the power of one signal is much lower than the others, a receiver cannot track that signal (far problem). Since received signal power is proportional to the inverse of squared range from transmitter and the receiver; small user movement does not affect the signal power from satellite, but it does influence that from the pseudolite, because it is installed in close vicinity to the receiving device. Therefore, even that there are "free" golden codes, which could be assigned to the pseudolites, the solution preferred nowadays consists in application of TDMS (Time Division Multiple Access) (tamże). Pseudolites using this system send pulsing signals at fixed cycle rates, which can take up, for example, 20% of the cycle period. Let us recall that the C/A code period is equal to 1 ms, then the pseudolite transmits its signal for 200 µs (20% of 1 ms).

In spite of the problems listed, the pseudolites have found many applications (WANG 2002, GREJNER-BRZEZINSKA and YI 2002). Generally, they can be used to design pseudolite-only indoor positioning systems, in positioning in deep mines or overbuilt urban areas, when there are too few GNSS satellites visible. They can also be used to strengthen geometry of the position derived, improving accuracy and reliability of satellite solution (see references given in Introduction).

Our general scientific and practical goal is application of integrated position determinations GPS and PLs to monitoring of bridges. In such application, it is desirable that the measuring system delivers equal precision in all position components, all the time. When using GPS to derive position, then the accuracy, continuity and reliability of the results depends very much on constellation visible. When the number of observed satellites changes, there can be seen jumps in accuracy of the data derived. What is more, due to the geometric distribution of the satellites all of which are over the point to be positioned (data from satellites below approximately 15^o are typically not used), the accuracy of the height component is generally 2 or 3 times worse than for the horizontal components. Additionaly, in areas located on latitudes higher than about 45^o, the North component is worse than the East one, due to the 55^o inclination of GPS orbits in respect to the equatorial plane (MENG et al. 2003). One possibilty to make the situation better is to apply pseudolites. Compared with satellites in space, pseudolites can be optimally located, which can significantly improve the geometric strength of positioning solutions, particularly for the height component, thus it is obvious that installation of pseudolites in locations that ensure good geometry is the key to good positional precision. Thus geometrcal analyses is needed in preliminary stage of pseudolite-base experiments or applications.

DOP and RDOP factors

The DOP factor is a measure for the geometry of solution in satellite positioning. It is defined as scalar, it can be used both in autonomous as well as in relative positioning (HOFMANN-WELENHOF 1997, ERICKSON 1992). This extension is refered to as RDOP (Relative DOP). DOPs are computed as the square root of the sum of the diagonal components of the covariance matrix of parameters (unknowns) $\mathbf{C}_{\mathbf{X},a}$ in single point positioning model, which is the inverse of the normal equations:

$$\mathbf{C}_{\mathbf{X},a} = (\mathbf{A}_a{}^{\mathbf{T}}\mathbf{C}_a^{-1}\mathbf{A}_a)^{-\mathbf{1}} \tag{2}$$

where $\mathbf{A}_a$ is the design matrix and $\mathbf{C}_a^{-1}$ is unscaled and represents relative weights of the observations. The index a recalls that the matrix refers to single point (autonomous) positioning. In case of single point positioning it has diagonal form, with all diagonal elements being equal (because of the assumption that observations to all satellites are of equal accuracy) thus it can be omitted in (2) and matrix $\mathbf{C}_{\mathbf{X},a}$ can be written as:

$$\mathbf{C}_{\mathbf{X},a} = \sigma_{0a}^2(\mathbf{A}_a{}^{\mathbf{T}}\mathbf{A}_a)^{-\mathbf{1}} \tag{2a}$$

where means observation accuracy. The design matrices for both the cases of single point and relative positioning may ba found in numerous references, see eg. in (HOFMANN-WELENHOF 1997). In case of single point positioning the matrix can be written in cartesian coordinates as:

$$\mathbf{C}_{\mathbf{X},a} = \sigma_{0a}^2 \begin{bmatrix} \sigma_{X,a}^2 & \sigma_{XY,a} & \sigma_{XZ,a} & \sigma_{Xt,a} \\ \sigma_{YX,a} & \sigma_{Y,a}^2 & \sigma_{YZ,a} & \sigma_{Yt,a} \\ \sigma_{ZX,a} & \sigma_{ZY,a} & \sigma_{Z,a}^2 & \sigma_{Zt,a} \\ \sigma_{tX,a} & \sigma_{tY,a} & \sigma_{tZ,a} & \sigma_{t,a}^2 \end{bmatrix}$$

In geodetic coordinate system the covariance matrix of parameters can be written as:

$$\mathbf{C}_{\mathbf{X},a}^g = \sigma_{0a}^2 \begin{bmatrix} \sigma_{B,a}^2 & \sigma_{BL,a} & \sigma_{Bh,a} & \sigma_{Bt,a} \\ \sigma_{LB,a} & \sigma_{L,a}^2 & \sigma_{Lh,a} & \sigma_{Lt,a} \\ \sigma_{hB,a} & \sigma_{hL,a} & \sigma_{h,a}^2 & \sigma_{ht,a} \\ \sigma_{tB,a} & \sigma_{tL,a} & \sigma_{th,a} & \sigma_{t,a}^2 \end{bmatrix}$$

Now we are in a position to define various DOP factors:

– geometrical dilution of precision GDOP:

$$GDOP = \sqrt{\sigma_{X,a}^2 + \sigma_{Y,a}^2 + \sigma_{Z,a}^2 + \sigma_{t,a}^2} = \sqrt{\sigma_{B,a}^2 + \sigma_{L,a}^2 + \sigma_{h,a}^2 + \sigma_{t,a}^2}\,,$$

– positional dilution of precision PDOP:

$$PDOP = \sqrt{\sigma_{X,a}^2 + \sigma_{Y,a}^2 + \sigma_{Z,a}^2} = \sqrt{\sigma_{B,a}^2 + \sigma_{L,a}^2 + \sigma_{h,a}^2}\,,$$

– horizontal dilution of precision HDOP:

$$HDOP = \sqrt{\sigma_{B,a}^2 + \sigma_{L,a}^2}\ ,$$

– vertical dilution of precision VDOP:

$$VDOP = \sigma_{h,a}\ ,$$

– time dilution of precision TDOP:

$$TDOP = \sigma_{t,a}\ .$$

In case of relative positioning the weight matrix for observations is not diagonal, thus it must be taken into allowance in forming equations for appropriate DOP factors. Let us assume that the relative position is derived on the basis of the pseudoranges measured or carrier phases with fixed ambiguities (for the case of float ambiguities refer to (ERICKSON 1992)), then there are only 3 parametrs in our adjustment task, it means the corrections to approximate coordinates of the unknown station. They may be written in both cartesian and geodetic systems. In case of relative positioning the design matrix does not contain the time components. Its explicit form may be found in (HOFMANN-WELENHOF 1997). The (non-diagonal) matrix **C** for (correlated) double differences is derived in (HOFMANN-WELENHOF 1997), too.

The formulae for $\mathbf{C}_{\mathbf{X},r}$ in cartesian and geodetic systems may be written symbolically as:

$$\mathbf{C}_{\mathbf{X},r} = \sigma_{0r}^2 \begin{bmatrix} \sigma_{X,r}^2 & \sigma_{XY,r} & \sigma_{XZ,r} \\ \sigma_{YX,r} & \sigma_{Y,r}^2 & \sigma_{YZ,r} \\ \sigma_{ZX,r} & \sigma_{ZY,r} & \sigma_{Z,r}^2 \end{bmatrix} \text{ and } \mathbf{C}_{\mathbf{X}}^{g} = \sigma_{0r}^2 \begin{bmatrix} \sigma_{B,r}^2 & \sigma_{BL,r} & \sigma_{Bh,r} \\ \sigma_{LB,r} & \sigma_{L,r}^2 & \sigma_{Lh,r} \\ \sigma_{hB,r} & \sigma_{hL,r} & \sigma_{h,r}^2 \end{bmatrix} \quad (3)$$

Similarly like in the case of single point positioning, now the RDOP factors may be defined:

– geometrical relative dilution of precision RGDOP:

$$RGDOP = \sqrt{\sigma_{X,r}^2 + \sigma_{Y,r}^2 + \sigma_{Z,r}^2} = \sqrt{\sigma_{B,r}^2 + \sigma_{L,r}^2 + \sigma_{h,r}^2}\ ,$$

– positional relative dilution of precision RPDOP:

$RPDOP = RGDOP$ (only in case of fixed ambiguities),

– horizontal dilution of precision HDOP:

$$HDOP = \sqrt{\sigma_{B,r}^2 + \sigma_{L,r}^2}\ ,$$

– vertical dilution of precision VDOP:

$$VDOP = \sigma_{h,r}\ .$$

The DOP factors are used to assess the effect that satellite geometry has on positioning results. Starting again with single point positioning, the following relation holds:

$$\sigma_a = \sigma_{0,a} \cdot DOP \tag{4}$$

When relative positions and double differenced observations are regarded, it can be written:

$$\sigma_r = \sigma_{0,r} \cdot RDOP \tag{4a}$$

In these relations σ_a and σ_r mean achievable accuracy of single point and relative double difference positioning, $\sigma_{0,a}$ and $\sigma_{0,r}$ mean measurement accuracy of undifferenced and double differenced observations (reflecting unmodelled and not-removed errors).

The covariance matrix $\mathbf{C_x}$ in (3) is computed on the basis of (2), but now the definitions of the design and weight matrices are different:

$$\mathbf{C}_{\mathbf{X},r} = (\mathbf{A}_r^T \mathbf{C}_r^{-1} \mathbf{A}_r)^{-1},$$

where:

$\mathbf{A}_r$ – the design matrix in double difference model, containing only positional terms (since the assumption of fixed ambiguities), the index r stands for "relative"

$\mathbf{C}_r^{-1}$ – the weight matrix of observations, non-diagonal in case of double difference model. The matrix $\mathbf{C}_r^{-1}$ depends on the number of satellites observed and is given as (eg. HOFMANN-WELENHOF 1997):

$$\mathbf{C}_r^{-1} = \frac{1}{2\sigma_{0r}^2} \frac{1}{n_{dd}+1} \begin{bmatrix} n_{dd} & -1 & .. & -1 \\ -1 & n_{dd} & .. & -1 \\ .. & .. & .. & .. \\ -1 & -1 & .. & n_{dd} \end{bmatrix}$$, where is number of double differences created.

The matrix $\mathbf{A}_r$ is closely related to the matrix $\mathbf{A}_a$ and the successive rows of it can be computed making differences between the row containing data for the reference satellite and the remaining ones. Such the way of computations gives a fixed relation between the RDOP and DOP factors, independent of the number of satellites observed and their configuration. It may be checked that each component of the matrix $\mathbf{C}_{\mathbf{X},r} = (\mathbf{A}_r^T \mathbf{C}_r^{-1} \mathbf{A}_r)^{-1}$ is exactly two times bigger than an appropriate component of $\mathbf{C}_{\mathbf{X},a} = (\mathbf{A}_a{}^{\mathbf{T}}\mathbf{C}_a^{-1}\mathbf{A}_a)^{-1}$. From this it follows that always RDOP= $\sqrt{2}\cdot$DOP for one epoch of observations.

The DOP and RDOP factors were used to perform the pre-analysis of positioning accuracy obtainable in the bridge experiment.

Accuracy analysis

On 1st October 1998 the motorway bridge over Vistula River, located near Toruń, was open. Its structure is a 13-span continuous box girder (BIEŃ 1999). Total length of the bridge is 955.4 m. For the needs of the bridge and near motorway construction, a control network was established, in the shape of geodetic triangles and quadrilaterals, connected to the detailed national network of II class. The coordinates of the bridge object were transformed into a local system of coordinates. The accuracy of coordinates determination is on the level of 1–2 mm.

For the pre-analysis performed 6 points from the network were chosen (see Fig. 1). Three of them are located on the bridge itself, and they are treated as points to be monitored, and the remaining three are located in the bridge vicinity, here they are used as possible locations for pseudolites.

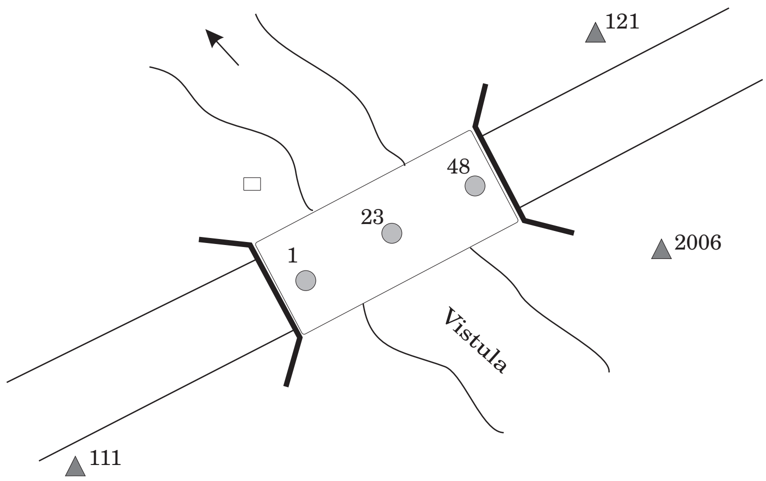

Fig. 1. Points chosen for analysis – the bridge and its vicinity (121, 111, 2006 – control points, used as possible locations for pseudolites; 1, 23, 48 – points to be observed

Table 1

Results of the pre-analysis

Point	ѕαριαντ	ΓΔΟΠ	ΠΔΟΠ	ΗΔΟΠ	ѕΔΟΠ	ΝΔΟΠ	ΕΔΟΠ	ΤΔΟΠ	ΡΠΔΟΠ	ΡΗΔΟΠ	ΡѕΔΟΠ	ΡΝΔΟΠ	ΡΕΔΟΠ
each	0	4.34	3.64	1.94	3.04	1.82	0.69	2.36	5.16	2.75	4.30	2.57	0.98
1	1-111	2.02	1.79	1.17	1.33	0.95	0.68	0.94	2.53	1.65	1.89	1.35	0.96
	1-121	3.27	2.82	1.79	2.15	1.64	0.72	1.65	3.99	2.53	3.03	2.32	1.02
	1-2006	2.08	1.86	1.28	1.32	1.04	0.76	0.94	2.63	1.82	1.87	1.46	1.07
	2-111+121	1.73	1.57	1.23	1.08	0.95	0.60	0.72	2.22	1.59	1.53	1.34	0.85
	2−111+2006	1.53	1.41	1.00	0.97	0.85	0.53	0.60	1.98	1.42	1.38	1.21	0.75
	2−121+2006	2.01	1.79	1.23	1.28	0.97	0.75	0.91	2.53	1.74	1.81	1.38	1.06
	3-111+121+2006	1.42	1.31	0.94	0.90	0.78	0.52	0.55	1.85	1.32	1.28	1.10	0.73
23	1-111	2.06	1.82	1.19	1.36	0.98	0.97	0.96	2.58	1.68	1.92	1.39	0.95
	1-121	3.35	2.88	1.82	2.21	1.67	0.72	1.70	4.08	2.57	3.12	2.36	1.02
	1-2006	2.03	1.82	1.25	1.29	1.00	0.76	0.91	2.57	1.77	1.83	1.41	1.07
	2-111+121	1.78	1.61	1.15	1.11	0.98	0.60	0.76	2.28	1.62	1.58	1.38	0.85
	2−111+2006	1.53	1.40	1.00	0.97	0.85	0.53	0.60	1.99	1.42	1.38	1.21	0.74
	2-121+2006	1.95	1.74	1.19	1.25	0.92	0.75	0.88	2.46	1.68	1.76	1.30	1.06
	3-111+121+2006	1.41	1.30	0.92	0.90	0.77	0.51	0.55	1.84	1.30	1.27	1.08	0.73
48	1-111	2.08	1.84	1.20	1.38	1.00	0.67	0.97	2.61	1.70	1.94	1.42	0.95
	1-121	3.48	2.98	1.85	2.30	1.71	0.72	1.78	4.22	2.62	3.26	2.41	1.02
	1-2006	1.95	1.75	1.20	1.25	0.93	0.76	0.87	2.47	1.70	1.77	1.32	1.08
	2−111+121	1.84	1.65	1.16	1.16	0.99	0.61	0.80	2.34	1.65	1.63	1.40	0.86
	2-111+2006	1.52	1.40	0.99	0.98	0.83	0.53	0.60	1.98	1.40	1.38	1.18	0.75
	2-121+2006	1.85	1.65	1.12	1.19	0.84	0.75	0.83	2.34	1.59	1.69	1.18	1.06
	3-111+121+2006	1.38	1.27	0.89	0.90	0.72	0.52	0.54	1.80	1.26	1.27	1.02	0.73

Explanation: in variant name the first number means the number of pseudolites assumed (thus variant 0 means that computations for satellites alone were performed), when it is not equal to 0, the point numbers being location(s) of pseudolite(s) follow.

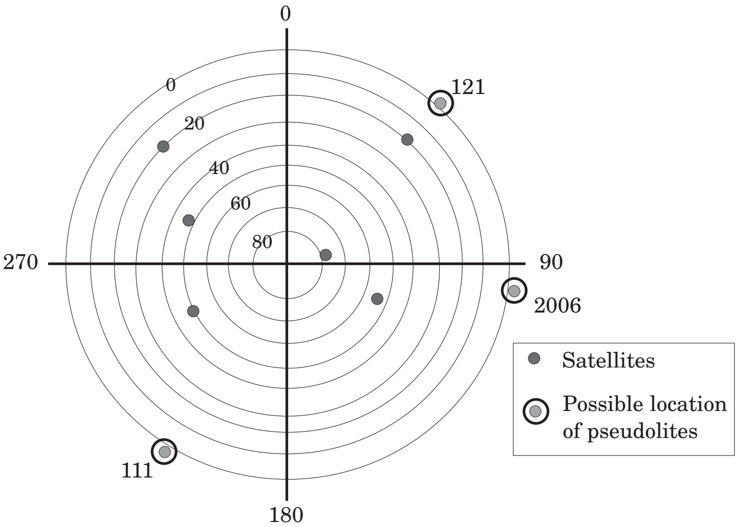

Fig. 2. Configuration of satellites and possible pseudolites – sky view

Example satellite configuration for 1st August 2004, 9h was used, basing on the almanach file alm 04.214. There are 6 satellites visible. The variants considered are schematically given in Fig. 2. Three, two or one pseudolites were assumed. The resulting DOP and RDOP factors were compared to the values obtained for the case without any pseudolite. Computations were performed for the points 1, 23, 48. At these points, since they are close to each other, the satellite configuration was admitted the same, while the azimuths and elevations of the pseudolites locations were computed separately for each point. The known values of azimuths and elevations (of both satellites and pseudolites assumed) were used to compute matrices $\mathbf{A}_a$ and $\mathbf{A}_r$. Matrices $\mathbf{C}_a^{-1}$ and $\mathbf{C}_r^{-1}$ were admitted as given in section 3. The results of the pre-analysis are shown in Table 1.

Looking through the above Table, the following conclusions can be derived:

a) The configuration of satellites (variant 0) gives the value of PDOP equal to 3.64. The smallest theoretical value of this factor, computed for 6 satellites is between 1.47 (the value obtained for minimum elevation=10^o) and 1.70 (the value obtained for minimum elevation=20^o) (CELLMER 2004). Thus the configuration chosen for the analysis is not a very good one. It was chosen on purpose - since the improvement observed when adding PLs is less for good configurations of satellites.

b) Let us look at the case of 1 PL - then the smallest PDOP possible (computed for 7 satellites, 0^o of the minimum elevation) is equal to 1.25. In case of analysis performed the best results were obtained when locating the PL at 111 (for points 1 and 23) or 2006 (for points 23 and 48). The values obtained fall into 1.79 to 1.84. In case of locating the PL at 121, the values of PDOP range from 2.82 to 2.98. It can be explained, since more satellites are located in northern semi-circle, thus it is better to locate the PL in the southern semi-circle (azimuths from 90° to 270°). It can be concluded, that having only 1 PL, one should have prepared some different locations for the PL, the satellite configuration predicted should be analysed for the period of observations, and the PL should be set at the point located as far as possible (in terms of azimuths) from most satellites.

c) Extrapolating, in case of 2 PLs, the best values of the DOP factors are obtained for PLs located at 111 and 2006, then the PDOP values are near 1.40, while the smallest PDOP possible (computed for 8 satellites, 0° of the minimum elevation) is equal to 1.17).
d) It is obvious that the smallest values of DOPs are obtained in case of 3 PLs – for PDOP the appropriate values are between 1.27 to 1.37 (minimum: 1.09).
e) It is interesting to notice that in some cases (in the columns of EDOP and RDOP) there occur small deterioration of results.
f) The biggest improvement is observed for the vertical (VDOP and RVDOP) and time components (TDOP).
g) When only satellites are considered, the east components (EDOP, REDOP) are the smallest, after adding PLs their change is very small (in some cases being worse with PLs).

Conclusions

1. In the paper the theoretical basis of pseudolites and DOP factors was given.

2. The analysis of accuracy in case of using pseudolites to improve positioning accuracy was performed.

3. In case of very good configuration of satellites, adding PLs is not so important as in cases of poor satellite configuration.

4. It seems that there should be some locations for PLs prepared – then the actual locations should be chosen on the basis of satellite configuration predicted for the period of observation campaign.

5. The biggest improvement is observed for the vertical (VDOP and RVDOP) and time components (TDOP).

6. When only satellites are considered, the east components (EDOP, REDOP) are the smallest, after adding PLs their change is very small (in some cases being worse with PLs).

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