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INTERDISCIPLINARY APPROACH TO DEFORMATION ANALYSIS IN ENGINEERING, MINING, AND GEOSCIENCES PROJECTS BY COMBINING MONITORING SURVEYS WITH DETERMINISTIC MODELING PART I

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Abstract

This presentation summarizes the author's developments and contributions to the interdisciplinary approach to integrated deformation analysis in engineering, mining and geosciences projects by combining monitoring surveys and deterministic analysis (numerical modeling using finite element method). The combination of monitoring and numerical modeling of deformations is essential for studying the processes occurring in engineering structures and in rock mass at the construction and post-construction stages.

Safety, economy, efficient functioning of man-made structures and fitting of structural elements, environmental protection, and development of mitigation measures in case of natural disasters require good understanding of causative factors and the mechanism of deformations, which can be achieved only through proper monitoring and analysis of deformable bodies. The author has developed the interdisciplinary approach to modeling, physical interpretation, and prediction of deformations. The approach is based on a combination (integration) of deterministic modeling (prediction) of deformations with the monitoring results obtained from geodetic and/or geotechnical measurements of displacements and deformations of the investigated object.

INTERDYSCYPLINARNE PODEJŚCIE DO ZINTEGROWANEJ ANLIZY DEFORMACJI DOTYCZĄCEJ PROBLEMÓW INŻYNIERYJNYCH, GÓRNICZYCH I GEOFIZYCZNYCH PRZEZ POŁĄCZENIE POMIARÓW GEODEZYJNYCH Z ANALIZĄ DETERMINISTYCZNĄ CZĘŚĆ I

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S ł o w a k l u c z o w e: analiza deformacji, zintegrowany monitoring, inżynieria, górnictwo, metoda elementów skończonych.

S t r e s z c z e n i e

Praca jest podsumowaniem osiągnięć naukowych autorki w zakresie interdyscyplinarnego podejścia do zintegrowanej analizy deformacji problemów inżynieryjnych, górniczych i geofizycznych, przez połączenie pomiarów geodezyjnych z analizą deterministyczną (numeryczna analiza metodą elementów skończonych). Połączenie monitorowania i metody deterministycznej w modelowaniu deformacji jest nieodzowne do przeprowadzenia analizy procesów zachowania się struktur inżynieryjnych i górotworu w czasie budowy oraz podczas eksploatacji obiektów. Bezpieczeństwo, ekonomika i właściwe funkcjonowanie całej budowli oraz współdziałanie jej elementów, uwzględnienie aspektów ochrony środowiska oraz stworzenie możliwości przeciwdziałania w przypadku naturalnych katastrof wymaga dobrego zrozumienia przyczyn i mechanizmu odkształceń. Zrozumienie to może być osiągnięte przez prowadzenie odpowiedniego monitorowania i analizy odkształceń obiektów. Autorka opracowała interdyscyplinarną metodologię modelowania, fizycznej interpretacji i przewidywania odkształceń. Metodologia ta jest oparta na integracji deterministycznego modelowania odkształceń z wynikami geodezyjnych lub geotechnicznych obserwacji przemieszczeń i odkształceń badanego obiektu.

1. Introduction

This presentation summarizes the author's developments and contributions to the interdisciplinary approach to integrated deformation analysis in engineering, mining and geosciences projects by combining monitoring surveys and deterministic analysis (numerical modeling using finite element method). The combination of monitoring and numerical modeling of deformations is essential for studying the processes occurring in engineering structures and in rock mass at the construction and post-construction stages.

Safety, economy, efficient functioning of man-made structures and fitting of structural elements, environmental protection, and development of mitigation measures in case of natural disasters require good understanding of causative factors and the mechanism of deformations, which can be achieved only through proper monitoring and analysis of deformable bodies. Development of new methods and techniques for monitoring and analysis of deformations and development of methods for optimal modeling and prediction of deformation is the subject of intensive international studies of several professional and scientific groups.

Within the most active international organizations, which are involved in deformation studies, one should list the International Federation of Surveyors (FIG) with their very active group on deformation measurements and analysis; International Association of Geodesy (IAG) with their commissions on geodynamics and recent crustal movements; International Society for Mine Surveying (ISM) with their commission on ground subsidence and surface protection in mining areas; International Society for Rock Mechanics (ISRM) with their overall interest in rock stability and ground control; International Commission on Large Dams (ICOLD); International Society of Soil Mechanics and Foundation Engineering, and International Association of Hydrological Sciences (IAHS), which has an interest in ground subsidence due to the withdrawal of underground liquids (water, oil, etc.). Most of the activities and studies of the various organizations focus, of course, on direct applications to their particular deformation problems according to their specialization. In order to monitor and model the deformations, an interdisciplinary effort is needed to develop generalized methods and techniques for integrated monitoring, analysis, and physical interpretation of deformations. This task was initiated by FIG in the early 1980s.

The author has been involved in the FIG activity on deformation measurements and analysis from the very beginning. Thanks to her background in mechanical engineering and in mining geomechanics and over 20 years of working together with geodetic engineers at the University of New Brunswick, the author has developed the interdisciplinary approach to modeling, physical interpretation, and prediction of deformations. The approach is based on a combination (integration) of deterministic modeling (prediction) of deformations with the monitoring results obtained from geodetic and/or geotechnical measurements of displacements and deformations of the investigated object.

In 1980s, the author's research focused on developing a methodology for modeling and predicting ground subsidence caused by mining activity in brittle rock. The methodology is based on a sequential-computation (S-C) method developed by the author (Szostak-Chrzanowski and Chrzanowski 1991a).

Since 1990, the author's research has focused on the development of an analytical approach to the identification of physical parameters of the deformable material to gain information on the material model, and to enhance understanding of the deformation mechanism of the investigated object (rock mass or structure). The main objective of the author's research has been to develop a methodology for improving the modeling of large-scale deformation problems, for example, modeling of the whole rock mass versus small scale problems when the rock behavior is investigated only in the immediate vicinity around the underground opening. Using the combination of deterministic modeling with the results of monitoring surveys and using a concept of separability, the author has developed a methodology for identifying the best model of the deformation mechanism from among several postulated models.

The analysis of deformation is based on continuum mechanics. Solving differential equations of equilibrium is the main problem in continuum mechanics. In many cases closed form solutions may be difficult or impossible to obtain. Therefore, numerical methods, such as the Finite Element Method (FEM), are used. To facilitate her research, the author developed FEMMA software (Szostak-Chrzanowski and Chrzanowski 1991b) for the FEM analysis including rigorous error propagation (Szostak-Chrzanowski et al. 1993a).

This presentation, after a general review of problems and literature related to the analysis of deformations, summarizes the author's achievements and applications of the integrated deformation analysis in engineering, mining and geosciences projects. More details are given in the following selected publications (Chrzanowski at al. 2000, Chrzanowski and Szostak-Chrzanowski 2004, Szostak-Chrzanowski et al. 1993a, 1994, 1996, 2002, 2005). Due to the interdisciplinary nature of the research, the author has to cooperate with specialists in geodetic engineering, rock mechanics, geotechnical and structural engineering, and geophysicists.

The author's developed methodology has found many practical applications and has been implemented in several industrial projects sponsored by:

POTACAN and PCS potash mining corporations in New Brunswick, Canada; KGHM Polish Copper; Metropolitan Water District of Southern California, USA; Hydro Quebec, Quebec, Canada; Canadian Centre for Mineral and Energy Technology; and the Geophysics Division of the Geological Survey of Canada

2. Background and basic definitions of deformation analysis

2.1. Review of problems

Integrated analysis of deformations of any type of a deformable body includes geometrical analysis and physical interpretation. Geometrical analysis describes the change in shape and dimensions of the monitored object, as well as its rigid body movements – translations and rotations – (Chrzanowski et al.

1990). The ultimate goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains (Chrzanowski et al. 1983, Chrzanowski et al. 1986). Physical interpretation is to establish the relationship between the causative factors (loads) and the deformations (Chen and Chrzanowski 1986). This can be determined either by a statistical method, which analyses the correlation between the observed deformations and loads, or a deterministic method, which utilizes information on the loads, properties of the material, and physical laws governing the stress-strain relationship (Fig. 1).

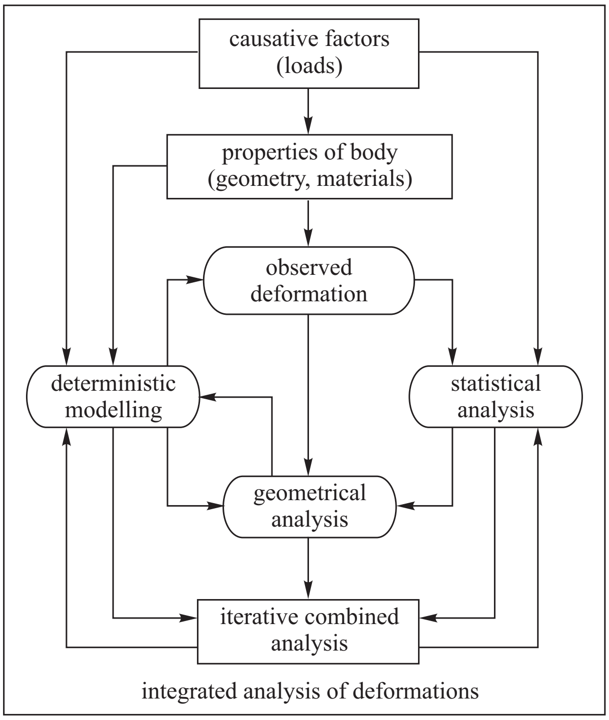

Fig. 1. Schematic load-deformation relationship

Most problems of geometrical analysis have been solved within the FIG activity. Many problems, however, remain to be solved in deterministic modeling of deformations particularly when modeling behavior of natural materials such as rock and soils.

In order to form a deterministic problem the following information must be defined: acting loads, material properties, model of the behaviour of the material, and boundary conditions.

The most critical problem in modeling and predicting rock deformations is to obtain real characteristics of in-situ rock mass. Collection of in-situ characteristics of rock is very difficult and very costly and the data is often incomplete. In laboratory testing, the selected samples may differ from one location to another, they may be disturbed during the collection, or the laboratory loading conditions may differ from natural conditions. This is especially valid in case of soil testing. The physical values obtained from laboratory testing require scaling in order to represent a rock mass. The process involves a degree of uncertainty. Generally, four types of scale are distinguished: (1) sample as an intact rock; (2) rock in the vicinity of opening with some joints; (3) rock with many joints; and (4) rock as a rock mass. The problem of scale-dependent properties is a main problem in modeling rock behavior (GLASER and DOOLIN 2000).

One of the important problems in rock mechanics is to model the Young modulus and the strength of the rock material, which vary through the in-situ rock mass. Generally, Young's modulus of in-situ rock masses is smaller than the values obtained in a laboratory (BIENIAWSKI 1984, SAKURAI 1997). The material strength decreases with the size of the model where large fractures are predominant. In case of soil or rockfilled material (e.g. embankment dams), the values of geotechnical parameters change during and after construction. During filling up a reservoir the earth dams undergo a process of wetting of the material in selected zones. The material properties may considerably differ between dry and wet conditions (TOUILEB et al. 2000).

Another important problem in numerical modeling is the selection of the material behavior model. The model of linear elasticity is still most widely used in modeling behavior of rocks, especially hard rocks (JING 2003). More sophisticated constitutive models used in rock mechanics are, for example, non-tensional model (ZIENKIEWICZ et al. 1968), anisotropic elasticity, plasticity, elasto-plasticity, and visco-elasticity. Plasticity and elasto-plasticity models used in rock mechanics are typically based on Mohr-Coulomb and Hoek-Brown failure criteria (HOEK and BROWN 1982). Visco-elastic models are used in salt rock or other weak rocks and may be based on a reological model (OWEN and HINTON 1986). The salt rock was also modeled as non-Newtonian liquid (DUSSEAULT et al. 1987). Behaviour of the soil material may be analysed using a hyperbolic non-linear model describing the behaviour of soil before failure (KONDNER 1963, KONDNER and ZELASKO 1963, and DUNCAN and CHANG 1970). Use of more sophisticated constitutive models in rock mechanics may be limited by a difficulty in obtaining necessary parameters.

A combination of a deterministic model with observed deformations may be used either in forward or back analysis. In forward analysis, the physical parameters are used as input data, while expected displacements, strains, and stresses are calculated. The displacements are calculated using the equilibrium equation:

$$\mathbf{K}\delta = \mathbf{r} - \mathbf{f}^{b} - \mathbf{f}^{\sigma o} - \mathbf{f}^{\varepsilon o} \quad (1)$$

where $\delta = (u1\ v_1\ w_1\ u_2\ v_2\ ...\ u_n\ v_n\ w_n)^T$ is the vector of displacements, **K** is the global stiffness matrix of the material, $\mathbf{f}^b$ is the loading vector of body forces, $\mathbf{f}^{\sigma o}$ is the loading vector from initial stresses, $\mathbf{f}^{\varepsilon o}$ is the loading vector from initial strains, and **r** is the vector of external forces.

In back analysis, measured displacements are taken as input data and they are used to calculate unknown parameters such as material properties, loads, and initial stresses. Recent developments in automatic geodetic data collection and remote sensing monitoring techniques allow for collecting of large data sets of high precision for the back analysis of deformations.

Back analysis can be divided into: 1) inverse analysis (with assumptions that the investigated material is uniform and that the initial stress field is uniform or linear); 2) optimization method. The concept of back analysis was presented by Sakurai and Takeuchi (1983); Gioda and Sakurai (1987). In most cases of back analysis, the modeled material is considered as elastic (Jing 2003). For instance, back-analysis method used in modeling of rock behavior around tunnels was presented by Zihifa at al. (2000). The only example of back analysis for large scale problem modeling was given by Chi et al. (2001). However, the analysis was based on an empirical formula, which was developed for ground movement due to tunneling. Optimization methods may be used in cases of modeling complex initial stresses and multiple strata types (Deng and Lee 2001). Gens et al. (1996) and Ledesma et al. (1996) used probabilistic formulation to estimate parameters from field instrumentation.

2.2. Finite Element Method

The basic concept of the finite element method (displacement approach) is that the continuum of the deformable body is replaced by an assemblage of individual small elements of finite dimensions, which are connected together only at the nodal points of the elements (Zienkiewicz and Taylor 1989). The elements may be of any shape, but usually eight nodal brick elements, three nodal elements and four nodal elements are chosen for a three- and two-dimensional analyses, respectively. For each element, one can establish the relationship between the nodal loads and displacement or strain field variables.

The global matrices and vectors in the equilibrium equation (1) are calculated through a superimposition of local (at each element or at each node of the FEM mesh) matrices $\mathbf{K}_e$ and vectors $\mathbf{f}_e^{\,b}$, $\mathbf{f}_e^{\,\sigma o}$, and $\mathbf{f}_e^{\,\varepsilon o}$. The local stiffness matrix and local loading vectors in individual elements are calculated from:

$$\mathbf{K}_e \int_\Omega \mathbf{B}_c^T \mathbf{D} \mathbf{B}_c dx\, dy\, dz \qquad (2)$$

$$\mathbf{K}_e \int_{\Omega} \mathbf{B}_c^T \mathbf{D} \mathbf{B}_c dx\, dy\, dz \tag{3}$$

where $\mathbf{B}_e$ is the matrix relating strains in the element to its nodal displacements, $\mathbf{N_e}$ is the shape function, **D** is the constitutive matrix of the material, which in case of the linear elastic analysis, contains the elasticity parameters such as Young modulus E and Poisson Ratio ν, and $\mathbf{b} = (b_x\, b_y\, b_z)^T$ is the vector of body force.

The transformation of the global stiffness matrix may be expressed as:

$$K = \sum_{e=1}^{ne} T_e^T K_e T_e \tag{4}$$

The functions in equations (2) and (3) are given in x, y, z global coordinates. By introducing local coordinates ξ, η, and ζ in an element, one can write:

$$\iiint F(x, y, z) dx\, dy\, dz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} G(\xi,\eta,\zeta) \det[J]\, d\xi\, d\eta\, d\zeta \tag{5}$$

where det[J] is the determinant of Jacobian matrix.

Using the Gaussian quadrature rule (ZIENKIEWICZ and TYLOR 1989), one can write:

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} G(\xi,\eta,\zeta) \det[J]\, d\xi\, d\eta\, d\zeta = \sum_{m=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n} W_i W_j W_m \det[J] G(\xi_i,\eta_j,\zeta_m) \tag{6}$$

where W are weights.

In order to evaluate matrix **G**, a transformation of the coordinates is necessary. The coordinate transformation is established using standard shape functions N given in terms of local coordinates.

Since the stiffness matrix **K** is singular, boundary conditions must be applied in order to solve equation (1) for the displacements. Assume that the displacement boundary conditions are given by

$$H^T \delta = d \tag{7}$$

where **d** is a vector of known displacements and **H** is a location matrix. Then, the solution of equation (1) is given by

$$\delta = \left(K + sHH^T\right)^{-1}\left(f + sHd\right) \tag{8}$$

where s is a scale factor which should be chosen to be sufficiently large according to the computer capability.

Most of the numerical codes for rock mechanics are suitable for small-scale studies (JING 2003). Commercial codes for large-scale studies in structural engineering may lack some important capabilities for the determination of the quality of calculated quantities. Among many commercial finite element programs one can list COMSOL Multiphysics™ and GEO-SLOPE (KRAHN 2004), which are used by the author in solving some forward problems.

The author developed FEMMA software (Finite Element Method for Multiple Applications) to investigate large scale problems in rock mechanics (SZOSTAK-CHRZANOWSKI et al. 1991b, 1994).

2.3. Accuracy of Deterministic Modeling

In order to perform a meaningful combined analysis of deformations, the accuracy of both geometrical and deterministic models must be known. This requirement applies, for instance, when different hypotheses regarding the deformation mechanism are put forward, and one would like to identify the most appropriate hypothesis on the basis of a comparison of observed deformations with deterministic values (CHRZANOWSKI et al. 1994). While the accuracy of geometrical models (CHRZANOWSKI et al. 1986) may be easily determined from the knowledge of the errors of the observed deformations, the accuracy determination of the finite element model poses some problems that have not yet been fully solved.

In both forward and back analyses, input data is affected by errors of measurements and estimation. The accuracy of FEM analyses depends mainly on: (1) discretization error, (2) computer round-off errors, (3) matrix ill-conditioning, and (4) errors in the physical parameters of the material and input boundary conditions (observed loads and/or deformation). These effects never allow for an exact solution to be obtained. Therefore, users of the finite element method must always be aware of the accuracy limitations and should always perform a quantitative analysis of the errors of the FEM results before using them for any further analysis and/or interpretation.

The influence of the first three factors has been extensively investigated and discussed in the literature by many authors (e.g., ZIENKIEWICZ and TAYLOR 1989). The influence of errors in the physical parameters of the material and errors in the boundary conditions, though generally recognized, particularly in rock and soil mechanics, as the main sources of errors in FEM analyses, has lacked a rigorous treatment. These errors may be classified into two groups, (a) systematic biases from the true values and (b) random uncertainties.

The influence of a systematic bias, for example a difference between the in-situ and laboratory values of Young modulus, E, can be introduced into the error analysis in a *trial and error* mode by performing the FEM analysis separately for each value of Young modulus and checking whether the difference in the output results is significant. Propagation of random errors, however, is much more complex and requires a rigorous propagation of variances and covariances of the parameters, which are treated as random variables. This approach is particularly important in any rock mechanics problems where mechanical properties of the same type of rock may be significantly changing from one location to another due to inhomogenities and discontinuities in the rock material. In this case, even if in-situ determination of the properties is performed, it can be done only at a few discrete points while in other places the values may differ randomly within a certain, statistically determined, confidence (±) interval. The concept of variances-covariances propagation in FEM was introduced by Szostak-Chrzanowski et al. (1994). The effect of input data errors for the failure envelope was investigated by Zambrano-Mendosa et al. (2003).

3. Developments in deformation analysis

3.1. Areas of Author's Research and Development

The Author's research has concentrated on:

1 – optimal use of deterministic modeling in predicting structural and ground deformations in engineering, mining, and geoscience projects;

2 – optimal combination (integration) of deterministic and geometrical models for the purpose of identifying the mechanism of deformation and explaining causes of the observed deformations;

3 – optimal combination of deterministic models with observed deformations for the verification of material properties of the deformable body at the construction and post-construction (operation) stages;

4 – propagation of variances-covariances in FEM;

5 – modeling of gravity changes.

The presented research has concentrated on the analysis of large-scale problems using finite element analysis. The author developed a concept of equivalent (averaged) medium which may be divided into a few interacting blocks. The concept is used in forward and back analyses for studying the behavior of the rock mass. The material model for the equivalent medium has been developed on the basis of the determination of the deformation modulus as a function of time.

Since the properties of the in-situ rock may significantly differ from the results of the laboratory tests, methods for calibrating mechanical properties of rock or soil mass have been developed using a calibrating function, which is based on the distribution of compressive and tensional stresses. In addition, the author developed a methodology for the identification of material parameters of both brittle and salt (viscous) rock with application to mining problems and for soil structures with application to embankment dams. Modeling of brittle rock deformation is based on a non-tensional model of the behavior, while modeling of salt rock deformation is based on the assumption that the salt rock behaves as non-Newtonian liquid.

The author, in her research has concentrated on the development of a methodology to model in-situ mechanical parameters for rock and for saturated soils. The scale ratio of laboratory Young modulus to in-situ Young modulus for intact rock is based on stress distribution in the rock mass. The investigated rock mass is divided into selected blocks: rock in the vicinity of the opening with some joints; rock with many joints; or rock as a total rock mass (SZOSTAK-CHRZANOWSKI and CHRZANOWSKI 1991a). On the basis of stress distribution, the author also derived the scale ratio of a laboratory value of tensional strength to the in-situ values (SZOSTAK-CHRZANOWSKI and CHRZANOWSKI 1991a). The assumptions regarding the behavior model of rock mass is used as a basis for the identification of boundaries of blocks within the rock mass with assigned to each block scaled parameters (CHRZANOWSKI et al. 2000).

3.2. Use of integrated analysis and separability concept in identification of deformation mechanism

In some cases of unexpected deformation of man-made or natural structures the causative factors may not be known. For example, the mechanism and causative factors of tectonic movements are usually not well known, and they are subjected to various hypotheses. The main purpose of the concept of integrated analysis developed by the author is to identify, on the basis of a comparison (or a simultaneous analysis) of geometrical and deterministic models, which of the postulated mechanisms can be optimally explained by the analysis. Depending on the design and accuracy of the monitoring surveys, the separation (discrimination) among various postulated mechanisms may not be possible (CHEN and CHRZANOWSKI 1994). For instance, if only geodetic levelling is used as the main tool to describe the surface effects of tectonic movements then one may be not able to deduce whether the observed vertical movements are caused by a thermal expansion of the earth's crust or by a subducting tectonic plate or by both. In order to distinguish between these mechanisms, measurements of relative horizontal movements or gravity changes could help in the identification of the best model (SZOSTAK-CHRZANOWSKI et al. 1993b).

In order to identify the best model, the author has combined the integrated analysis with the concept of separability (CHEN et al. 1994). The concept and methodology require a priori knowledge of the expected deformation pattern. This can be obtained from deterministic modeling of deformations using, for example, the FEM analysis with assumed material parameters and boundary conditions. The computed displacements or their derived quantities, e.g., strains, are then compared with the measured ones. The correlation between them is calculated, and the most appropriate deformation mechanism is identified. On the other hand, based on deterministic patterns of expected deformations for various hypotheses of deformation mechanisms, a monitoring scheme can be optimally designed using the criterion of separability (CHEN et al. 1994, CHRZANOWSKI et al. 1994).

When m postulated models are considered then the separability criteria are:

$$\lambda_{\min}\left(M_{ij}\right) \geq \frac{\sigma_0^2 \delta_0}{b_i^2} \quad \text{for all} \quad i, j (i \neq j) \tag{9}$$

with

$$\mathbf{M}_{ij} = \mathbf{B}_i^T \mathbf{P_d} \mathbf{B}_i - \mathbf{B}_i^T \mathbf{P_d} \mathbf{B}_j \left(\mathbf{B}_j^T \mathbf{P_d} \mathbf{B}_j\right)^{-1} \mathbf{B}_j^T \mathbf{P_d} \mathbf{B}_i \tag{10}$$

where $\mathbf{P_d}$ is the weight matrix (which is a function of the configuration and survey errors of the monitoring scheme) of the displacement vector, ${\sigma_0}^2$ is the variance factor, δ_0 is the boundary value of the noncentrality parameter, b_i ($i = 1, 2, ..., m$) is the smallest deformation to be correctly detected, and $\mathbf{B}_i$ is the deformation matrix, which is formulated from the displacements calculated using FEM. If the accuracy of monitoring surveys is inadequate to achieve a separation between two or more models than the surveys must be redesigned to change $\mathbf{P_d}$ (i.e. change the type, configuration, and accuracy of the surveys) in a such way that a better separation between different deformation models could be achieved.

The author adapted and implemented the concepts of integrated analysis and separability to studies of earth crustal movements (SZOSTAK-CHRZANOWSKI et al. 1993b, CHRZANOWSKI et al. 1994) including modeling of tectonic movements in Western Canada (SZOSTAK-CHRZANOWSKI et al. 1996).

3.3. Software FEMMA

In order to implement developments in the back analysis, the separability analysis, and the error analysis the author developed FEMMA software. The software has been developed in two main versions, FEMMA 2.0 and FEMMA 3.0, for two- and three-dimensional: forward and back deformation analysis; thermal analysis: steady state heat-transfer; and error propagation analysis. Modeling of gravity changes is accomplished in an additional version of software. All versions of FEMMA are supported by the automatic mesh generation software MESHGEN 2.0 and 3.0.

Special characteristics of FEMMA are:

1 – optional use of either two-nodal (bars), or three-nodal (triangular), or four-nodal elements in the two-dimensional analyses and use of eight nodal ('bricks') elements in three-dimensional problems;

2 – modelling of discontinuities using either the split node technique (MELOSH and RAEFSKY 1981) or anisotropic elements;

3 – use of no-tension criteria and Hoek-Brown criteria (HOEK and BROWN 1982) in rock deformation studies.

The modular structure of the software allows for an easy introduction of additional changes and adaptations for various applications.

3.4. Propagations of Errors in FEM

3.4.1. Propagation of random errors in forward analysis

Results of any measurements (treated as random variables) or quantities derived from random variables are meaningless unless they are accompanied by information on their accuracy. Terms such as variance, covariance, standard deviation, confidence level, and probability, are commonly used in accuracy analysis (e.g., MIKHAIL 1976).

If a set of n unknowns u_i (vector $\mathbf{u}$) is computed from a set of k known variables z_i (vector $\mathbf{z}$) that have known variances and covariances (matrix $\mathbf{C_z}$), then according to the general formula, known as the propagation of variance--covariance matrices, one can calculate the variance-covariance matrix $\mathbf{C_u}$ of the unknowns from

$$\mathbf{C_u} = \mathbf{A}\ \mathbf{C_z}\ \mathbf{A}^{\mathbf{T}} \tag{11}$$

where $\mathbf{A}$ is the design matrix containing partial derivatives of the functions $u_i = u_i(z_1, z_2, \ldots, z_k)$, where $i = 1, 2, \ldots, n$ at the approximate points of variables.

In forward analysis the Young modulus E, Poisson Ratio ν, initial stress $\boldsymbol{\sigma}_o$, initial strain ε_o, and body forces b are typical input data. If one treats the variables E, ν, $\boldsymbol{\sigma}_o$,$\boldsymbol{\varepsilon}_o$, $\mathbf{r}$, and body forces b in each element as random variables with a known variance covariance matrix $\mathbf{C_z}$, then, using the general rule (equ.11) of the error propagation, one can calculate the variance-covariance matrix $\mathbf{C}_\delta$ of the nodal displacements. Where

$$A = \left(\frac{\partial \delta}{\partial E}, \frac{\partial \delta}{\partial \nu} \cdots \frac{\partial \delta}{\partial b_y} \cdots \right) \tag{12}$$

with

$$\frac{\partial \delta}{\partial E} = -\left(K + sHH^T\right)^{-1} - \delta \sum_{e=1}^{ne} T_e^T T_e \iint B_e^T \frac{\partial D}{\partial E} B_e t\, dx\, dy \tag{13}$$

$$\frac{\partial \delta}{\partial \nu} = -\left(K + sHH^T\right)^{-1} - \delta \sum_{e=1}^{ne} T_e^T T_e \iint B_e^T \frac{\partial D}{\partial \nu} B_e t\, dx\, d \tag{14}$$

$$\frac{\partial \delta}{\partial b_x} = \left(K + sHH^T\right)^{-1} \sum_{e=1}^{ne} T_e^T \iint N_e^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} t\, dx\, dy \tag{15}$$

In case of a plane strain, two-dimensional analysis

$$\frac{\partial D}{\partial E} = \frac{1}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{pmatrix} \tag{16}$$

$$\frac{\partial D}{\partial \nu} = \frac{E(1+4\nu)}{(1+\nu)(1-2\nu)^2} \begin{pmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{pmatrix} + \frac{E}{(1+\nu)(1-2\nu)} \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{17}$$

More details on the developed methodology are given in SZOSTAK-CHRZANOWSKI et al. (1993a).

3.4.2. Propagation of random errors in back analysis

The author developed a methodology for modeling large-scale problems using back analysis. The principal equation of back analysis is based on equation (1) of equilibrium. Let $\mathbf{d}_1$* be the vector of measured displacements and $\mathbf{d}_2$ is a vector of the remaining (unknown) nodal displacements. Equation (1) is rewritten as

$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} d_1^* \\ d_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \tag{18}$$

Eliminating $\mathbf{d}_2$ results in

$$\left(\mathbf{K}_{11} - \mathbf{K}_{12}\mathbf{K}_{22}^{-1}\mathbf{K}_{21}\right)\mathbf{d}_1^* = \mathbf{f}_1 - \mathbf{K}_{12}\mathbf{K}_{22}^{-1} \tag{19}$$

which is rewritten as

$$\mathbf{K}_{11}^*\mathbf{d}_1^* = \mathbf{f}_1^* \tag{20}$$

Equation (20) represents the basic equation governing the back analysis problem.

If there are u unknown parameters to be estimated in the left-hand side of the equation or in matrix $\mathbf{K}_{11}^*$, and m observed displacements, where $m > u$, then the unknown parameters can be solved by using the least-squares technique.

Let u unknowns in matrix $\mathbf{K}_{11}^*$ be denoted by $\mathbf{p}$, and let $\mathbf{p}^0$ be their approximate values, then the least-squares solution for $\mathbf{p}$ is performed by minimizing the quadratic form

$$\left[\mathbf{d}_1^* - \mathbf{d}(\mathbf{p})\right]\mathbf{C}^{-1}\left[\mathbf{d}_1^* - \mathbf{d}(\mathbf{p})\right] \tag{21}$$

where $\mathbf{C}$ is the variance covariance matrix for the observed displacements, and

$$\mathbf{d}(\mathbf{p}) = \left[\mathbf{K}_{11}^*(\mathbf{p})\right]^{-1}\mathbf{f}_1^* = \min \tag{22}$$

Since $\mathbf{K}_{11}^*$ is a non-linear function of the unknown parameters $\mathbf{p}$, linearization with respect to $\mathbf{p}^{(0)}$ is made. The corrections to the approximate values $\mathbf{p}^{(0)}$ are then estimated from

$$\Delta\mathbf{p} = \left[\mathbf{A}^T\mathbf{C}^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^T\mathbf{C}^{-1}\left[\mathbf{d}_1^* - \mathbf{d}(\mathbf{p}^0)\right] \tag{23}$$

where **A** is the partial derivatives of **d**(**p**) with respect to **p** and is evaluated at the values of $\mathbf{p}^0$. Since this is a nonlinear problem, an iteration process is needed.

Finally, the variance covariance matrix of the estimated parameters is calculated from:

$$\mathbf{C_p} = \left[\mathbf{A}^T \mathbf{C}^{-1} \mathbf{A}\right]^{-1} \tag{24}$$

The same back analysis procedure holds for unknown parameters in the right-hand side of equation (20). Examples of the error analysis in back analysis are given in SZOSTAK-CHRZANOWSKI et al. (1994).

3.5. Modeling of ground subsidence in mining areas as a large-scale problem

3.5.1. Earlier methods of mining subsidence modeling

Modeling of ground subsidence in mining areas may be performed using either geometrical (empirical) theories or deterministic analysis. Geometrical theories for predicting the ground subsidence have been developed in central Europe and the United Kingdom. The first theories were developed by KEINHORST (1925), followed by BALS (1932). Within that group of theories, KNOTHE'S (1957) 'influence function' theory, developed in Poland, has gained the most popularity and has been used (sometimes with modifications) till now in many countries, including adaptations in the U.S.A. (e.g., LUO and PENG 1993), and in P.R. China.

In geometrical models, some parameters (coefficients) of the functions must be determined (calibrated) empirically through a comparison with the observed subsidence. Since other parameters, such as mechanical properties of the rock and tectonic stresses, are not taken into account, the prediction theories are applicable only to the areas where the mining, geological, and tectonic conditions are the same or very similar to the area where the empirical data for the theory had been collected. Since the conditions in different areas are never the same, any attempt to adapt, for instance, the European geometrical methods for ground subsidence prediction on the North American continent requires the calibration of the model parameters through many years of comparisons with the observed deformations in the new area. Usually, this approach is unrealistic because very few mines in North America have a well organized and systematic program of monitoring surveys. It also should be stressed that the geometrical theories are generally not reliable in cases of complicated geometry of mined deposits, in the presence of faulting, and in areas of previous extensive mining operations.

Most of the 150 or so currently active underground mines in Canada are in hard rocks and are located in sparsely populated areas. Therefore, except for a few coal and potash mines, the effects of mining on the surface infrastructure have never been a major concern. The leading geomechanical problem in conditions of Canadian mining has been to understand damage and failure around mining openings (small scale problems) excavated in brittle rock mass under high in situ stresses (CASTRO 1997). Most of the research and analyses of the rock strata behavior around mine openings in mining areas were based on small-scale problems using numerical modeling (UDD and YU 1993, CORKUM et al. 1991, RIZKALLA and MITRI 1991, SURIYACHAT and MITRI 1991).

Over the past 30 years, most Canadian mines have started recognizing the importance of improving their monitoring techniques and methods of modeling and predicting of rock strata deformation not only for safety and environmental protection purposes but first of all for a better understanding of the mechanism of rock strata deformation leading to the development of more economical and safer mining methods.

The first attempts to use numerical FEM in ground subsidence were made in 1975 (VONGPAISAL and COATES 1975) and applied to the Falconbridge Sudbury mining operation. FEM code was developed to model stress distribution in potash mines (FOSSUM 1985). At the same time, the author initiated research on the development of a deterministic model of deformations and stress distribution in rock mass also using FEM (Szostak-CHRZANOWSKI 1988). In 1989, the author presented a method (known as S-C method) to model and predict ground subsidence in brittle rock (SZOSTAK-CHRZANOWSKI 1989)

3.5.2. Development of methodology for subsidence modeling based on large scale problem modeling

The author developed a methodology (known as S-C method), based on large-scale problem modeling, for brittle rock deformation. The method is based on the assumption that the brittle rock behaves as non-tensional material (SZOSTAK-CHRZANOWSKI and CHRZANOWSKI 1991b). The S-C method was successfully implemented in modeling and predicting of ground subsidence in several coal, copper, lead and zinc mines (SZOSTAK-CHRZANOWSKI 1988, SZOSTAK-CHRZANOWSKI and CHRZANOWSKI 1991b) including modelling of seafloor subsidence over offshore coal mining in Nova Scotia (CHRZANOWSKI et al. 1998); and modelling of ground subsidence and identification of a fault over a steeply inclined coal seam near Sparwood, B.C. (SZOSTAK-CHRZANOWSKI and CHRZANOWSKI 1991b).

The S-C method was expanded to model deformations in salt rocks. The application to the prediction of ground subsidence in potash and salt mining at two mines in New Brunswick is described in CHRZANOWSKI and SZOSTAK-CHRZANOWSKI (1997) and CHRZANOWSKI and SZOSTAK-CHRZANOWSKI (2004). The method is supported by software FEMMA.

In the the author's deterministic analysis of subsidence of salt and potash rock mass, the rock material is considered as a non-Newtonian liquid with high and not constant viscosity (Dusseault et al. 1987). The initial stress in the intact salt rock was assumed to be isotropic lithostatic. Development of shearing stresses due to mining activity causes the flow of the salt mass into the excavated areas in order to achieve a new equilibrium state of stresses. Shearing stresses are developed due to de-stressing of the salt rock over a mining opening and due to an increase in compressive stresses at the sides of the mining cavity. The 'flow-in' zone is identified within the zone delineated by maximum shearing stresses. In modeling the final maximum subsidence at the top of the salt formation, the value of E in the 'flow' zone is scaled to give the same volume of the subsidence basin (under the cap rock) as is the volume of mining openings. Once the equivalent subsidence trough at the top of the salt is determined, the response of the brittle rock is modeled (Szostak-Chrzanowski and Chrzanowski 1991a).

3.5.3. Modeling of subsidence due to withdrawal of liquids

A change in underground fluid (water or oil) conditions may cause surface subsidence. The change of water condition may be due to pumping to the surface or due to an inflow into the mining openings. Vertical flow of ground water, horizontal flow, and inelastic compaction of the material of aquifer systems are important parameters.

The aquifer system from which the water flows may be composed of layers of coarse or fine-grained material (sediments, sand, porous sandstone). The compaction of the material may be elastic or inelastic and is characterized by an elastic or inelastic storage coefficient of the compaction. The depth of the aquifer also has an influence on subsidence. The total stress in the aquifer is caused by the weight of the overlying rock and water. The effective stress is the resulting stress of total stress and acting upward stress caused by water pressure. In confined aquifers, there can be large changes in pressure with little change of thickness of the saturated water column. Therefore, the total stress may remain practically constant, but the change in pressure will result in change in effective stress. The aquifer may consolidate or compact due to increased stress. For example, a confined aquifer with an initial thickness of 45 m consolidates 0.20 m when the head is lowered by 25 m (Fetter 1942). The deformation may be calculated using the theory of consolidation of one dimension. The subsidence model derived by Bravo et al. (1991) uses the principle of relationship of elastic compaction of soil and ground-water piezometric head.

The author incorporated the principles of compaction analysis to integrated analysis of subsidence due to water inflow into the aforementioned PCA mine (Szostak-Chrzanowski et al. 2005). Currently, the author is developing a methodology for modelling ground subsidence in oil fields (Szostak-Chrzanowski et al. 2006b).

4. Numerical modeling of gravity changes

Relocation of rock masses and/or change of density, and change of height due to human (e.g., mining) activities or tectonic activity may produce significant local changes to the gravity field in the vicinity of activity. The gravity changes result in local tilts of the level surface (equipotential surface of gravity) and, consequently, changes of the direction of the plumblines to which the majority of geodetic and some geotechnical measurements are referenced. Thus, if any geodetic measurements of high precision are required during the mining operation, for example, for the purpose of monitoring the stability of surface structures, they should be corrected for changes of gravity and deflection of the vertical as a function of time. Otherwise, differences in the repeated observations (angles, distances, gyroazimuths, height differences, tilts, etc.) caused by the gravity changes could be misinterpreted as deformations of the observed object. In addition, by comparing the expected (modeled) gravity changes with the values observed with precision gravimeters, one may gain information on the behaviour of the rock masses disturbed by mining activity or affected by activity of tectonic origin. Various aspects of the mining 'microgravimetry' are discussed in FAJKLEWICZ (1980).

The calculation of gravity changes involves solving complex integrals (summation of influences of point masses). The theory of gravity and various numerical methods used in the gravity calculations, are given in textbooks on geodesy, for example in VANICEK and KRAKIWSKY (1986), TORGE (1991), HEISKANEN and MORITZ (1967).

The author has developed a method for numerical modeling (using FEM) of gravity changes caused by the simultaneous effects of mass relocation and rock deformations. The method is based on dividing the investigated object into the finite number of elements and on summation of gravity due to change of density at the point caused by each of the finite elements.

Derivatives of the potential of the gravitation with respect to x, y, z, in a Cartesian coordinate system give the components of the gravity. For example, x component is given as:

$$g_x = -G\rho_1 \int_V \frac{(x' - x)}{R^3} dv \qquad (25)$$

where ρ_1 is the initial density, and R is the radius calculated from

$$R = \left[(x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{1/2} \qquad (26)$$

and x, y, z are the coordinates of mass m_1 and x', y', z' are the coordinates of mass m_2.

The gravity may change due to the changes of mass or changes of density with constant mass, due to the deformation caused by acting loads. The difference between the gravity values at the two epochs may be a result of the mass change due to, e.g., open pit mining activities or a change in the density caused by the change in the strain field.

If the gravity were calculated at two epochs, the gravity change would be a difference between the gravity values at time 1 and time 2, i.e.:

$$\Delta g = g_1 - g_2 \tag{27}$$

The new density based on an assumption of constant mass is expressed by

$$\rho_2 = \rho_1 \frac{V_1}{V_2} \tag{28}$$

where ρ_1 and ρ_2 are the densities and V_1 and V_2 are volumes before and after the deformation, respectively.

Using equation (25) and equation (28) the gravity change, dg_d, due to the change of density is calculated from

$$g_2 = -G\rho_1 \frac{V_1}{V_2} \int_{V_2} \frac{(z' - z)}{R^3} dv \tag{29}$$

The gravity change is calculated as a sum of the effects calculated for each element of the finite element model using Gaussian quadrature rule. The volume of each element is calculated from changes of the coordinates of the nodal points obtained from the stress-strain finite element analysis. The same finite element mesh is used to calculate gravity change components.

The method has been used in modeling regional deformations and gravity changes of tectonic origin (Szostak-Chrzanowski et al. 1996) and in modeling expected gravity changes in a large open pit mine, Belchatow, in Poland (Szostak-Chrzanowski et al. 1995).

5. Verification of material parameters of earth dams – integrated approach

During construction, earthen dams undergo a settlement that consists of two components: crossarm settlement and pre-crossarm settlement. The crossarm settlement is defined as a settlement at a given location and elevation due to the weight of overlying material. It reaches its maximum value approximately at the mid-elevation of the dam. The pre-crossarm settlement is the settlement at a given location and elevation due to weight of the underlying material.

At the stage of filling the reservoir, two main effects must be considered: pressure of water and effect of wetting. During the process of wetting, the values of geotechnical material parameters and the derived values of Young modulus decrease. Young modulus of the material in the submerged sections of the structure becomes smaller and buoyancy force is developed producing dam deformation. The wet parameters are smaller than dry parameters for the same soil. The rock mass on which the embankment dam is located may be assumed to behave as a linear-elastic material under the load of the weight of the dam and the weight of water in the reservoir.

The behaviour of the earth material may be determined using a hyperbolic non-linear model describing the behaviour of soil before failure developed by KONDNER (1963) and KONDNER and ZELASKO (1963). In the hyperbolic model, the non-linear stress- strain curve is a hyperbola in σ_1–σ_3 versus axial strain plane. The relationship takes the form modified by DUNCAN and CHANG (1970):

$$\begin{pmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{pmatrix} = \frac{3B}{9B-E} \begin{bmatrix} (3B+E) & (3B-E) & 0 \\ (3B-E) & (3B+E) & 0 \\ 0 & 0 & E \end{bmatrix} \begin{pmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{pmatrix} \tag{30}$$

where $\Delta\sigma$ and $\Delta\tau$ are stress increments and $\Delta\varepsilon$ and $\Delta\gamma$ are strain increments, E is Young modulus, and B is bulk modulus.

The relation of initial tangent modulus E_i and confining stress σ_3' is given by JANBU (1963):

$$E_i = KP_a \left(\frac{\sigma_3'}{P_a} \right)^n \tag{31}$$

Similarly the relation between bulk modulus B and confining stress σ_3' can be determined (DUNCAN et al. 1980):

$$B = K_b P_a \left(\frac{\sigma_3'}{P_a} \right)^m \tag{32}$$

where P_a is atmospheric pressure, K is loading modulus number, and n is exponent for loading behaviour. K_b is bulk modulus number and m is bulk modulus exponent.

The author developed a methodology for modelling and verification of geotechnical parameters of large earth dams built using different technologies. In the process of the calculation of displacements the author determined the change of geotechnical parameters and Young modulus of the zones in a dam between dry and wet conditions. Results of geodetic deformation surveys have been used in verifying design geotechnical parameters of various earth dams (SZOSTAK-CHRZANOWSKI et al. 2002, 2006a, SZOSTAK-CHRZANOWSKI and MASSIERA 2004)

The agreement of deformations obtained from FEM solution and monitoring may confirm that the geotechnical parameters and the values of Young modulus, as used in the FEM analysis are correct. This is an important conclusion for a possible use of the verified parameters in future analyses of possible effects of additional loads arising, for example, from tectonic movements. A good understanding of the deformations occurring in embankment dams allows for minimizing the effects such as transverse cracking, longitudinal fissuring, arching effect and stress concentration, hydraulic fracturing, development of plastic zones, and damages in the instrumentation.

6. Conclusions

The author's new developments in integrated analysis of deformations significantly increased the role of geodetic monitoring surveys in physical analysis of deformations. They also have enhanced the process of understanding the mechanism of deformations in civil engineering, mining, and natural deformations such as tectonic movements.

The development of a method for modeling and predicting rock deformations due to mining activity, hydrological changes, or oil withdrawal is a specially important contribution. The S-C method developed by the author for modeling of deformations of rock mass is universal and gives good results.

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