TECHNICAL SCIENCES
Abbrev.: Tech. Sc., Pap. And Rep., No 9, Y. 2006

ON THE LOCALIZATION OF THE FAILURE AT THE BOUNDARY OF A HOLE IN THE ORTHOTROPIC STRETCHED DISC

Aleksandra Wieromiej-Ostrowska
University of Warmia and Mazury in Olsztyn

Key words: elastic plane stress problem, failure initiation, orthotropy.

Abstract

The localization of failure initiation in a simple tension test applied to the infinite orthotropic disc with a circular hole is considered. The considerations are based on the Tsai and Wu model. According to the author's supposition, the angular coordinate of the failure at the hole boundary should depend on the form of the polynomial failure criterion. The computations based on accessible handbook data for the elastic and strength properties of chosen typical composite material have not confirmed the above supposition, giving evidence of surprising insensitivity of to changes in both elastic and strength characteristics of the material.

O LOKALIZACJI ZNISZCZENIA NA BRZEGU OTWORU KOŁOWEGO W ROZCIĄGANEJ TARCZY ORTOTROPOWEJ

Aleksandra Wieromiej-Ostrowska
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: płaski stan naprężenia, zniszczenie kompozytów, ortotropia.

Streszczenie

Rozważania nad zniszczeniem kompozytów oparto na kryterium Tsai i Wu. Zbadano przydatność testu prostego rozciągania dla nieskończonej tarczy z otworem kołowym. Miejsce zniszczenia na obwodzie tarczy wyznacza kąt φ_0, który mógłby zależeć od postaci funkcji wytężenia F. Analizę przeprowadzono na podstawie danych literaturowych dla wybranego typowego kompozytu zbrojonego jedną rodziną włókien. Otrzymane wyniki pozwalają wnioskować, że kąt φ_0 zależy od właściwości badanego kompozytu dla pewnej klasy materiałów.

Introduction

Tsai and Wu (CHAWLA 1987) proposed a criterion of failure for orthotropic material in a plane stress state, described by the function of the following form:

$$F(\sigma_1,\sigma_2,\sigma_6)=F_1\sigma_1+F_2\sigma_2+F_{11}\sigma_1^2+F_{22}\sigma_2^2+F_{66}\sigma_6^2+2F_{12}\sigma_1\sigma_2=1 \quad (1)$$

where:

σ_1 – tensile and compressive stress along the fiber direction,
σ_2 – tensile and compressive stress across the fiber direction,
σ_6 – shear stress for in-plane shearing.

According to Tsai and Wu, the value of F_{12} the term has to be either evaluated by means of a biaxial test (CHAWLA, KRISHAM 1987) or assumed in the following form:

$$F_{12}=F_{12}^{*}\sqrt{F_{11}F_{22}} \quad (2)$$

where the unknown F_{12}^{*} term satisfies the inequality $-0.5\le F_{12}^{*}\le 0$.

Biaxial tests are quite inconvenient experimentally, while the relation (3) has no convincing theoretical foundations.

In the present paper a simple tension test for the orthotropic infinite disc with a circular hole is proposed. The place of the failure at the boundary of the hole can be described by its angular co-ordinate. As mentioned above, this angle may depend on the failure surface in stress space. Numerical data for a chosen composite (epoxy resin + kevlar fiber, reinforced with one family of fibers) were based on the relation and experimental results given in ŚLEZIONA (1998).

Basic relations

Let us consider an orthotropic infinite elastic disc with a circular hole, uniaxially stretched with stress σ applied along the fiber direction, see Fig. 1. Such a case was solved by LECHNICKI (1957) the circumferential stress at the boundary of the hole can be, expressed as follows:

$$\sigma_{\varphi\varphi}=\sigma\frac{E_\varphi}{E_{11}}\left[-\gamma_1\gamma_2\cos^2\varphi+(1+\gamma_1+\gamma_2)\sin^2\varphi\right] \quad (3)$$

where

γ_1 and γ_2 are dimensionless elastic constants defined by the following relations[1],

[1] Notation according to Blinowski and Ostrowska-Maciejewska (1995).

$$\gamma_1^2 \gamma_2^2 = \frac{E_{11}}{E_{22}} \tag{4}$$

$$\gamma_1^2 + \gamma_2^2 = 2\left(\frac{E_{11}}{2G_{12}} - \nu_{12} \right) \tag{5}$$

where:

E_{11}, E_{22} – Young moduli of the composite along and across the fibers,
E_{φ} – Young modulus in the tangent direction at the boundary of the hole,
G_{12} – shear modulus for shearing parallel to the fibers,
ν_{12} – corresponding Poisson ratio.
(For details of the calculations see Appendix I).

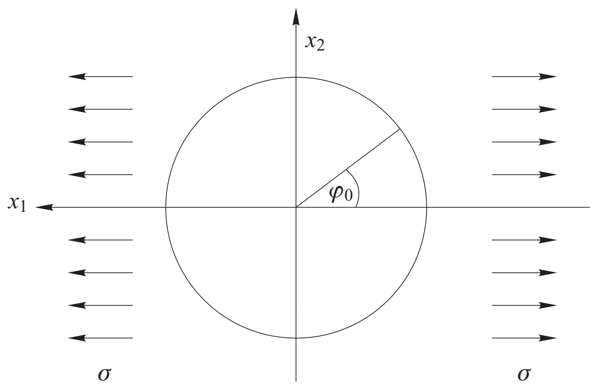

Fig. 1. A disc with a circular hole stretched along fibers

The ratio $\dfrac{E_{\varphi}}{E_{11}}$ can be found from relation (Lechnicki 1957):

$$\frac{E_{11}}{E_{\varphi}} = \sin^4 \varphi + \left(\gamma_1^2 + \gamma_2^2\right)\sin^2 \varphi \cos^2 \varphi + \gamma_1^2 \gamma_2^2 \cos^4 \varphi \tag{5a}$$

where

φ – an angle co-ordinate calculated from the direction of fibers.

Substituting relation (5a) into (3) we obtain:

$$\sigma_{\varphi\varphi} = \frac{\sigma\left[- \gamma_1 \gamma_2 \cos^2 \varphi + (1 + \gamma_1 + \gamma_2)\sin^2 \varphi\right]}{\sin^4 \varphi + \left(\gamma_1^2 + \gamma_2^2\right)\sin^2 \varphi \cos^2 \varphi + \gamma_1^2 \gamma_2^2 \cos^4 \varphi} \tag{6}$$

Using a well-known transformation formula:

$$\begin{aligned}
\sigma_1 &= \sigma_{rr}\cos^2\varphi + 2\sigma_{r\varphi}\sin\varphi\cos\varphi + \sigma_{\varphi\varphi}\sin^2\varphi \\
\sigma_2 &= \sigma_{rr}\sin^2\varphi - 2\sigma_{r\varphi}\sin\varphi\cos\varphi + \sigma_{\varphi\varphi}\cos^2\varphi \\
\sigma_6 &= \left(\sigma_{\varphi\varphi} - \sigma_{rr}\right)\sin\varphi\cos\varphi + \sigma_{r\varphi}\cos 2\varphi
\end{aligned} \tag{7}$$

and taking into account that radial σ_{rr} and shear stresses $\sigma_{r\varphi}$ should vanish at the free boundary:

$$\left.\begin{aligned}\sigma_{rr} &= 0\\ \sigma_{r\varphi} &= 0\end{aligned}\right\} \tag{8}$$

the following expressions are obtained for stress field components in the Cartesian co-ordinate system oriented along the axes of orthotropy:

$$\begin{aligned}
\sigma_1 &= \frac{\sigma\left[-\gamma_1\gamma_2\cos^2\varphi\sin^2\varphi + (1+\gamma_1+\gamma_2)\sin^4\varphi\right]}{\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi} \\
\sigma_2 &= \frac{\sigma\left[-\gamma_1\gamma_2\cos^4\varphi + (1+\gamma_1+\gamma_2)\sin^2\varphi\cos^2\varphi\right]}{\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi} \\
\sigma_6 &= \frac{\sigma\left[-\gamma_1\gamma_2\cos^3\varphi\sin\varphi + (1+\gamma_1+\gamma_2)\sin^3\varphi\cos\varphi\right]}{\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi}
\end{aligned} \tag{9}$$

Substituting expressions (9) into (1), we obtain the following (rather awkward) relation:

$$F\left[\sigma_1(\sigma,\varphi),\sigma_2(\sigma,\varphi),\sigma_6(\sigma,\varphi)\right] = \Phi(\sigma,\varphi) = \frac{\sigma^2}{\left[\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi\right]^2} \times$$
$$\times\left\{\begin{aligned}
&F_1\left[-\gamma_1\gamma_2\cos^2\varphi\sin^2\varphi + (1+\gamma_1+\gamma_2)\sin^4\varphi\right]\left[\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi\right] \\
&F_2\left[-\gamma_1\gamma_2\cos^4\varphi + (1+\gamma_1+\gamma_2)\sin^2\varphi\cos^2\varphi\right]\left[\sin^4\varphi + \left(\gamma_1^2+\gamma_2^2\right)\sin^2\varphi\cos^2\varphi + \gamma_1^2\gamma_2^2\cos^4\varphi\right] + \\
&F_{11}\left[-\gamma_1\gamma_2\cos^2\varphi\sin^2\varphi + (1+\gamma_1+\gamma_2)\sin^4\varphi\right]^2 + \\
&F_{22}\left[-\gamma_1\gamma_2\cos^4\varphi + (1+\gamma_1+\gamma_2)\sin^2\varphi\cos^2\varphi\right]^2 + \\
&F_{66}\left[-\gamma_1\gamma_2\cos^3\varphi\sin\varphi + (1+\gamma_1+\gamma_2)\sin^3\varphi\cos\varphi\right]^2 + \\
&F_{12}\left[-\gamma_1\gamma_2\cos^2\varphi\sin^2\varphi + (1+\gamma_1+\gamma_2)\sin^4\varphi\right]\left[-\gamma_1\gamma_2\cos^4\varphi + (1+\gamma_1+\gamma_2)\sin^2\varphi\cos^2\varphi\right]
\end{aligned}\right\} \tag{10}$$

Preliminary results and conclusions

The numerical value of the function $\Phi(\sigma,\varphi)$ at the boundary of a hole depends on both angle φ and the value of σ. We look for such a value of $\sigma = \sigma_0$, for which at some angle φ_0 the limiting criterion $\Phi(\sigma,\varphi)=1$ is attained. This point can be obtained as an intersection of the curves $\Phi(\sigma,\varphi)=1$ and $\frac{\partial\Phi(\sigma,\varphi)}{\partial\varphi}=0$. (In Cartesian co-ordinates $\{\sigma, \varphi\}$ at the minimum point the vector gradient is normal to the φ – axis).

Plots of the functions $\Phi(\sigma,\varphi)=1$ and $\frac{\partial\Phi(\sigma,\varphi)}{\partial\varphi}=0$ for different values of the term F_{12} are shown in Fig. 2. (values of F_i and F_{ij} were fixed).

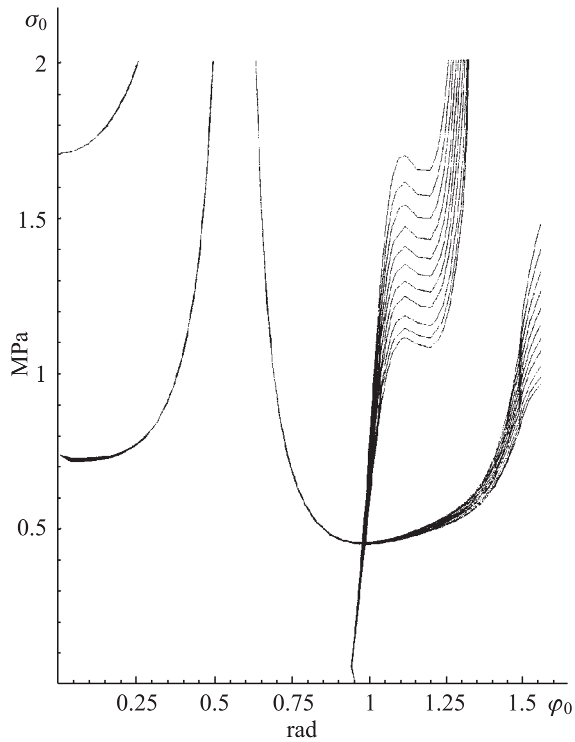

Fig. 2. Plots of the functions $\Phi(\sigma,\varphi)=1$ and $\frac{\partial\Phi(\sigma,\varphi)}{\partial\varphi}=0$ for different values of the F_{12} term. Values of F_i and the rest of F_{ij} were fixed

The values taken for the F_1, F_2, F_{11}, F_{22}, F_{66} terms were based on relations (CHAWLA, KRISHAM 1987)

$$F_1 = \frac{1}{X_1^T} - \frac{1}{X_1^C}, \; F_2 = \frac{1}{X_2^T} - \frac{1}{X_2^C},$$
$$F_{11} = \frac{1}{X_1^T X_1^C}, \; F_{22} = \frac{1}{X_2^T X_2^C}, \; F_{66} = \frac{1}{S^2} \tag{11}$$

where:

$X_1^T, X_1^C, X_2^T, X_2^C$ – longitudinal and transverse tensile and compressive strength,

S – shear strength,

According to ŚLEZIONA (1998), the following values were assumed:

$X_1^T = 1258$ MPa, $X_1^C = 205$ MPa, $X_2^T = 19$ MPa, $X_2^C = 73$ MPa, $S = 53$ MPa.

For the material under considerations (epoxy resin + kevlar fiber), expression (2) can be substituted the term F_{12}. Finally, the following data were obtained.

$F_1 = -0.004$, $F_2 = 0.038$, $F_{11} = 0.0000038$, $F_{22} = 0.00072$, $F_{66} = 0.00035$, $F_{12} = -0.000026$ (for $F_{12}^* = -0{,}5$). For the values γ_1 and γ_2 we obtain: $\gamma_1 = 0.59$ and $\gamma_2 = 4.01$, (see Appendix I).

The results of numerical experiments are presented in Tables 1–4. The values of one parameter only were changed in every Table, while the remaining ones stayed constant. Changes of σ_0 and φ_0 versus F_{ij} are shown in Figs. 3–6 (Appendix II).

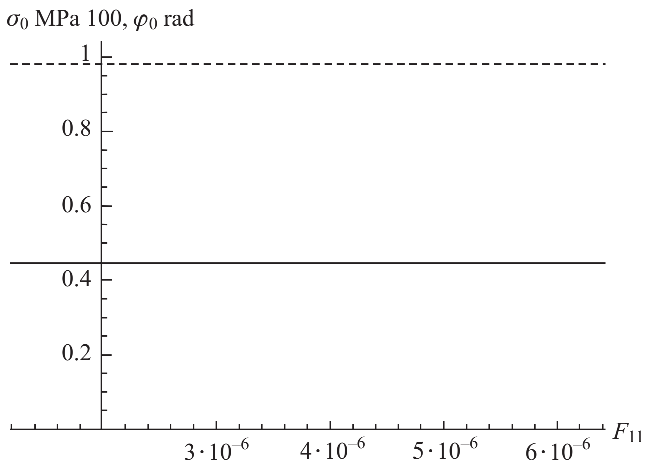

Fig. 3. Relation between φ_0 (dashed curve) and σ_0 (solid curve) and the value of F_{11} the term (compare Table 1)

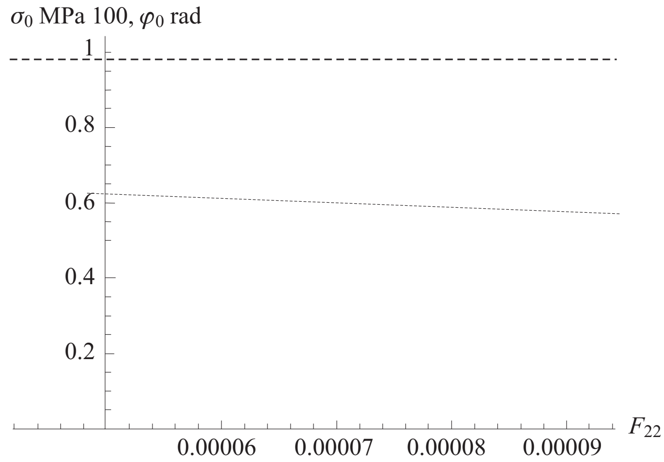

Fig. 4. Relation between φ_0 (dashed curve) and σ_0 (solid curve) and the value of F_{22} the term (compare Table 2)

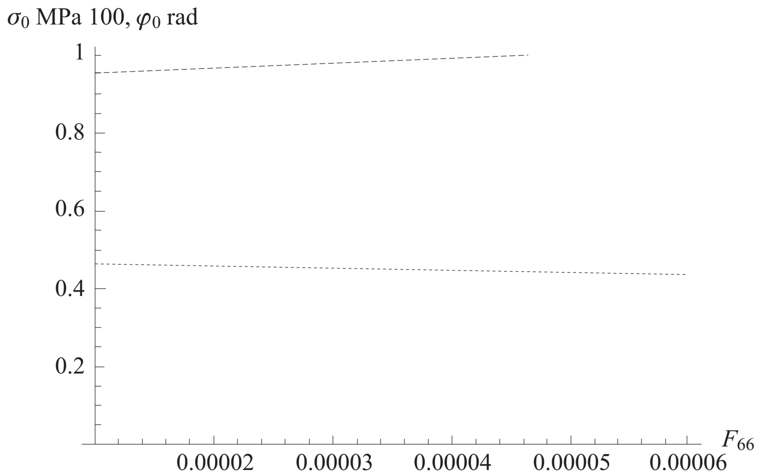

Fig. 5. Relation between φ_0 (dashed curve) and σ_0 (solid curve) and the value of F_{66} the term (compare Table 3)

It can be easily noticed that both the angle of localization φ_0 and critical stress σ_0 are surprisingly insensitive to the changes in the terms F_{11}, F_{22}, F_{66}, F_{12}. Knowledge on the localization of the minimum point for this class of composites seems to be not very useful for the evaluation of the constants F_{ij} and particularly F_{12}. Thus, it can be concluded that failure initiation for this class of materials should always occur near the point $\varphi_0 = \frac{\pi}{3}$. This conclusion

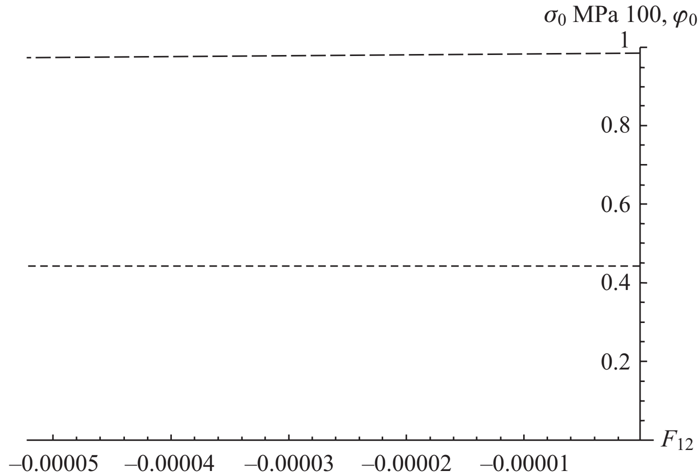

Fig. 6. Relation between φ_0 (dashed curve) and σ_0 (solid curve) and the value of F_{12} the term (compare Table 4)

remains valid independently on the value of elastic constants. Changes in the values γ_1 and γ_2 have no practical impact on the angle φ_0 and stress σ_0.

Acknowledgements

The author wishes to express her gratitude to Professor Andrzej Blinowski for helpful discussions on the subject of the present paper.

References

Blinowski A., Ostrowska-Maciejewska J. 1995. *On the stress distribution in bending of strongly anisotropic beams*. Engng.Trans., 43(1-2): 83-89.

Boczkowska A., Kapuściński J., Puciłowski K., Wojciechowski S. 2000. *Kompozyty*. Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa.

Chawla P., Krisham K. 1987. *Composite materials: science and engineering*. Springer, New York.

Lechnicki S.G. 1957. *Anizotropnye płastinki*. Gos. Izd. Techniko-Teoretičeskoj Literatury, Moskva.

Śleziona J. 1998. *Podstawy technologii kompozytów*. Wydawnictwo Politechniki Śląskiej, Gliwice.

Reviewed linguistically by Aleksandra Poprawska Accepted for print 2005.09.22

Appendix I

The elastic constants of the composite according to Śleziona (1998) can be estimated as follows:

(I)
$$E_{11} = E_{f1} V_f + E_m V_m,$$

(II)
$$E_{22} = \frac{E_m}{1 - \sqrt{V_f}\left(1 - \frac{E_m}{E_{f2}}\right)},$$

(III)
$$G_{12} = \frac{G_m}{1 - \sqrt{V_f}\left(1 - \frac{G_m}{G_{f12}}\right)},$$

(IV)
$$\nu_{12} = \nu_{f12} V_f + \nu_m V_m,$$

where:

E_{f1} – Young modulus of the fibers in the fiber direction,
E_{f2} – Young modulus of the fibers across the fiber axis,
E_m – Young modulus of the matrix,
V_f – volumetric part of fibers, $V_f = 0.7$ was assumed for the material under considerations,
$V_f = 1 - V_f$ – volumetric part of the matrix,
G_m – shear modulus of the matrix,
G_{f12} – shear modulus for the shearing parallel to the fibers,
ν_{f12} – Poisson ratio,
ν_m – Poisson ratio of the matrix.

The following numerical data were taken (Śleziona 1998): $E_{f1} = 130$, $E_{f2} = 130$, $E_m = 3$, $G_m = 1$, $G_{f12} = 50$, $\nu_{f12} = 0.3$, $\nu_m = 0.35$. Thus, $E_{11} = 91.1$, $E_{22} = 15.87$, $G_{12} = 5.37$, $\nu_{12} = 0.315$.

Dimensionless elastic constants γ_1 and γ_2 according to relations (4) and (5) are the following: $\gamma_1 = 0.59$ and $\gamma_2 = 4.01$.

Appendix II

Table 1

F_{11}	φ_0	σ_0
0.0000013	0.98005	0.447436
0.0000018	0.980266	0.44736
0.0000023	0.980483	0.447284
0.0000028	0.980701	0.447207
0.0000033	0.980921	0.447131
0.0000038	0.981142	0.447055
0.0000043	0.981364	0.446978
0.0000048	0.981588	0.446901
0.0000053	0.981813	0.446824
0.0000058	0.982039	0.446747
0.0000063	0.982266	0.44667

Table 2

F_{66}	φ_0	σ_0
0.00010	0.956691	0.463928
0.00015	0.960988	0.460583
0.00020	0.965537	0.457222
0.00025	0.970377	0.453857
0.00030	0.975557	0.450468
0.00035	0.981142	0.447055
0.00040	0.987215	0.443611
0.00045	0.993892	0.440129
0.00050	1.00134	0.436598
0.00055	1.0098	0.433006
0.00060	1.01966	0.429332

Table 3

F_{22}	φ_0	σ_0
0.00047	0.98442	0.483898
0.00052	0.983589	0.47567
0.00057	0.982861	0.467914
0.00062	0.982219	0.460586
0.00067	0.981649	0.453644
0.00072	0.981142	0.447055
0.00077	0.980688	0.440787
0.00082	0.980279	0.434814
0.00087	0.979911	0.429113
0.00092	0.979577	0.423663
0.00097	0.979274	0.418444

Table 4

F_{12}	φ_0	σ_0
-0.000051	0.97633	0.450566
-0.000046	0.977272	0.449863
-0.000041	0.978224	0.44916
-0.000036	0.979187	0.448458
-0.000031	0.980159	0.447756
-0.000026	0.981142	0.447055
-0.000021	0.982136	0.446353
-0.000016	0.983141	0.445653
-0.000009	0.984568	0.444672
-0.000004	0.985602	0.443971
0	0.986438	0.443411