TECHNICAL SCIENCES
Abbrev.: Tech. Sc., Pap. And Rep., No 9, Y. 2006

ON THE SINGULARITIES AT THE TIPS OF ORTHOTROPIC WEDGES IN PLANE ELASTICITY PART TWO

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Key words: anisotropic elasticity, stress singularities.

Abstract

Part Two of the present paper contains more detailed studies of the subject, including cases of arbitrary symmetry. The order of singularity λ changes with the change in the properties of at least one material as well as with the wedge rotation angle ψ and its opening angle φ. Relations $\lambda - \varphi$ for different sets of elastic constants corresponding to composites of epoxy resin and kevlar fiber, epoxy resin and boron fiber and real metallic cubic crystal (aluminum and tungsten) were studied. For some cases of arbitrary symmetry, modes of stress distribution for different values of λ were determined. It was found that the real solutions for λ are solitary in a complex plane. No complex solutions corresponding to finite elastic energy in the vicinity of a wedge tip ($0 \leq \mathrm{Re}\lambda \leq 2$) were found.

O RZĘDZIE OSOBLIWOŚCI W OTOCZENIU WIERZCHOŁKA ORTOTROPOWEGO KLINA W PŁASKIM ZAGADNIENIU TEORII SPRĘŻYSTOŚCI (CZ. 2)

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Słowa kluczowe: anizotropia sprężysta, osobliwości pól naprężeń.

Streszczenie

W niniejszej pracy rozpatrzono przypadek klina niesymetrycznego, zorientowanego niezgodnie z osiami ortotropii.

Zbadano przebiegi zmienności rzędu osobliwości λ ze zmianą kąta obrotu ψ jednego materiału przy kątach rozwarcia klina $\varphi = \dfrac{\pi}{2}$ i $\varphi = \dfrac{\pi}{3}$ dla różnych kombinacji stałych sprężystych odpowiadających takim materiałom, jak: kompozyt żywicy epoksydowej i włókna kewlarowego, kompozyt żywicy epoksydowej i włókna borowego i rzeczywisty kryształ metaliczny w układzie regularnym (aluminium i wolfram).

Poszukiwano rozwiązań pierwiastków rzeczywistych i zespolonych. Otrzymane rozwiązania są izolowanymi punktami na płaszczyźnie zespolonej, nie znaleziono rozwiązań zespolonych w obszararze ($0 \leq \mathrm{Re}\lambda \leq 2$) odpowiadających skończonym wartościom energii sprężystej w otoczeniu wierzchołka klina.

Dla wybranego przypadku niesymetrycznego znaleziono rozkłady naprężeń odpowiadających wartościom λ. Rozkłady te nie wykazywały ani symetrii, ani antysymetrii.

Introduction

Strength of singularity at the tip of an orthotropic wedge embedded into an infinite two-dimensional elastic orthotropic body was considered in Part One of the present paper (Blinowski and Wieromiej-ostrowska 2005). The considerations were restricted to wedges symmetrically oriented with respect to the axes of orthotropy. They concerned primarily the relations between the order of singularity and the wedge opening angle and/or the elastic properties of materials. Mixed boundary value conditions were assumed: continuity of both tractions and displacements at the interfaces was demanded.

Two modes of stress distribution corresponding to different values of λ, symmetric and skew-symmetric, were found. In the case of nearly isotropic materials, quantitative results roughly repeated those obtained by the authors for isotropic materials, reported in a paper by Blinowski and Wieromiej (2004), where the symmetries of solutions were assumed in advance.

In Part Two of the paper the considerations are generalized to wedges arbitrarily oriented with respect to the axes of orthotropy. The mathematical procedures applied do not differ from those used in Part One.

Results and conclusions

Values of the order of singularity l are to be determined from the condition:

$$\det \mathbf{A} = 0. \tag{1}$$

Matrix **A** describes the system of eight homogenous equations generated by the boundary conditions for the determination of unknown multipliers $\left[A_1^I, B_1^I, A_2^I, B_2^I, A_1^{II}, B_1^{II}, A_2^{II}, B_2^{II}\right]$ of a linear combination of particular solutions (29) to equation (3) (compare Part One of the present paper).

The calculated numerical values of λ can be substituted into the appropriate expressions for the components of matrix **A**, and next their numerical values, corresponding to the subsequent values of λ, can be calculated. Using a standard procedure of determining eigenvectors of matrix **A**, corresponding to null eigenvalues, we can find normalized vectors **x** representing sets of unknown multipliers $\left[A_1^I,\ B_1^I,\ A_2^I,\ B_2^I,\ A_1^{II},\ B_1^{II},\ A_2^{II},\ B_2^{II}\right]$.

As expected, the order of singularity λ changes with the change in the properties of at least one material, as well as with the wedge rotation angle ψ and its opening angle φ. Calculations were performed for different values of elastic constants corresponding to some chosen composites: epoxy resin and kevlar fiber, epoxy resin and boron fiber as well as real metallic cubic crystal (aluminum and tungsten). Initially the authors looked for real solutions only.

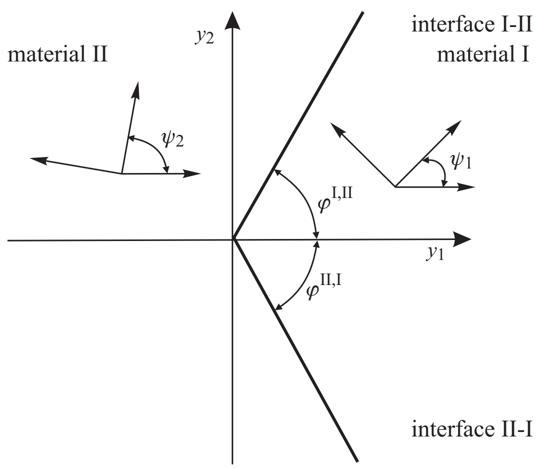

Fig. 1. Orientation of an orthotropic wedge embedded in an orthotropic medium with respect to the observer frame axes $\{y_1, y_2\}$

Quite interesting is the case when an orthotropic wedge is embedded into an infinite two-dimensional elastic orthotropic body made of the same material (corresponding to a composite of epoxy resin and kevlar fiber). The material of the wedge was rotated by the angle $\psi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with respect to the axes of orthotropy of the other body. For the wedge opening angle $\varphi = \frac{\pi}{2}$, only single solutions for the angles of rotation ψ belonging to the interval $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ were found, while no solutions for ψ belonging to the intervals $\left(-\frac{\pi}{2},-\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ were observed, see Fig. 2.

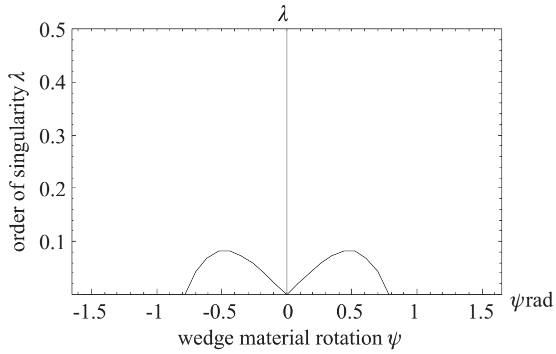

Fig. 2. The order of singularity λ versus the rotation angle ψ for the opening angle $\varphi = \frac{\pi}{2}$. An orthotropic wedge is embedded into an infinite two-dimensional elastic orthotropic body made of the same material corresponding to a composite of epoxy resin and kevlar fiber (Table 1)

Table 1

Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material I	91.1	15.87	0.59	4.01	2.92
Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material II	91.1	15.87	0.59	4.01	2.92

The same problem was considered for a composite of epoxy resin and boron fiber, and similar results were obtained, compare Fig. 3.

The case of an orthotropic wedge made of an epoxy-kevlar composite embedded into epoxy-boron was considered. The material of the wedge was rotated by the angle ψ from the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. For the wedge opening angle $\varphi = \frac{\pi}{3}$, double solutions for angles of rotation ψ belonging to the interval $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ were found, and no solutions for ψ belonging to the intervals $\left(-\frac{\pi}{2},-\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ were observed, see Fig. 4.

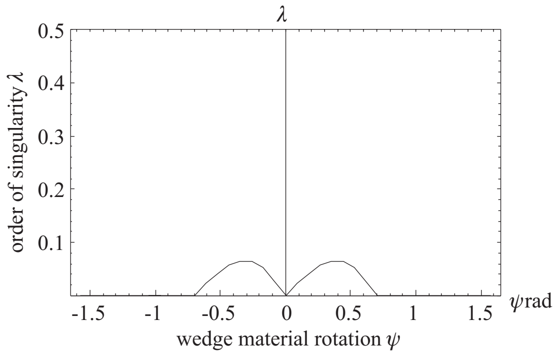

Fig. 3. The order of singularity λ versus the rotation angle ψ for the opening angle $\varphi = \frac{\pi}{2}$. An orthotropic wedge is embedded into an infinite two-dimensional elastic orthotropic body made of the same materials corresponding to a composite of epoxy resin and boron fiber (Table 2)

Table 2

Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material I	266.9	16.6	1.05	3.79	2.82
Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material II	266.9	16.6	1.05	3.79	2.82

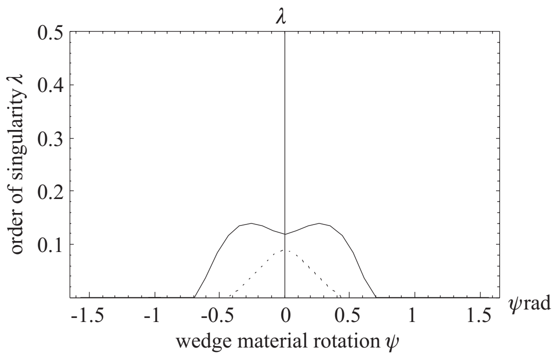

Fig. 4. The order of singularity λ versus the rotation angle ψ for the opening angle $\varphi = \frac{\pi}{2}$. An orthotropic wedge made of an epoxy-kevlar composite embedded into an epoxy-boron composite (Table 3)

Table 3

Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material I	266.9	16.6	1.05	3.79	2.82
Elastic constants	E_1	E_2	γ_1	γ_2	γ_3
Material II	66.5	9	0.6	4.51	3.27

The case of real metallic cubic crystal (aluminum and tungsten), where the wedge and elastic plane were made of the same material, was studied. The material of the wedge was rotated by the angle ψ belonging to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for the wedge opening angles $\varphi = \frac{\pi}{2}$ and $\varphi = \frac{\pi}{3}$. No real solution for λ in the interval (0, 1) corresponding to finite elastic energy was observed.

Complex solutions $\lambda = a + ib$ were looked for as well. For this purpose the following function was defined:

$$\mathrm{F}(a, b) = |\mathbf{A}| \tag{2}$$

It was found that all solutions to the equation

$$F(a, b) = 0 \tag{3}$$

were real and solitary in a complex plane. The case of a wedge made of a composite of epoxy resin and kevlar fiber embedded into a composite of epoxy

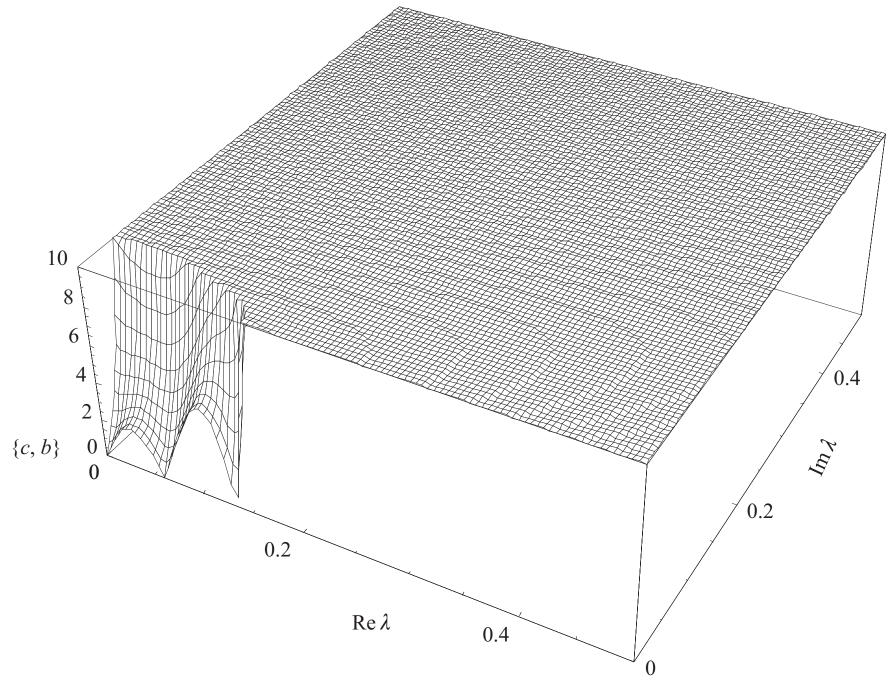

Fig. 5. Real solutions are solitary in the complex plane region corresponding to finite energy in the vicinity of a wedge tip ($0 \leq \mathrm{Re}\lambda \leq 1$). An orthotropic wedge made of an epoxy-kevlar composite embedded into an epoxy-boron composite. The wedge rotation angle $\psi = \frac{\pi}{18}$ and the opening angle $\varphi = \frac{\pi}{3}$

resin and boron fiber is shown in Fig. 5, where the truncated value of the function $F(a, b)$ is depicted. In Fig. 6 the same situations is shown in a broader region.

For the problems considered in Part One of this paper the symmetric and skew symmetric distributions of stress were found. For instance, in the case where the wedge and elastic plane were made of different material, λ_1 was found to be equal to 0.362. The corresponding eigenvector of matrix **A** had the following form: $[0;-0.146;0;0.271;0;-0.481;0;0.820;]$, for the second value of the corresponding eigenvector was: $[0.058;0;-0.070;0;0.605;0;-0.790;0;]$ (compare Fig. 4 in BLINOWSKI and WIEROMIEJ-OSTROWSKA 2005).[1]

For the lack of symmetry of the boundary value problem no particular symmetry of the solution can be expected. For example, in the case of an epoxy-kevlar wedge and elastic space of different material corresponding to an epoxy-boron composite and for the rotation angle $\psi = \frac{\pi}{12}$ we obtain: $\lambda_1 = 0.0383868$. The corresponding eigenvector is the following [–0.867; 0.134;

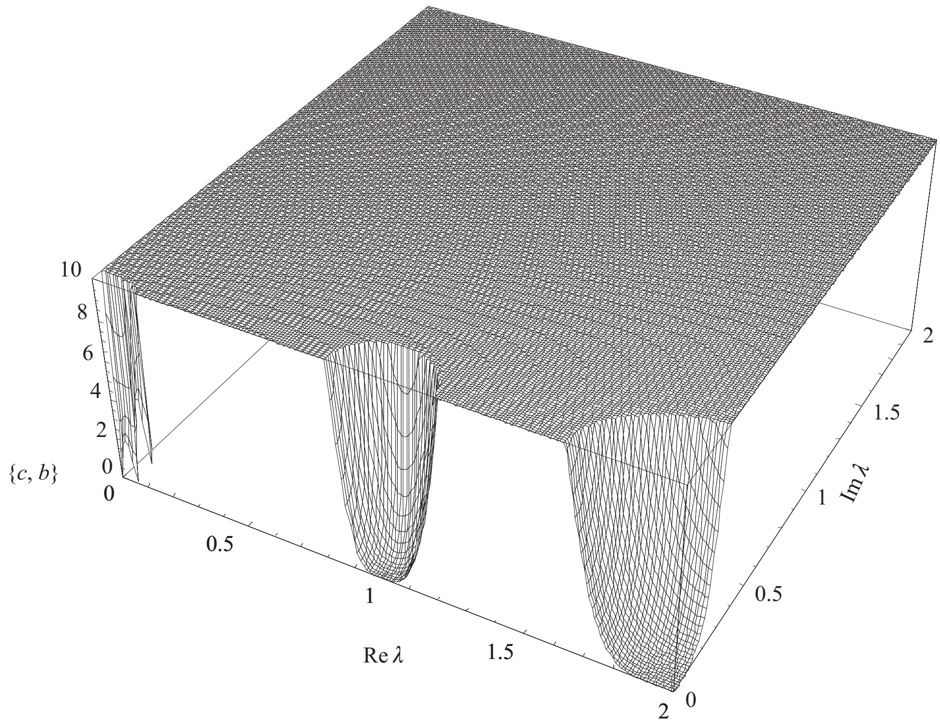

Fig. 6. The same case as in Fig. 5 shown in a broader domain

[1] Zeroes at the even positions in eigenvectors of matrix **A** describe the symmetric distribution of stress, while zeroes of the odd positions correspond to a skew symmetric stress field.

0.053; –0.009; –0.475; 0.033; 0.006; –0.004;], for the second value of $\lambda_2 = 0.139507$ the corresponding eigenvector looks as follows: [–0.830; –0.079; 0.064; 0.027; –0.467; –0.283; 0.001; 0.012;]. It is not difficult to notice that none of them describes a symmetric or skew-symmetric stress field.

The examples discussed above indicate that, depending on the problem geometry and on the properties of the materials, two, one or none solutions for the order of singularity can be obtained. For some cases singularities do not occur. The method of analysis proved to be useful and can be employed to other problems of a theoretical and/or practical interest, omitted in the present paper.

The particular problem of the order of singularity at the vicinity of a triple point of contact of three wedges made of the same (rotated) or different materials can be also mentioned here. The solution to such a problem may make a valuable contribution to a better understanding of the material fatigue process. Mathematical treatment of such a problem does not differ from the present one: making use of bigger, 12 x 12, matrices does not seem to cause additional troubles. In view of the results reported above it is not clear if any singularities can be expected in the case of commonly used polycrystalline engineering materials. The authors will discuss this problem in their next paper.

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Reviewed linguistically by Aleksandra Poprawska Accepted for print 2006.06.16