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DETERMINATION OF THE PARAMETERS OF FLUID MOTION IN TAYLOR-COUETTE FLOW

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Key words: Taylor-Couette flow, numerical simulation, CFD.

Abstract

According to the Taylor-Couette theory, the flow of a fluid takes place in the gap between two concentric cylinders as a result of rotation of one of them. If the rotational speed is sufficient, centrifugal forces overcome forces of internal friction present between fluid particles, which causes the formation of toroidal vortices filling the gap between the cylinders. The vortices are axi-symmetrical at a low rotational speed of the cylinders, and take the shape of a wave if the rotation rate is further increased. Taylor-Couette flow is still under investigation, since the physical aspects of this phenomenon have not been described in detail to date.

The paper presents the results of flow simulations carried out using the commercial software FLUENT. The aim of the numerical computations presented in the paper was to determine the critical values of the angular velocity of cylinders, at which the characteristic vortices are formed, as well as to estimate the impact of cylinder length on the course of the phenomenon analyzed.

WYZNACZANIE PARAMETRÓW RUCHU CIECZY W PRZEPŁYWACH TAYLORA-COUETTE'A

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Słowa kluczowe: przypływy Taylora-Couette'a, symulacja numeryczna, CFD.

Streszczenie

Przepływ cieczy według teorii Taylora-Couette'a powstaje w przestrzeni między koncentrycznymi cylindrami w wyniku ruchu obrotowego jednego z cylindrów. Przy wystarczająco dużej prędkości obrotowej siły odśrodkowe pokonują siły lepkości występujące między cząsteczkami cieczy, co wywołuje kształtowanie się wirów toroidalnych napełniających przestrzeń między cylindrami. Wiry są osiowosymetryczne przy niskiej prędkości obrotowej cylindrów i przy zwiększaniu prędkości przyjmują kształt fali. Przepływy Taylora-Couette'a są wciąż badane ze względu na niepełny opis fizycznej strony zjawiska.

W pracy przedstawiono wyniki symulacji przepływów z zastosowaniem programu komercyjnego FLUENT. Celem prezentowanych obliczeń numerycznych było wyznaczenie wartości krytycznych prędkości kątowych cylindrów, dla których zaczynają się tworzyć charakterystyczne wiry, oraz wpływu długości cylindrów na przebieg badanego zjawiska.

Introduction

According to the Taylor-Couette theory, the flow of a fluid takes place in the gap between two concentric cylinders as a result of rotation of one or both of them. In the simplest case the outer cylinder is at rest, and the inner cylinder rotates. If the rotational speed of the inner cylinder is sufficient, centrifugal forces overcome forces of internal friction present between fluid particles, which causes the formation of vortices characteristic of Taylor-Couette flow (TILLMARK 1994, MIN 1994). As the rotational speed of the inner cylinder is increased, the vortices start to fill the gap between the cylinders. The vortices are toroidal and axi-symmetrical at a low rotational speed of the cylinders, and take the shape of a wave if the rotation rate is further increased (Fig. 1).

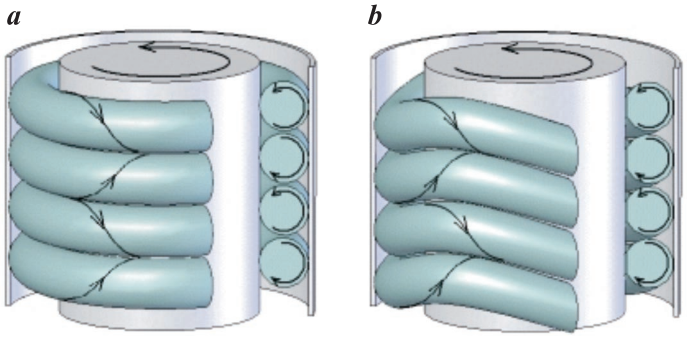

Fig. 1. Two patterns of Taylor-Couette vortex flow, indicating the direction of motion of the cylinders and fluid particles, *a* – axi-symmetrical vortices, *b* – wavy vortices

The phenomenon under analysis is used in the construction of rotating filters since it facilitates the transportation of pollutant molecules at axial flow of a fluid between rotating cylinders (WERELEY, LUEPTOW 1999). The wavy system of vortices enables to construct devices for suspension production in liquids used for industrial purposes (RUDOLPH et al. 1998). The systems based on vortex formation are also applied in bioengineering, e.g. to store blood in a liquid state (AMEER et al. 1998).

The aim of the numerical computations presented in the paper was to determine the critical values of the angular velocity of cylinders, the shape of vortices, as well as to estimate the impact of cylinder length on the course of the phenomenon analyzed.

Theoretical basis of the phenomenon

Due to the viscosity of a fluid, a boundary layer is formed as a result of the difference between the average velocity of a fluid and the velocity of the surface of a solid body. In the case studied the wall of a movable cylinder moves. Due to the phenomenon of adhesion, fluid elements that have direct contact with the surface of a solid body adhere to this surface, so that their relative velocity is equal to zero (ALI et al. 2002). The transition from the velocity of a fluid element at the surface of a solid body to the velocity of a fluid element situated at a certain distance from this surface must be continuous, with no slip, since in a viscous fluid slip would be accompanied by infinitely big tangent forces.

The phenomenon of boundary layer separation is a consequence of changes in the distribution of the velocity of fluid elements in subsequent sections perpendicular to the surface of a solid body. The velocity distribution in the boundary layer changes so that fluid elements in this layer gradually lose speed. As a result of these changes fluid elements subjected to the operation of the largest tangent forces in the immediate vicinity of the surface of a solid body completely lose their kinetic energy. Under such conditions the fluid elements closest to the surface of a solid body display a tendency to retrace their orbits. Such a velocity system is conducive to boundary layer separation, accompanied by the periodic formation of vortices, which flow off the surface of a solid body.

Computational model

The following computational model was used:

- cylinders in a vertical system (direction of gravitation),
- standard turbulence model k-ε of SST type,
- energy equation.

Turbulence modeling is presented below (RODI 1993).

- turbulence kinetic energy equation *k*

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\rho\varepsilon-Y_M+S_k,$$

– dissipation velocity

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_i}(\rho\varepsilon u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right]+C_{1\varepsilon}\frac{\varepsilon}{k}P_k-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}+S_\varepsilon .$$

Turbulent viscosity:

$$\mu_t=\rho C_\mu\frac{k^2}{\varepsilon}.$$

Production *k*:

$$P_k=\mu_t S^2,$$

where *S* is a module of the mean value of the determinant of a stress tensor defined as:

$$S\equiv\sqrt{2S_{ij}S_{ij}}$$

Values of constants

$$C_{1e}=1.44,\quad C_{2e}=1.92,\quad C_\mu=0.09,\quad \sigma_k=1.0,\quad \sigma_e=1.3.$$

Numerical computations

Numerical computations were performed using FLUENT 6 software. A rotation of the inner cylinder without fluid inflow into the computational space was considered. Cylinder dimensions were modeled according to the actual cylinder dimensions on the test stand:

– diameter of the inner cylinder $d_1=\phi$ 50 mm,
– diameter of the outer cylinder $d_2=\phi$ 80 mm.

In order to determine the impact of cylinder length on the formation of vortices in Taylor-Couette flow, two cases were taken into account:

– cylinder length $l=200$ mm,
– cylinder length $l=800$ mm.

Figure 2 shows the generated finite-volume based computational grid of 200 000 cells. Figures 3–9 illustrate the results of numerical computations.

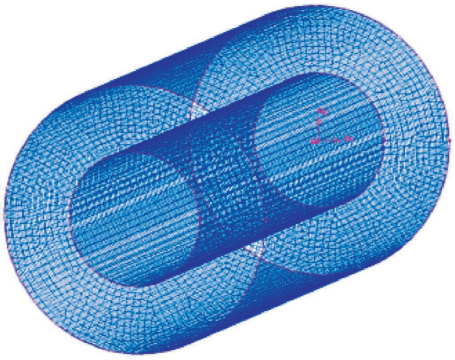

Fig. 2. Grid discretizing the computational space

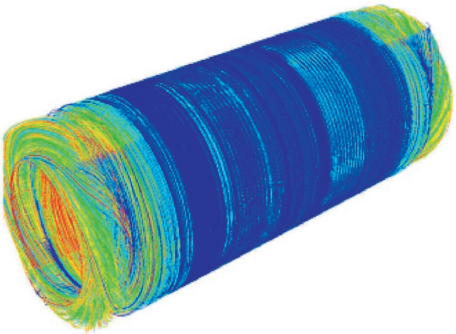

Fig. 3. Streamlines for velocity ω = 100 rad/s, cylinder length 200 mm

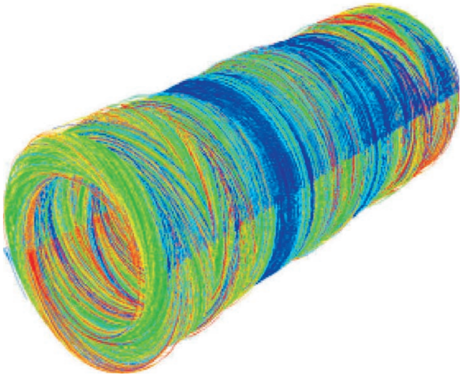

Fig. 4. Streamlines for velocity ω = 150 rad/s, cylinder length 200 mm

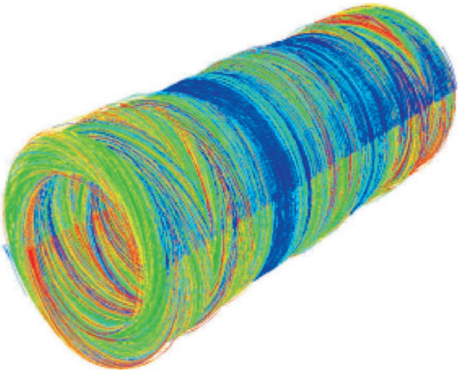

Fig. 5. Streamlines for velocity ω = 180 rad/s, cylinder length 200 mm

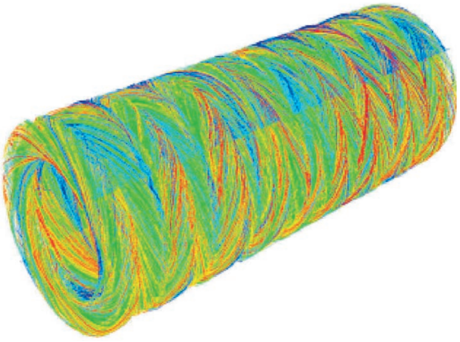

Fig. 6. Streamlines for velocity ω = 200 rad/s, cylinder length 200 mm

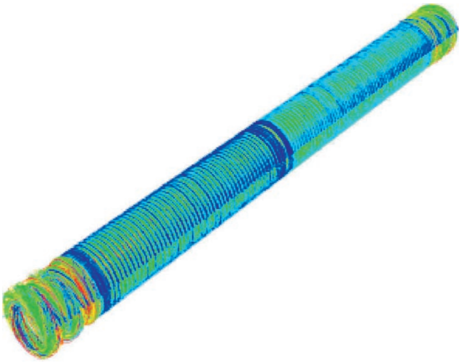

Fig. 7. Streamlines for velocity ω = 180 rad/s, cylinder length 800 mm

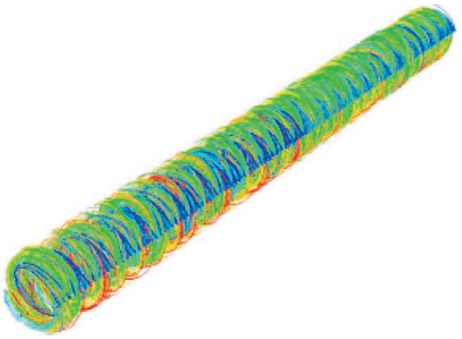

Fig. 8. Streamlines for velocity ω = 200 rad/s, cylinder length 800 mm

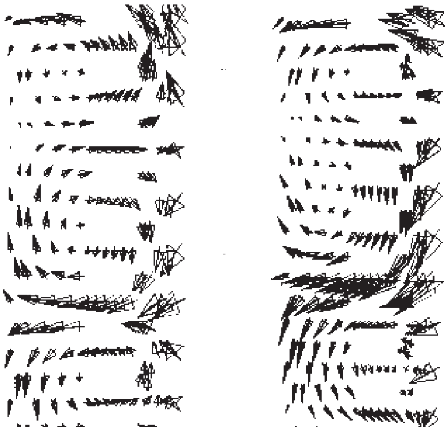

Fig. 9. System of velocity vectors in the longitudinal section for angular velocities ω = 200 rad/s (on the left) and ω = 400 rad/s

Conclusions

The results of the above computations enable to formulate the following conclusions:

1. Vortex formation starts at an angular velocity of about ω = 100 rad/s.

2. As the rotational speed is increased, the vortices spread over the entire gap occupied by the fluid. At an angular velocity of ω = 150–200 rad/s the entire gap is filled with vortices.

3. Cylinder length had no effect on vortex formation in the range studied.

4. At ω = 400 rad/s the velocity distribution is more complex, representing the beginning of vortex formation in the wavy system.

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