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A PROBABILISTICALLY APPROACHED FORECAST OF THE FATIGUE LIFE OF NOTCHED MEMBERS

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K e y w o r d s: fatigue life, deterministic description of cracking, fatigue crack propagation.

A b s t r a c t

The aim of the study was present a model of a probabilistic estimation of the fatigue life of structural members, based on a deterministic description of cracking. Components with notches in the form of centrally located holes with side cuts were analyzed. In the method based on a probabilistic approach to crack propagation, some dependences were used that take into account the presence of areas showing plastic strains in front of crack tips. It was assumed that cracking can be modeled on the grounds of some general-purpose quantity used to describe the energy state in the area of a crack tip, i.e. the Rice's integral (J). The formulated computational model was employed to estimate the fatigue life of model components made of some selected aluminum alloys for aeronautical structures. Experimental work was carried out using some flat specimens with centrally positioned holes. They were exposed to flat bending at $R = 0$. Analytical and experimental results show pretty good conformity.

PROGNOZOWANIE TRWAŁOŚCI ZMĘCZENIOWEJ ELEMENTÓW KONSTRUKCYJNYCH Z KARBEM W UJĘCIU PROBABILISTYCZNYM

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S ł o w a k l u c z o w e: trwałość zmęczeniowa, opis deterministyczny pękania, propagacja pęknięć zmęczeniowych.

S t r e s z c z e n i e

Przedstawiono model probabilistycznego szacowania trwałości zmęczeniowej elementów konstrukcyjnych oparty na deterministycznej charakterystyce pękania. Analizie poddano elementy z karbem w postaci centralnego otworu z bocznymi nacięciami. W przyjętej metodzie probabilistycznego opisu rozwoju pęknięć wykorzystano zależności uwzględniające obecność stref odkształceń plastycznych przed czołem pęknięcia. Założono, że zjawisko pękania można zamodelować na podstawie uniwersalnej wielkości do opisu stanu energetycznego w strefie czoła pęknięcia, jaką jest całka Rice'a (J). Opracowany model obliczeniowy wykorzystano do oceny trwałości zmęczeniowej elementów modelowych z wybranych lotniczych stopów aluminium. Badania doświadczalne przeprowadzono na próbkach płaskich z centralną szczeliną, poddanych płaskiemu zginaniu przy $R = 0$. Uzyskano dość dobrą zgodność wyników obliczeń i doświadczenia.

Introduction

Fatigue crack propagation and fatigue life are issues of great significance to structural components and their getting failed/damaged, not yet thoroughly recognized. The deepest knowledge possible in this field proves of vital importance because of serious effects of any potential failure/damage (aircraft, chemical pipelines, water-supply systems in nuclear power plants, etc.) Fatigue crack propagation depends on a variety of stochastic factors, such as loading course, component geometry, properties of the material, etc. Therefore, the application of probabilistic models of crack propagation seems strongly advisable. This approach to the problem of fatigue life is represented, among others, by the authors of CALEYO et al. (2002), AHAMMED (1997, 1998), AHAMMED, MELCHERS (1997). The determination of the fatigue life of a component, from the probabilistic standpoint, requires knowledge of service-induced cracking, including a complete probabilistic description of crack propagation, with information on stochastic factors of cracking. Hence, the probabilistic model assumed should both take into account the fundamental dependences that describe the cracking dynamics, and be based on experimental findings.

The aim of the study was to present a model making use of a difference equation that describes the crack growth dynamics approached probabilistically. This approach was suggested by Prof. H. Tomaszek, and discussed in more detail in (KOCAŃDA et el. 1999). For the instance of cracking given consideration, a differential equation that takes into account the dependences specific to a deterministic model was introduced. The Gaussian probability distribution of crack lengths offers a solution to this equation. Outlined is the way of estimating the relevant coefficients with well-known methods (the maximum likelihood method, the least squares method, the method of bisection) described in the literature on the subject. The fatigue life of a given component was estimated using probability distribution that includes known parameters and assuming the risk of exceeding the acceptable crack-length limit.

Analytical considerations were supported with exemplary computations based on experimental findings from tests of model members made of aeronautical structure-dedicated aluminum alloys, 2024-T3 and D16, used for aircraft skins.

Deterministic description of cracking

The model presented in the paper is based on a description of cracking, with the integral J in the form of (1) suggested by Dowling in:

$$J_{\max} = \left[1{,}24 \frac{(\sigma_{\max})^2}{E} + 1{,}02 \frac{\sigma_{\max} \varepsilon_{\max}}{\sqrt{n'}} \right] l \tag{1}$$

where:

$J_{\max}$ – maximum value of the integral J,
E – Young's modulus,
$\sigma_{\max}$ – maximum stress,
$\sigma_{\max}$ – maximum strain,
n' – coefficient of cyclic strain-hardening,
l – actual length of a crack.

Two empirical coefficients are used in the above formula. From the standpoint of the formulated model, specific values are of no or little importance. Their remaining independent of the current length of a crack is what actually matters. Predicted is estimation – on the grounds of experimental findings – of quantities indispensable in the model, with no need to give, e.g. the values of stress. Hence, all dependences presented below can also be used for e.g. the range of effective stress.

The following formula was accepted to describe the rate of cracking:

$$\frac{dl}{dN} = c(J_{\max})^m \tag{2}$$

where:

N – the number of loading cycles,
c, m – constants.

The following relationship was assumed to relate strains and stresses:

$$\varepsilon_{\max} = a\sigma_{\max} \tag{3}$$

where:

a – factor of proportionality, further on assumed to remain constant.

Due to transformations, the integral J takes the following form:

$$J_{\max} = \left[\frac{1{,}24}{E} + 1{,}02 \frac{a}{\sqrt{n'}} \right] (\sigma_{\max})^2 \, l \tag{4}$$

Having substituted (4) for $J_{\max}$ in (2), the following is arrived at:

$$\frac{dl}{dN} = c \left[\frac{1{,}24}{E} + 1{,}02 \frac{a}{\sqrt{n'}} \right]^m (\sigma_{\max})^{2m} \, l^m \tag{5}$$

Using the following notification:

$$A = c \left[\frac{1{,}24}{E} + 1{,}02 \frac{a}{\sqrt{n'}} \right]^m (\sigma_{\max})^{2m} \tag{6}$$

we get:

$$\frac{dl}{dN} = A l^m \tag{7}$$

While setting to solve this equation, we have to take into account the value of the exponent m.

I. For $m = 1$:

$$\int \frac{dl}{l} = \int A dN$$

$$\ln l = AN$$

Allowing for the condition that the initial crack length $l(N = 0) = l_0$, we get:

$$l = l_0 e^{AN} \tag{8}$$

II. For $m \neq 1$:

$$\int \frac{dl}{l^m} = \int A dN$$

$$\frac{1}{1-m} l^{1-m} = AN$$

Allowing for the initial condition:

$$l=\left[(1-m)AN+l_0^{1-m}\right]^{\frac{1}{1-m}}$$

Taking into consideration the fact that the exponent m of value greater than unity, the above relation can be written in the following form:

$$l=\frac{l_0}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}}} \tag{9}$$

Therefore, the current length of a crack l is described with the following equation:

$$l=\begin{cases} l_0e^{AN} & , \ m=1 \\ \dfrac{l_0}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}}} & , \ m\neq 1 \end{cases} \tag{10}$$

It might prove useful to check whether the solution to the above equation remains continuous towards the value of the exponent m. Left- and right-hand limits of the function $l(m)$ at the point $m = 1$ are equal to each other ($g_l = g_p = g$). The value of the g limit is:

$$g=\lim_{m\to 1}\frac{l_0}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}}}$$

Expression in the denominator gives an indeterminate symbol of type 1^{∞}. To make use of the de l'Hospital theorem, we applied the transformation $u^v = e^{v\ln u}$

$$g=\frac{l_0}{\lim\limits_{m\to 1}\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}}}\overset{1^{\infty}}{=}\frac{l_0}{\lim\limits_{m\to 1}e^{\frac{1}{m-1}\ln\left[1-(m-1)ANl_0^{m-1}\right]}}=$$

$$=\frac{l_0}{e^{\lim\limits_{m\to 1}\frac{1}{m-1}\ln\left[1-(m-1)ANl_0^{m-1}\right]}}$$

$$g_H = \frac{l_0}{e^{\lim\limits_{m\to 1}\frac{\frac{-ANl_0^{m-1}-(m-1)ANl_0^{m-1}\ln l_0}{1-(m-1)ANl_0^{m-1}}}{1}}} = \frac{l_0}{e^{\lim\limits_{m\to 1}\frac{-ANl_0^{m-1}-(m-1)ANl_0^{m-1}\ln l_0}{1-(m-1)ANl_0^{m-1}}}} = \frac{l_0}{e^{\frac{-AN}{1}}} =$$

$$= \frac{l_0}{e^{-AN}} = l_0 e^{AN}$$

The calculated limit of a function equals its value at point $m = 1$; hence, the solution given with eq (10) remains continuous towards the exponent m, and two ranges of applicability of this solution result only from the limitations of the mathematical notation.

The checkup of the correctness of the solution found:

I. For $m = 1$:

$$\frac{dl}{dN} = l_0 A e^{AN} = Al$$

Having compared the above expression to formula (7) for $m = 1$, we can easily find that the solution is correct.

II. For $m \neq 1$:

$$\frac{dl}{dN} = \frac{-l_0 \frac{1}{m-1}\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}-1}(-1)(m-1)Al_0^{m-1}}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{2}{m-1}}} =$$

$$= \frac{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{2-m}{m-1}} Al_0^m}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{2}{m-1}}} =$$

$$= \frac{Al_0^m}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{m}{m-1}}} = A\left[\frac{l_0}{\left[1-(m-1)ANl_0^{m-1}\right]^{\frac{1}{m-1}}}\right]^m = Al^m$$

Again, having compared the above expression to formula (7) for $m \neq 1$, we can easily find that the solution is correct.

Probabilistic description of cracking

The following assumptions were made while setting to formulate a probabilistic model:

1. Loading cycles of Dt duration may occur rather randomly than continuously, at the rate λ i.e. $\lambda \Delta t \leq 1$.
2. Consideration is given to such lengths of fatigue cracks that for some interval (or for a certain number of loading cycles) the probability that a catastrophic fracture occurs in a structural member equals zero.
3. The rates of fatigue cracking are described by the deterministic approach with the above-presented relations.

Thus, we obtain the following difference equation that describes the crack growth dynamics in probabilistic terms:

$$U_{l,t+\Delta t} = (1 - \lambda \Delta t) U_{l,t} + \lambda \Delta t U_{l-\Delta l,t} \tag{11}$$

where:

$U_{l,t}$ – probability that at time instant t the crack length is *l*.

After transformation to functional notation, we get the following equality:

$$U(l, t + \Delta t) = (1 - \lambda \Delta t) U(l,t) + \lambda \Delta t U(l - \Delta l, t) \tag{12}$$

where:

U(l,t) – density function of crack lengths.

The above-shown crack length increment during one cycle Δl will be found from the relations provided by the deterministic model.

Applying eq (7), after determination of increments for one loading cycle ($\Delta N = 1$), and allowing for $N = \lambda t$, here is what resulted:

$$\Delta l = \alpha l^m$$

Taking into account expression (10) that defines the crack length:

$$\Delta l = \begin{cases} \alpha l_0 e^{\alpha \lambda t} & , m = 1 \\ \dfrac{\alpha l_0^m}{\left[1 - (m-1)\alpha \lambda t l_0^{m-1}\right]^{\frac{m}{m-1}}} & , m \neq 1 \end{cases} \tag{13}$$

The coefficient a introduced at this point is of a slightly different sense than *A* used earlier. Thus, the coefficient *a* takes into account the probability P_{th} that in case a loading cycle occurs, the stress $(\sigma_{max})^{2m}$ will exceed some threshold value and the crack growth will result.

$$\alpha = c\left[\frac{1{,}24}{E} + 1{,}02\frac{a}{\sqrt{n'}}\right]^m P_{th}(\sigma_{max})^{2m} \tag{14}$$

To arrive at a differential equation, terms of eq (12) need to be subjected to series expansion, with reduction to several terms of expansion:

$$U(l, t+\Delta t) \approx U(l,t) + \frac{\partial U(l,t)}{\partial t}\Delta t$$

$$U(l-\Delta l, t) \approx U(l,t) - \frac{\partial U(l,t)}{\partial l}\Delta l + \frac{1}{2}\frac{\partial^2 U(l,t)}{\partial l^2}\Delta l^2$$

After substitution of these dependences for the terms of eq (12) and allowing for the expression to determine the crack length increment Δl, the following was obtained:

$$U(l,t) + \frac{\partial U(l,t)}{\partial t}\Delta t = (1-\lambda\Delta t)U(l,t) + \lambda\Delta t\left[U(l,t) - \frac{\partial U(l,t)}{\partial l}\Delta l + \frac{1}{2}\frac{\partial^2 U(l,t)}{\partial l^2}\Delta l^2\right]$$

$$\frac{\partial U(l,t)}{\partial t} = -\dot{b}(t)\frac{\partial U(l,t)}{\partial l} + \frac{1}{2}\dot{w}(t)\frac{\partial^2 U(l,t)}{\partial l^2} \tag{15}$$

where:

$$\dot{b}(t) = \lambda\Delta l = \begin{cases} \lambda\alpha l_0 e^{\alpha\lambda t} & , \ m=1 \\ \dfrac{\lambda\alpha l_0^m}{\left[1-(m-1)\alpha\lambda t l_0^{m-1}\right]^{\frac{m}{m-1}}} & , \ m \neq 1 \end{cases}$$

$$\dot{w}(t) = \lambda\Delta l^2 = \begin{cases} \lambda\alpha^2 l_0^2 e^{2\alpha\lambda t} & , \ m=1 \\ \dfrac{\lambda\alpha^2 l_0^{2m}}{\left[1-(m-1)\alpha\lambda t l_0^{m-1}\right]^{\frac{2m}{m-1}}} & , \ m \neq 1 \end{cases}$$

In the next step, a solution to eq (15) should be found in the form of the Dirac equation, so as to make the function integral equal one.

The solution searched for shows the Gaussian distribution:

$$U(l,t)=\frac{1}{\sqrt{2\pi w(t)}}e^{-\frac{(l-b(t))^2}{2w(t)}} \tag{16}$$

In the above formula, $b(t)$ stands for the expected value of crack length, whereas $w(t)$ for the variance of crack length distribution.

I. For m = 1:

$$b(t)=l_0 e^{\alpha\lambda t}$$

$$w(t)=\frac{\alpha l_0^2}{2}\left(e^{2\alpha\lambda t}-1\right)$$

II. For $m \neq 1$:

$$b(t)=\frac{l_0}{\left[1-(m-1)\alpha\lambda t l_0^{m-1}\right]^{\frac{1}{m-1}}}$$

$$w(t)=\frac{\alpha l_0^{m+1}}{m+1}\left[\frac{1}{\left[1-(m-1)\alpha\lambda t l_0^{m-1}\right]^{\frac{m+1}{m-1}}}-1\right]$$

Estimation of parameters of distribution

The determination of the parameters of distribution (16) requires an evaluation of the coefficient α and exponent m. To estimate the parameters of distribution, the maximum likelihood and the least squares methods were used (the descriptions of both methods were neglected, since they remain in common use). Let us assume that we have service- or lab-tests-originated data on crack propagation. The data have the following form:

$$[(l_0,\ t_0),\ (l_1,\ t_1),\ ...,\ (l_n,\ t_n)] \tag{17}$$

Estimation of the coefficient α

With the use of the likelihood function algorithm and after differentiation against the parameter of interest and transformation into functional notation, the following equations were obtained:

$$\begin{cases} \sum_{k=0}^{n-1}\left(l_{k+1}-l_k\right)=l_0\sum_{k=0}^{n-1}\left(e^{\alpha\lambda t_{k+1}}-e^{\alpha\lambda t_k}\right) & m=1 \\ \sum_{k=0}^{n-1}\left(l_{k+1}-l_k\right)=l_0\sum_{k=0}^{n-1}\left(\frac{1}{\left[1-(m-1)\alpha\lambda t_{k+1}l_0^{m-1}\right]^{\frac{1}{m-1}}}-\frac{1}{\left[1-(m-1)\alpha\lambda t_k l_0^{m-1}\right]^{\frac{1}{m-1}}}\right), & m\neq 1 \end{cases} \tag{18}$$

and after transformation thereof:

$$\begin{cases} l_n=l_0 e^{\alpha\lambda t_n}, & m=1 \\ l_n=l_0\frac{1}{\left[1-(m-1)\alpha\lambda t_n l_0^{m-1}\right]^{\frac{1}{m-1}}}, & m\neq 1 \end{cases} \tag{19}$$

Having solved the above equations vs a, the following is found:

$$\begin{cases} \alpha=\frac{1}{\lambda t_n}\ln\frac{l_n}{l_0}, & m=1 \\ \alpha=\frac{1-\left(\frac{l_0}{l_n}\right)^{m-1}}{(m-1)\lambda t_n l_0^{m-1}}, & m\neq 1 \end{cases} \tag{20}$$

The above dependences enable to estimate the value of the coefficient α for some known value of the exponent m.

Estimation of the exponent *m*

With the least squares method applied, the following relation was found:

$$\sum_{k=0}^{n}\left(l_k - B_k\right)C_k = 0 \tag{21}$$

where:

$$C_k = \frac{B_k}{1-m}\left[\ln\frac{B_k}{l_0} - \frac{\lambda t_k}{\lambda t_n}\left(\frac{l_n}{B_k}\right)^{1-m}\ln\frac{l_n}{l_0}\right] \tag{22}$$

$$B_k = l_0\left\{1 - \frac{\lambda t_k}{\lambda t_n}\left[1-\left(\frac{l_n}{l_0}\right)^{1-m}\right]\right\}^{\frac{1}{1-m}} \tag{23}$$

The result has an awkward form. It is difficult to find the analytical form of the estimator m. While verifying the computational model (see section 6), it was decided to numerically solve eq (21) with the method of bisection.

The estimation of the relevant coefficients that occur in the resultant probability distribution can be divided into the following stages:

1. Assuming the value of the exponent $m = 1$, or estimation thereof according to (21).
2. Estimation of the value of the coefficient a according to (20), in compliance with the value of m.

Fatigue life of a member

Using the probability distribution with known parameters, we can estimate the fatigue life of a given member. Therefore, the probability $R(t)$ that the assumed acceptable crack length l_d is not exceeded should be determined. This should take the following form:

$$R(t) = P\left(l \le l_d\right) = \int_{-\infty}^{l_d} U(l,t)\,dl \tag{24}$$

The probability $R(t)$ can be treated as the reliability of a given member in terms of fatigue cracking. It would be convenient to use tables of Gaussian distribution to determine $R(t)$. Beyond a doubt, the operation requires the random variables to be standardized. With some specific change in reliability with time, for the assumed minimum level of reliability R^*, the fatigue life of member T can be determined from the following condition:

$$R(t) \geq R^* \tag{25}$$

Computations are to be carried out for subsequent values of t, for which condition (25) is satisfied. The greatest value $t = T$ that satisfies condition (25) is the estimate of the member's life.

Experimental verification of the computational model

The correctness of the presented computational model was verified on the grounds of experimental findings from tests intended to investigate fatigue crack propagation in model specimens made of aluminum alloys, 2024-T3 and D16,

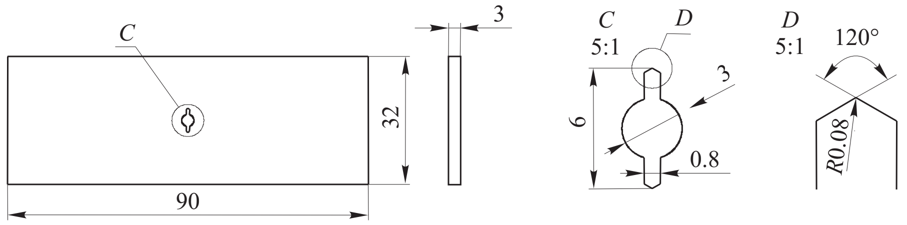

Fig. 1. Specimens to study fatigue crack growth

used for aircraft skins. Specimens to test fatigue crack propagation were cut out of delivered sheets, along the direction of rolling. Fig. 1 shows the dimensions of the specimens. In the central parts of the specimens initial orifices were made in the form of holes, 3 mm in diameter, with side cuts of a total length of 6 mm (see Detail C in Fig. 1). Detail D shows the vertical angle of the cut tip and the radius of the notch bottom. The orifices were initially cut out using a suitably profiled hook tool.

The tests of fatigue crack propagation were carried out under flat bending conditions, for the stress ratio $R = -1$ and the frequency of variations in loading – 25 Hz. The specimens were tested at three values of amplitude of nominal bending stress: σ_{gna} = 80 MPa, 90 MPa, and 100 MPa. The crack lengths on the specimen surfaces were recorded using acetyl-cellulose replicas, by means of ladder-type strain gauges, as well as by an optical method, using a light microscope NEOPHOT 21 with a digital image analyzer LUCIA.

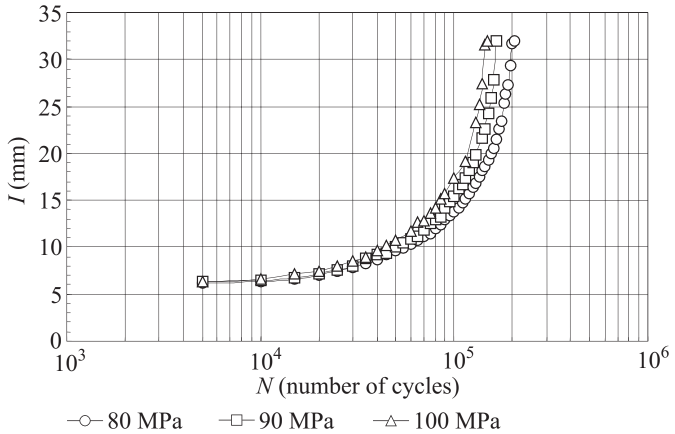

Fig. 2. Crack propagation in specimens made of alloy 2024-T3, examined at σ_{gna} = 80 MPa, 90 MPa, and 100 MPa

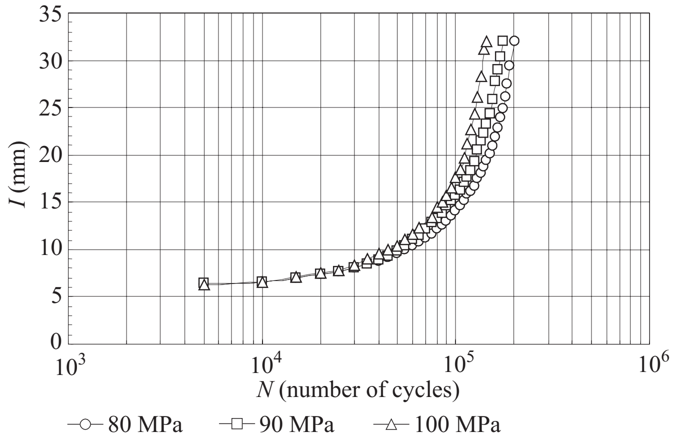

Fig. 3. Crack propagation in specimens made of alloy D16, examined at σ_{gna} = 80 MPa, 90 MPa, and 100 MPa

Measurements of crack lengths l for some fixed number of cycles N provided the basis for plotting graphs $l = f(N)$ shown in Figs. 2 and 3.

The dependences derived in sections 2÷5 gave grounds for a probabilistic description of crack propagation and an estimation of the fatigue life of the tested specimens. The plots that illustrate measurements of crack lengths and the courses of the expected values of crack lengths $b(t)$ for some selected specimens made of alloys 2024-T3 (specimen P1_2024) and D16 (specimen P1_D16) are presented in Figs 4 and 5, respectively.

While calculating fatigue life, it was assumed that the initial crack length $l_0 = 6$ (mm) and the acceptable crack length $l_d = 32$ (mm), both resulting from

90%

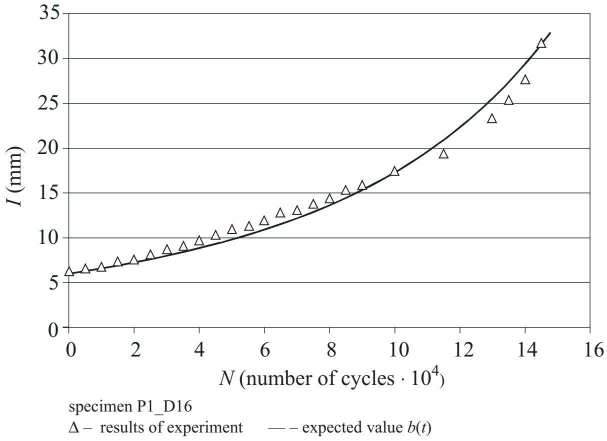

Fig. 4. Recorded and predicted crack lengths in the specimen P1_2024

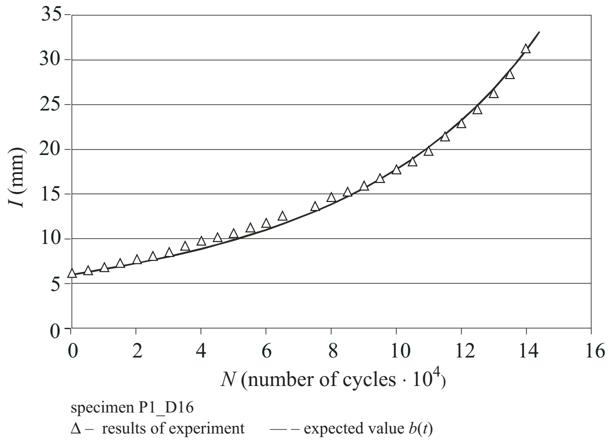

Fig. 5. Recorded and predicted crack lengths in the specimen P1_D16

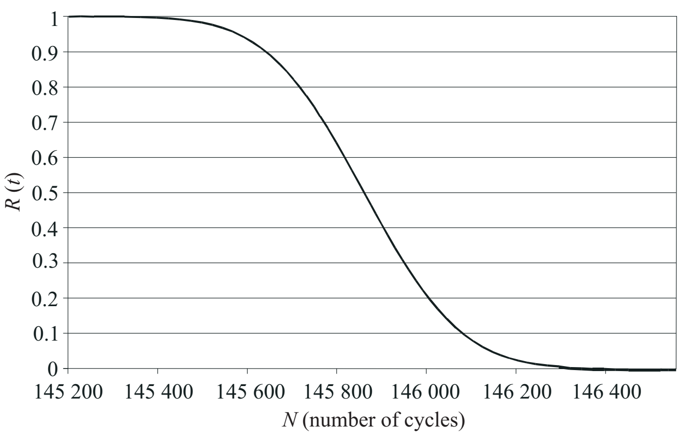

Fig. 6. Change in the probability $R(t)$ that the acceptable crack length l_d in the specimen P1_2024 is exceeded

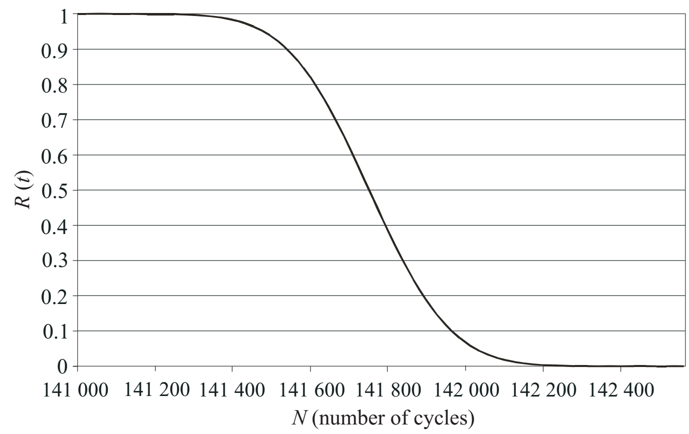

Fig. 7. Change in the probability $R(t)$ that the acceptable crack length l_d in the specimen P1_D16 is exceeded

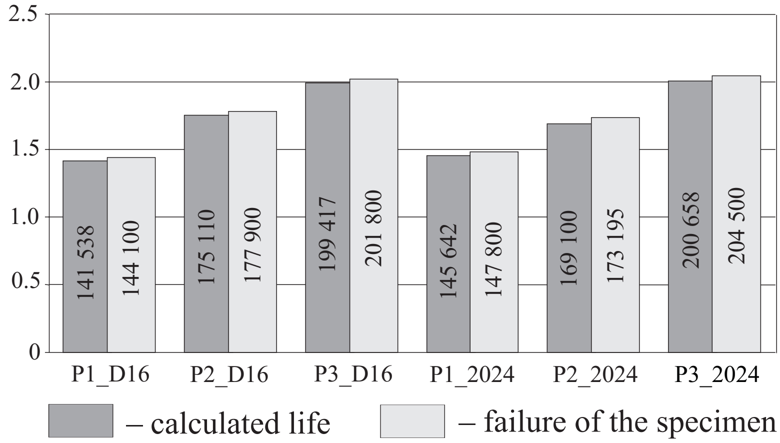

Fig. 8. Calculated and experimentally found fatigue life of specimens made of alloys 2024-T3 and D16

the geometric dimensions of the specimens used in fatigue tests. The assumed minimum reliability level was $R^* = 0.9$ (–).

Figs. 6 and 7 show plots that illustrate the courses of probability $R(t)$ for the analyzed specimens P1_2024 and P1_D16.

Both the calculated and experimentally found fatigue life of all analyzed specimens are shown in Fig. 8.

Summary

The probabilistic model presented in the paper enables to forecast the fatigue life of structural members. What is needed for the model is the knowledge of service-induced crack growth, which comprises a full probabilistic description of crack propagation, and therefore is the source of information on stochastic factors of cracking. The data indispensable for computations were collected in the course of experimental work on fatigue crack growth and propagation in model members made of aluminum alloys 2024-T3 and D16.

The recorded (during experimental work) and expected crack lengths in the specimens under examination showed satisfactory compatibility. A safe estimate of fatigue life was found. The computed fatigue life of specimens was expressed with the number of loading cycles to failure, which were lower and differed by 1.2–2.4% from those found experimentally.

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