OLSZTYN ECONOMIC JOURNAL
Abbrev.: Olszt. Econ. J., 2009, 4(1)

DOI 10.2478/v10021-009-0002-6

ESTIMATION OF RISK–RETURN RELATION PARAMETERS IN THE CONTEXT OF THE APT MODEL

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K e y w o r d s: APT model, non-typical observations, depth measures.

A b s t r a c t

Pricing of capital instruments is one of the important problems in the theory of finance. Theoretical studies resulted in appearance of *Multi-Index Models*, defining the correlation between the profitability of individual securities and a number of systematic risk factors. In the basis of those models another, different from the classical Markowitz theory, method for determining the risk of investment was given specifying at the same time the risk measure appropriate for that model. As a result of further works the *Arbitrage Pricing Theory* – APT was formulated.

The article shows an attempt at pricing capital investments in shares of innovation SiTech segment companies determined by means of the APT model. It was assumed that the rates of return are generated by two-index model in which the general stock exchange market situation and the teleinformation sector market situation are the sources of risk. Analysis of the relation described by that model was supported by the non-typical observations elimination methods based, among others on the measures of depth of the observations in the sample.

Estimations of the cross section regression, following the elimination of non-typical observations indicate that investments in modern technology securities are characterized by positive and statistically significant premium for market risk. On the other hand, it was determined that the influence of sectoral risk on the expected tares of return for analyzed companies was insignificant statistically.

SZACOWANIE PARAMETRÓW RELACJI RYZYKO–DOCHÓD W KONTEKŚCIE MODELU APT

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S ł o w a k l u c z o w e: model APT, obserwacje nietypowe, miary zanurzania.

Abstrakt

Jednym z istotnych problemów teorii finansów jest wycena instrumentów kapitałowych. Opracowania teoretyczne zaowocowały pojawieniem się modeli wielowskaźnikowych (*Multi-Index Model*), określających zależność rentowności pojedynczych walorów od wielu czynników ryzyka systematycznego. Na podstawie tych modeli podano inny, w odróżnieniu do klasycznej teorii Markowitza, sposób wyznaczenia ryzyka inwestycji, precyzując przy okazji właściwą dla tego modelu miarę ryzyka. W wyniku dalszych prac sformułowano teorię arbitrażu cenowego (*Arbitrage Pricing Theory*-APT).

Artykuł ukazuje próbę wyceny inwestycji kapitałowych w akcje spółek segmentu technologii innowacyjnych SiTech, określonej przez model APT. Założono, że stopy zwrotu są generowane przez model dwuwskaźnikowy, w którym źródłami ryzyka jest ogólna koniunktura na giełdzie i koniunktura sektora teleinformatycznego. Analizę relacji opisanej tym modelem wspomożono metodami eliminacji obserwacji nietypowych, opartymi m.in. na miarach zanurzania obserwacji w próbie.

Oszacowania regresji przekrojowej po eliminacji obserwacji nietypowych wskazują, że inwestycje w walory nowoczesnych technologii charakteryzują się dodatnią i statystycznie istotną premią za ryzyko rynkowe. Stwierdzono jednakże statystycznie nieistotny wpływ ryzyka sektorowego na oczekiwane stopy zwrotu analizowanych spółek.

Introduction

The systematic risk plays a special role in the securities risk analysis. In the developed capital markets such as the New York Stock Exchange (NYSE), an attempt at describing the relation between the expected profitability and the systematic risk was undertaken during 1960s and 1970s. Work on specifying the pricing of assets resulted in introduction of the CAPM (*Capital Asset Pricing Model)* model, independently by W. Sharpe in 1964 (SHARPE 1964), J. Lintner in 1965 (LINTNER 1965) and J. Mossin in 1966 (MOSSIN 1966), and the APT theory (*Arbitrage Pricing Theory*) by S. Ross published in 1976 (ROSS 1976).

Capital market equilibrium models, as they are frequently called, as a consequence of their design represent the method for determining the securities equilibrium price depending on the risk represented by them. The CAPM model defines the investment risk resulting from the behavior of all securities as the entire market, i.e. the general market situation. The level of sensitivity of individual securities to changes of indexes characterizing the status of a given capital market is the measure of that risk. According to that theory, the investor is remunerated in the form of the market premium only for the risk systematically influencing the level of the rates of return on the stocks.

The arbitrage pricing theory on the other hand enriches significantly the structure of capital assets pricing. It is a competitive theory on one hand and the theory expanding the CAPM model on the other. The APT theory allows determining the equilibrium conditions based on the process generating the rates of return assumed in advance. According to that theory the return on stocks depend on numerous factors that are the source of the systematic risk. That theory, however, does not define those factors and does not provide information on the value and direction of influence by those factors on the rate of return on securities.

The correlation expressed by the APT theory is a theoretical linear correlation and in reality it never happens that all (or a significant majority) of securities are spread along that straight line. Generally, the equilibrium is seen as a dynamic process and the majority of securities will be characterized by overpricing or underpricing relative to the level determined by that model. In case of some companies the deviation from the equilibrium plateau observed during certain periods of time can be so large that the security can be considered a non-standard observation in the sense of both the deviation from the market equilibrium level and in the sense of the statistical sample. Non-typical observations can be the cause for deviation of information obtained as a result of studies. Currently, the methodology of statistical studies on non-typical data has developed rapidly and it has become one of the more important problems of statistical analysis. Non-typicality is generally caused by heterogeneity of the statistical population from which the sample was taken or caused by the error made by the researcher. This is of major importance in, e.g. forecasting on the bases of estimated correlations. That fact made many authors undertake the search for effective procedures to solve that uneasy and at the same time very important issue. One of such solutions are the methods based on the measures of observation depth in the sample. The notion of depth was introduced by Tukey in 1975 and it was extensively developed by numerous researchers, including: (ROUSSEEUW, RUTS 1996), (LIU, PARELIUS, SINGH 1999). In this study the measure of depth of the observation in the sample was used for analysis of the relation described by the APT theory.

The article presents modeling of the correlation between the rates of return and market and sectoral risks of stocks of companies from the SiTech companies sector determined by means of arbitrage pricing theory. In the analysis conducted non-typical observations were eliminated by applying the method based on the Mahalanobis depth measure for depth of observations in the sample and the method using the values of standardized residues in linear regression (DOMAŃSKI, PRUSKA 2000).

Arbitrage Pricing Theory Model

The arbitrage pricing model is the theory introduced without restricting assumptions concerning the ideal capital market on which the CAPM model was based. There are no strong assumptions concerning the function of the investor's wealth usefulness. Additionally, it is not assumed that the investors take their decisions on the basis of two parameters: the expected revenue and the risk. The initial assumption of the model is that the rates of return on the securities in the market are generated by multifactor linear model in the form of (ELTON, GRUBER 1998):

$$R_{it} = \alpha_i + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \ldots + \beta_{id} f_{dt} \ (i = 1, \ldots, n) \ (t = 1, \ldots, T) \quad (1)$$

where:
R_{it} – rate of return on the i security during the period t,
α_i – free expression of the model,
β_{il} – load of the l factor for that i security, that is the parameter of sensitivity of the rate of return of that security to the influence of factor l,
f_{lt}, $(l = 1, \ldots, d)$ – factor l, systematically influencing the rates of return of i security during the period t,
ε_{it} – disturbing component representing a specific part of the rate of return on the i security during the period t.

Equation (1) is the process satisfying the following assumptions of stochastic structure:

1. $E(\varepsilon_{it}) = 0, E(\varepsilon_{it}\varepsilon_{jt}) =$ for $i \neq j$, $E(\varepsilon_{it}\varepsilon_{jt}) = \sigma^2$ for $i = j$, 2. $E(f_{lt}\varepsilon_{it}) = 0$.

The first assumption defines the specific components of risk as random variables with zero expected values and non-zero variations and that the random components of the model equations for i security and j security are uncorrelated, which means that the only cause for identical, systematic changes in the rates of return on the securities is their common, similar reaction to unexpected changes of factors. The second assumption concerns independence of systematic and specific factors.

Under the conditions of equilibrium, assuming that the rates of return are generated by the multifactor model described by equation (1), the model resulting from the arbitrage pricing theory assumes the format (HAUGEN 1996):

$$E(R_i) = \overline{R_i} \approx \gamma_0 + \gamma_1\beta_{i1} + \gamma_2\beta_{i2} + \ldots + \gamma_d\beta_{id} \ (i = 1, \ldots, n) \quad (2)$$

where:
γ_0, γ_l $(l = 1, \ldots, d)$ – constant parameters of the equation. The values of parameters γ_l are defined as premiums for the risk caused by factor f_l.

Method for elimination of non-typical observations

While analyzing a numeric data set there is always concern that observations, which do not match the others will appear in the set. Sometimes it is difficult to identify in ex post analysis the cause for the doubtful result and then numerous simple statistical procedures are available that will allow removing the non-typical result or further statistical analysis.

We call an observation a non-typical observation when it does not fit the configuration (core) of the entire set of individual observations. The correlation graph for a two-dimensional sample can present various configurations of points on a plane. In a two-dimensional case the observations can be presented on a plane and the initial, visual analysis of the entire set can be conducted. This can become one of the methods for identification of non-typical observations in two-dimensional sets.

Residues from the estimated linear regression function can be used for detecting non-typical observations. In the theory of linear regression, in addition to typical observations the following types are also identified

- non-typical,
- influential,
- distant from other observations.

A non-typical observation in linear regression is one for which a relatively large residue is obtained

$$e_i = y_i - \hat{y}_i, \ (i = 1,2,..., n) \tag{3}$$

that is one that does not fit within the specified vicinity of the estimated regression line.

Standardized residues can be used for identifying non-typical observation in linear regression

$$\tilde{e}_i = \frac{e_i}{S_e}, \ (i = 1,2,..., n) \tag{4}$$

where:

$\tilde{e}_i$ – standardized residue for observation i,
e_i – regression residue i,
n – number of observations,
S_e – standard deviation of regression residues determined according to the formula

$$S_e = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n - k}} \tag{5}$$

where:
k – defines the number of estimated regression function parameters (DOMAŃSKI, PRUSKA 2000).

Let $P_n^2 = \{x_1, x_2, ..., x_n\}$ be the system of observable vectors expressing a two-dimensional sample with population n originating from a certain two-dimensional distribution defined by the distribution function F_2 and let $\theta \in R^2$ be a certain point in real space R^2. In particular, it can belong to the system of points from sample P_n^2.

The criterion that uses Mahalanobis distance of point x_i relative to the vector of averages $\bar{x}$ is one of the criteria for determining the observation depth in the sample measure.

Mahalanobis depth measure Mzan_2 for point θ in two-dimensional sample P_n^2 is computed according to the following formula:

$$\mathrm{Mzan}_2\ (\theta; P_n^2) = [1 + Q(\theta, P_n^2)]^{-1} \tag{6}$$

where:
$Q(\theta, P_n^2)$ – is the Mahalanobis distance of point θ relative to the vector of averages $\bar{x}$, determined as

$$Q(\theta, P_n^2) = (\theta_1 - \bar{x}_1)^2\, s^{11} + 2(\theta_1 - \bar{x}_1)(\theta_2 - \bar{x}_2)s^{11} + (\theta_2 - \bar{x}_2)^2\, s^{22} \tag{7}$$

while

$$\theta = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix},\ \bar{x} = \begin{bmatrix}\bar{x}_1\\ \bar{x}_2\end{bmatrix},\ \bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j,\ S = \frac{1}{n-1}\sum_{i=1}^{n}(x_j - \bar{x})(x_j - \bar{x})^T,\ S^{-1}\begin{bmatrix}s^{11} & s^{12}\\ s^{21} & s^{22}\end{bmatrix}.$$

The depth measure allows organizing the observations according to the distance from the central concentration, which in this case is represented by two-dimensional median vector. The observation that corresponds to the highest value of the dept measure determines the two-dimensional median vector. Observation with the higher values of depth measure are positioned more centrally in the sample than those for which the depth measure assumes low values situated outside the "data cloud". Observations with the lowest depth values may be treated as deviating (non-typical).

Characteristics of the data

Analysis of multindex models and tests of correlations between the rates of return on capital investments and the risk expressed by beta index covered the period of three years, 2004–2006. The analysis encompassed observations for 28 continually listed securities belonging to the SiTech segment. The study used time series of monthly rates of returns for the securities (36 observations). The choice of the sample period, stabile as concerns the general positive stock exchange market allows to a certain extent stabile estimation of single index model indexes.

The beta parameters of multindex models were estimated in relation to the major Warsaw Stock Exchange index WIG and teleinformation sector index independent of the WIG index.

Results

For every security the KMNK parameters of the two-index model were estimated. The tested format of that model was as follows:

$$R_{it} = \alpha_i + \beta_{i1} R_{Mt} + \beta_{i2} R_{St} + \xi_{it}, \quad (i = 1,..., n = 28)\ (t = 1,..., T = 36) \quad (8)$$

where:

R_{it} – rate of return for company i during the period t;

R_{Mt}, R_{St} – corresponding rate of return of the WIG index and teleinformation sector index;

α_i, β_{i1}, β_{i1} – model parameters;

ξ_{it} – random component of the model.

Table 1 presents the expected $\overline{R}$ values, standard deviation of the rate of return for the analyzed companies $\hat{\sigma}$, estimated values for parameters $\hat{\alpha}$, $\hat{\beta}_{i1}$, $\hat{\beta}_{i1}$, of models (8), determination coefficients R^2 and results of the Durbin–Watson test (DW).

The majority of analyzed companies achieved the positive average monthly rate of return ranging from 0,094% to 8,368% during the studied period of 2004–2006. Assessment of parameter and corresponding values of t-Student statistics indicate statistical insignificance of that parameter, which is consistent with the theory as parameter α defines the part of the rate of return independent of the market situation.

Parameter β_1 indicates the degree of sensitivity of the rate of return for the stocks of a given company to changes in the market rate of return. Companies

Table 1

Assessment of two-index model indexes for SiTech segment companies listed during the period of 2004–2006

Abbreviation	Full company name	$\overline{R}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	R^2	Test DW
ABG	ABG	-1.717	12.087	-3.844 (-1.92^c)	0.817 (2.22^b)	0.961 (2.79^a)	0.28	1.608
ACP	ASSECOPOL	4.303	7.984	2.072 (1.58)	0.890 (4.02^a)	0.631 (2.63^b)	0.47	1.862
ATM	ATM	8.368	12.272	7.304 (2.75^b)	0.456 (1.02)	0.572 (1.18)	0.08	2.203
BCM	BETACOM	-0.622	12.181	-1.640 (-0.72)	0.521 (1.24)	0.633 (1.55)	0.11	1.473
BMP	BMPAG	1.708	7.993	0.884 (0.50)	0.288 (1.02)	-0.516 (-1.62)	0.17	1.509
CMP	COMP	2.811	9.052	1.189 (0.58)	0.569 (1.73^c)	-0.040 (-0.10)	0.13	1.541
CMR	COMARCH	4.105	9.307	2.572 (1.59)	0.589 (1.98^c)	0.610 (2.21^b)	0.21	1.476
CSS	CSS	1.491	8.908	-0.196 (-0.13)	0.649 (2.33^b)	0.569 (2.18^b)	0.24	1.998
ELZ	ELZAB	4.160	11.525	3.336 (1.59)	0.316 (0.82)	0.742 (2.06^b)	0.13	2.662
EMX	EMAX	0.447	8.854	-0.892 (-0.57)	0.515 (1.79^c)	0.532 (1.98^c)	0.18	2.421
IBS	IBSYSTEM	-1.129	12.798	-4.314 (-2.28^b)	1.224 (3.54^a)	1.144 (3.54^a)	0.43	2.275
IGR	IGROUP	7.190	23.452	4.695 (1.23)	0.959 (1.37)	2.338 (3.57^a)	0.31	1.687
INT	INTERIA	7.236	18.383	2.694 (0.94)	1.747 (3.34^a)	1.407 (2.88^a)	0.37	2.736
MCI	MCI	7.516	22.749	3.419 (0.85)	1.576 (2.15^b)	1.262 (1.84^c)	0.20	2.189
MCL	MCLOGIC	5.242	15.768	6.640 (2.24^b)	-0.537 (-0.99)	0.668 (1.31)	0.08	2.435
MNI	MNI	4.445	17.553	1.151 (0.49)	1.121 (1.95^c)	0.915 (1.71^c)	0.17	2.169
MTL	MEDIATEL	1.037	22.586	0.082 (0.02)	0.560 (0.65)	1.348 (1.46)	0.09	2.010
NET	NETIA	0.933	7.658	-0.789 (-0.59)	0.662 (2.68^b)	-0.176 (-0.76)	0.19	1.876
OPT	OPTIMUS	-1.165	9.847	-2.455 (-1.72^c)	0.496 (1.89^c)	1.184 (4.83^a)	0.45	1.697
PKM	PROKOM	-0.225	9.439	-3.646 (-2.81^a)	1.315 (5.52^a)	0.384 (1.72^c)	0.50	2.038
SGN	SYGNITY	0.591	7.261	0.058 (0.05)	0.205 (0.88)	0.616 (2.85^a)	0.21	2.314
SME	SIMPLE	4.845	18.222	1.437 (0.50)	1.310 (2.50^b)	1.693 (3.45^a)	0.35	2.507
SPN	SPIN	0.094	12.683	-3.020 (-1.49)	1.197 (3.23^a)	0.868 (2.51^b)	0.33	1.776
TEX	TECHMEX	-0.159	11.075	-1.366 (-0.67)	0.728 (2.02^c)	0.867 (2.30^b)	0.24	2.150
TLX	TALEX	-0.177	13.401	-2.572 (-1.08)	0.920 (2.11^b)	0.673 (1.65)	0.18	2.529
TPS	TPSA	1.624	7.540	-1.224 (-1.20)	1.095 (5.88^a)	-0.240 (-1.38)	0.52	1.770
TVN	TVN	5.189	8.480	2.672 (1.66)	0.884 (3.40^a)	-0.191 (-0.66)	0.38	2.404
WAS	WASKO	2.769	12.757	-0.212 (-0.11)	1.146 (3.20^a)	1.082 (3.22^a)	0.38	2.052

Source: Own computations.
a, b, c – model parameter of test statistic significant at the significance level of: $\alpha = 0.01$; $\alpha = 0.05$; $\alpha = 0.1$ respectively.
In brackets the values of t statistic is given. Critical values for the DW test are $d_l = 1.353$ and $d_u = 1.587$ respectively.

of SiTech segment are characterized by large variability of coefficients β_1 (from -0,53 to 1,747), but the majority of them reacted to the changes in the stock exchange market slower than the market (coefficients β_1 lower than 1). The company with the weakest reaction to market changes was SYGNITY (β_1 = 0,205), while INTERIA company with coefficient β_1 equal to 1,747 was the security most sensitive to such changes. The security reacting in average in the direction opposite to that of the market was MCLOGIC, for which coefficient β_1 was -0,537. In case of 20 companies changes in the WIG index had a statistically significant influence on changes to the rate of return on the stocks of companies studied.

Statistically significant influence of sectoral risk on the individual companies expressed by coefficient β_2 was observed in case of 18 companies. IGROUP (β_2 = 2,338) showed the strongest reaction to changes in the sector market situation while OPTIMUS (β_2) showed the weakest reaction. The values of DW test statistics show satisfying the assumption concerning absence of autocorrelation between the random components for the majority of the estimated models.

The average rates of return for the analyzed companies determined during the first stage and the estimated coefficients beta were used for testing the significance of the coefficients of the APT model empirical form:

$$\overline{R_i} = \gamma_0 + \gamma_1 \hat{\beta}_{i1} + \gamma_2 \hat{\beta}_{i2} + \varepsilon_i; \ (i = 1,..., n = 28) \tag{9}$$

where:

$\gamma_0, \gamma_1, \gamma_2$ – model parameters,
$\overline{R_i}, \hat{\beta}_{i1}, \hat{\beta}_{i2}$ – expected value and coefficients beta for security i,
ε_i – random component of the equation.

The assessments of relation (9) parameters are presented in table 2. The graphic spread of points is presented in Figure 1.

Table 2

Estimations of parameters for relation $\overline{R_i} = \gamma_0 + \gamma_1 \hat{\beta}_{i1} + \gamma_2 \hat{\beta}_{i2} + \varepsilon_i$ for 28 companies of SiTech segment listed during the period of 2004–2006

$\hat{\gamma}_0$	t_{γ_0}	$\hat{\gamma}_1$	t_{γ_1}	$\hat{\gamma}_2$	t_{γ_2}	R^2
1,447	1,23	0,415	0,33	1,035	1,06	0,061

Source: Own computations.

The dispersion of observations in figure 1 shows lack of correlation between the average rates of return and coefficients beta for the highlighted companies. This is confirmed by the results of regression analysis presented in Table 2, which indicate lack of statistical significance of the equation parameters and low level of explanation for the expected rates of return provided by the APT model (R^2 = 0,061).

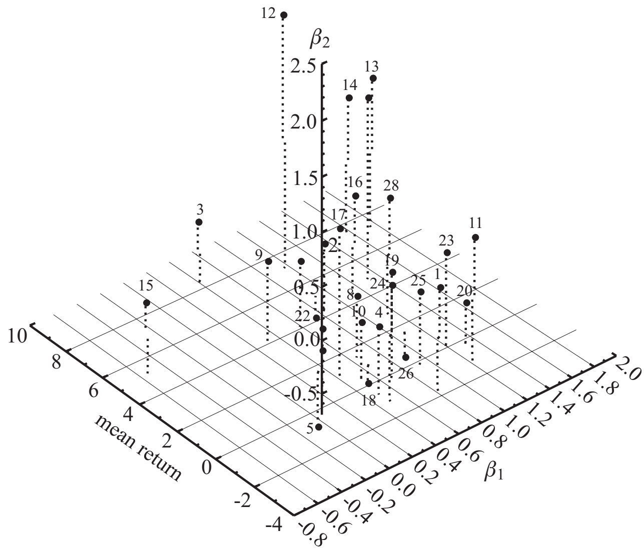

Fig. 1. Values of the average rates of return and coefficients beta for 28 companies of SiTech segment with the line of the securities market

Source: Own work.

Table 3

Values of standardized residues determined for 28 companies of teleinformation sector on the basis of relation (9)

Company abbreviation	Standardized residues	Company abbreviation	Standardized residues
ABG	-1.520	MCL	1.124
ACP	0.627	MNI	0.536
ATM	2.081	MTL	-0.670
BCM	-0.989	NET	-0.205
BMP	0.219	OPT	-1.366
CMP	0.394	PKM	-0.884
CMR	0.602	SGN	-0.533
CSS	-0.275	SME	0.372
ELZ	0.612	SPN	-0.928
EMX	-0.596	TEX	-0.940
IBS	-1.442	TLX	-0.913
IGR	0.988	TPS	-0.010
INT	1.218	TVN	1.203
MCI	1.389	WAS	-0.093

Source: Own computations.

Table 3 presents the values of standardized residues values determined for 28 companies. It can be noticed that for companies ABG, ATM, IBS the values of computed residues were relatively the highest.

For the purpose of computing the Mahalanobis depth measures the 28 objects (companies), characterized by three characteristics, β_1, β_2 and $\overline{R_i}$ were divided into two-element sets: $Z_1 = \{\overline{R_i}\ \beta_1\}$, $Z_2 = \{\overline{R_i},\ \beta_2\}$, $Z_3 = \{\beta_1,\ \beta_2\}$. The three numeric data sets established are treated as two-dimensional samples. For companies belonging to each of those subsets the Mahalanobis depth

Table 4

Organized Mahalanobis depth measures for two-dimensional sets

Company	Mzan (1)	Company	Mzan (2)	Company	Mzan (3)
MCL	0.244053	IGR	0.264297	MCL	0.255042
INT	0.291024	ATM	0.320068	IGR	0.274582
ATM	0.306447	BMP	0.329452	INT	0.323477
MCI	0.308754	TVN	0.335343	BMP	0.328337
IBS	0.374884	MCI	0.362385	TPS	0.334817
IGR	0.385794	INT	0.362403	MCI	0.36873
PKM	0.388881	OPT	0.378043	SME	0.371843
ABG	0.404862	ABG	0.381455	TVN	0.372709
OPT	0.428206	SME	0.382692	MTL	0.400004
SGN	0.429776	IBS	0.385823	NET	0.401858
SPN	0.435846	TPS	0.387759	PKM	0.403121
SME	0.437632	NET	0.402076	CMP	0.443528
ELZ	0.452442	CMP	0.431596	SGN	0.44528
BCM	0.458753	MTL	0.442923	OPT	0.46569
TLX	0.497237	BCM	0.481347	ELZ	0.488105
BMP	0.519959	TEX	0.495246	IBS	0.497353
TEX	0.520073	PKM	0.499404	SPN	0.533303
TVN	0.522292	MCL	0.504853	WAS	0.543868
MNI	0.525009	SPN	0.517274	ATM	0.55251
MTL	0.537782	TLX	0.517953	MNI	0.583325
EMX	0.538323	EMX	0.578754	BCM	0.613151
TPS	0.565578	ACP	0.594645	EMX	0.627562
WAS	0.566381	MNI	0.600058	CMR	0.702944
CMR	0.579003	SGN	0.601882	TEX	0.721481
ACP	0.623861	CMR	0.612644	ABG	0.728868
NET	0.63433	WAS	0.637917	TLX	0.741789
CMP	0.679284	ELZ	0.638252	CSS	0.744343
CSS	0.699277	CSS	0.713819	ACP	0.812514

Source: Own computations.

measures were computed according to formula 4 and they were presented in Table 4 in non-decreasing order.

The lowest depth measure values for two out of three two-dimensional subsets were obtained for companies MCL, ATM and BMP respectively. This means that those companies had much lower or much higher values of the tested variables β_1, β_2 or $\overline{R_i}$. Considering the values of the standardized residues and the values of depth measures computed for each of the two-dimensional samples those companies for which in each of two-dimensional samples low depth measure values were obtained and those with relatively high standardized residues. The eliminated companies were: ABG, ATM, BMP, IBS and MCL. For the remaining 23 companies the structural parameters of the APT model were estimated. Assessments of the model parameters are presented in Table 5.

Table 5

Estimations of relation parameters $\overline{R_i} = \gamma_0 + \gamma_1 \hat{\beta}_{i1} + \gamma_2 \hat{\beta}_{i2} + \varepsilon_i$ for 23 companies of SiTech segment companies listed during the period of 2004–2006

$\hat{\gamma}_0$	t_{γ_0}	$\hat{\gamma}_1$	t_{γ_1}	$\hat{\gamma}_2$	t_{γ_2}	R^2
-0.836	-0.69	2.892	2.28	1.163	1.40	0.325

a, b, c – model parameter of test statistic significant at the significance level of: $\alpha = 0.01$; $\alpha = 0.05$; $\alpha = 0.1$ respectively.
Source: Own computations.

Assessment of the free expression γ_0, reflecting the rate of return free from risk gave a negative result and it statistically insignificantly differs from zero. On the other hand, coefficient γ_1, expressing the market premium for risk related to stock exchange market situation proved statistically significant. Its value means that the increase in the risk of the individual company (coefficient β_1) by one percent point corresponds to the average increase in the expected monthly rate of return by 2,892%, ceteris paribus. Assessment of parameter γ_2, determining the premium for risk related to the teleinformation sector market situation proved statistically insignificant.

The determination coefficient at 0,325 is much higher than the value of that coefficient for estimations made prior to elimination of companies (0,061). Nevertheless, its still low value indicates that the version of the two-index APT model expressed by the equation (9) is insufficient for description of the correlation between the expected profit of portfolios and the systematic risk.

Conclusion

Results of the analysis show that elimination of non-typical observations facilitates improvement of the matching of the model to the empirical data. Regression analysis of the APT model for innovation technology segment companies supported by the methods of non-typical observations elimination indicates partial significance of the relations described by the model. SiTech segment companies are characterized by positive and statistically significant premium for systematic risk expressed by the stock exchange market situation. On the other hand insignificant influence of sectoral risk on the expected rates of return for the analyzed companies was determined. Relatively low value of the determination coefficient indicates that the level of expected rates of return on analyzed assets does not result from the sensitivity of those securities to changes of the stock exchange index and sectoral index only. The other potential exogenous factors in relation to the capital market influencing the general stock exchange market situation and the levels of listed prices for individual securities could include macroeconomic variables such as inflation, global production or interest rates and indexes of global stock exchanges describing the trends in the global capital market.

Translated by Jerzy Gozdek

Accepted for print 12.05.2009

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