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**POSSIBLE APPLICATIONS OF NETWORK METHODS  
TO OPTIMALIZATION OF CERTAIN ASPECTS  
IN THE CONSTRUCTION INDUSTRY MANAGEMENT**

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**Key words:** construction industry, construction industry management, decision problems, theory of networks and graphs.

**A b s t r a c t**

The construction industry, in a broad scope of works and services it renders, encounters many problems where optimization of an outcome is needed. At all the stages of a construction project, such as planning, designing and building, there are many problems which are difficult to solve directly. However, they can be handled with an aid of mathematical methods designed to support decision making processes. Such processes, apart from being space- and time-dependent, are very complex as they involve many factors which condition success or failure of a given task. Hence, methods based on the network theory are very useful. Decision-supporting network methods are applicable at any stage of a construction project. This paper presents how the theory of networks can be applied to solving certain decision problems in the construction industry.

**MOŻLIWOŚCI ZASTOSOWANIA METOD SIECIOWYCH W OPTYMALIZACJI  
NIEKTÓRYCH ZAGADNIENI Z DZIEDZINY ZARZĄDZANIA W BUDOWNICTWIE**

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**Sł o w a k l u c z o w e:** budownictwo, zarządzanie w budownictwie, problemy decyzyjne, teoria sieci i grafów.

**A b s t r a k t**

W szeroko pojętej działalności budowlanej istnieje wiele zagadnień wymagających optymalizacji efektów podjętej działalności. Zarówno na etapie planowania i projektowania, jak i w trakcie realizacji zaprojektowanych obiektów, pojawia się wiele problemów, które trudno rozwiązać bezpośrednio.

Można je rozwiązać, stosując matematyczne metody wspomaganie procesów decyzyjnych. Bardzo przydatne są metody oparte na teorii sieci, ze względu na złożoność procesów, wiele czynników warunkujących powodzenie przedsięwzięć, ich przestrzenny charakter oraz fakt, że procesy te przebiegają w czasie. Są one możliwe do wykorzystania na każdym etapie działalności inwestycyjnej. W artykule przedstawiono możliwości wykorzystania teorii sieci do rozwiązania niektórych problemów decyzyjnych w budownictwie.

## **Introduction**

Typically, problems occurring while designing a construction project can be solved in a variety of ways, i.e. the expected aim can be attained via alternative solutions (variants). Thus, the designing stage often includes the question of selecting a proper design variant. A similar task of choosing a variant solution will arise in the pre-design phase, and the choice of one of the variants will often have a significant influence on the economic effects of the project based on the solution chosen.

The need to choose a variant solution occurs also during the construction works, when it is necessary to select, for example, a building materials supplier, building equipment and tools.

A typical example of optimization during a decision making process in the construction industry is the question of locating industrial plants which make building materials (SZAFRANKO 1999). The construction and building materials industry is one of these branches of industries which are vulnerable to space-related conditions, with the issue of localization being one of the major aspects. It is so because the construction industry uses large quantities of building materials, and some of the construction materials, such as prefabricated components or construction elements, can be huge and heavy, which means they need to be transported using heavy-duty, specialist vehicles. Consequently, wrong location (not optimal) of construction materials factories can result in the costs of transport being exponentially high versus the value of the building materials (SZAFRANKO 1999).

Another common problem is the choice of a land plot on which to raise a designed object (SZAFRANKO 1999). Wrong location can add to the costs of the project. As before, several variant solutions need to be considered including all possible factors which can affect the final outcome.

All the above problems have some features in common. They all involve the need to choose one, potentially the best solution, and the choice relies on a number of factors, for example constraints regarding the availability of resources (materials, means, land, etc.). Besides, there can be various dependencies between particular variants, such as the execution of certain elements of one variant exclude others and vice versa – the realization of some other elements does not make sense unless other elements are realized too.

What is characteristic of all these problems is that most of them occur at a specific time and can be modified during an ongoing construction project. When a project comprises large objects or compounds of buildings, for practical reasons the whole undertaking is divided into stages, each of which is designated specific execution requirements (KORZAN 1978).

For such projects, the best decision supporting method seems to be the one which will take into account the dynamic character of the problem and allow the decision makers to observe the effects of potentially possible modifications of the processes while they are being executed. At the same time, it should make it possible to consider certain restricting conditions and logical dependencies. It seems that a method based on the theory of traditional networks and neural networks, the latter being increasingly more frequently recommended in the relevant literature, is an answer (FORD, FULKERSON 1967).

An example of the application of network theories to an optimization task can be presented as an investment project consisting of several objects, which are constructed over a certain time period as the need arises. Each object is a separate entity in the whole construction project and can be built and opened independently from the others. As the expected technical parameters change while buildings are being used, several stages can be distinguished. It is assumed that the number of stages and their duration are known. All the objects which are put to use at a given stage should be characterized by the technical parameters no lower than the minimal requirements specified for this stage.

The whole investment project can be realized in a number of ways and it is possible to use any strategy that will fulfill the demands of the subsequent stages. Each project execution method is different from the others in costs and cost distribution during the execution of the project.

### **Formulation of a problem using the network theory**

Preparation and development of land parcels is a process of certain duration. The decisions made during a land development project are affected by such factors as the growth rate in residential space demand, which is a consequence of effective demand, or a time when particular land plots are made available for development. Another important issue is the distribution of capital expenditures and cost of the capital.

The time factor is included by dividing the time horizon into several sections (stages), not necessarily equal in length. The number and duration of such stages depend on the forecasted growth rate of the town, the growing demand for residential space and the corresponding demand for land available

for development. All these values are time distributed and estimated for each time period. In every time period, the number of wanted flats is determined. The land plots available for development can be used completely or in part, in one or a few stages. It is assumed that the land plots will be made available for development at the beginning of each stage. The realization time is the time moment corresponding to the half-time of a subsequent stage.

### Designations

- $k = \overline{1, m}$  – subsequent number of a project execution stage,  
 $T_0$  – onset of the project execution,  
 $T_k$  – termination of stage  $k$  and onset of stage  $k+1$ ,  
 $T_m$  – termination of the project execution (time horizon of the analysis)  
 $Q_k$  – number of flats to be built during stage  $k$   
 (in the time period  $T_{k-1} \div T_k$ )  
 $j = \overline{1, n}$  – subsequent numbers of land plots  
 $q_j$  – number of flats which can be built on land plot  $j$ ,  
 $d_j$  – fixed costs of making available land plot  $j$ ,  
 $c_j$  – variable costs related to building up land plot  $j$ ,  
 $d_j^k$  – costs of making available land plot  $j$  at stage  $k$ ,  
 ( $d_j^k = d_j \alpha_{T_{k-1}}$ ),  
 $c_j^k$  – variable costs of building up land plot  $j$  at stage  $k$   
 ( $c_j^k = c_j \alpha_{T_k}$ ),  
 $x_{jl}^k$  – number of flats built on land plot  $j$  during stage  $l$  when the land plot was made available at stage  $k$ ,  
 $T_k'$  – contractual time for variable cost accounting at  $k^{\text{th}}$  stage of the project realization,  
 $y_j^k$  – binary variable which determines the intensity of preparing land plot  $j$  for development during stage  $k$ .

### Assumptions

A complete urban development programme is expected to be realized in a long time perspective, e.g. several years, which means that land plots to be developed can be made available to contractors successively. For the evaluation of particular variant solutions and, ultimately, choosing the optimum one, it is important to know when particular land plots will be made available for development and how the capital outlays will be distributed in time.

In order to take into account the time factor, the project realization time is divided into  $m$  stages. Stage  $k$  is between  $T_{k-1}$  and  $T_k$  moments of time. For each

stage, the demand for flats is determined. At stage  $k$  ( $k = 1, m$ ) it is  $Q_k$ . It is assumed that the investment project begins at  $T_0$  moment and terminates at  $T_m$ . Any land plot, for example land plot  $j$ , can be made available for development at any moment in the time period  $(T_0, T_m)$  (KUBALE 2002, TARAPATA 2003).

Another assumption is that the moment any stage, e.g.  $k^{\text{th}}$  stage, begins is the latest moment for making the land available for development at this stage. As it is not economically valid to prepare land plots for development earlier than that, it is assumed that land plots are made available for development at the early time of each project realization stage, that is at  $T_k$  ( $k = 0, m-1$ ) moments. A plot which is made available at stage  $k$  can be used for development at all the later stages which remain until the termination of the project. Thus, plot  $j$  is available at  $T_0$  moment and can be successively developed and built up throughout the whole investment project execution time. The same land plot made available at stage  $k$  can be used at all the successive stages.

It is assumed that each land plot can be made available at any of the possible time moments. The dates when the land plots are made available for development will affect the fixed costs and the dates when they are built up will influence the variable costs. The effect of time on costs is included in the analysis using the discount technique. For comparison of the outlays in different time periods, the starting moment of the investment project execution  $T_0$  is taken as a reference.

For determination of the discount factors, it is assumed that the all-in outlays involved in making the land plot available for development at stage  $k$  are incurred at the moment when this stage begins, that is at  $T_{k-1}$  moment. Thus,

$$d_j^k = \alpha_{T_{k-1}} d_j \quad (1)$$

where:

$\alpha_{T_{k-1}}$  – discount factor calculated for  $T_{k-1}$  moment

$$\alpha_{T_{k-1}} = (1 + r)^{-T(k-1)} \quad (2)$$

$r$  – cost of the capital involved,

$d_j$  – capital expenditures (in nominal value) on preparing the land plot for development,

and:

$$c_j^k = \alpha_{T_K} c_j \quad (3)$$

where:

$T_K$  corresponds to the half duration of the stage.

We designate the following as:

$x_{jl}^k$  – extent of the development of land plot  $j$  at the  $l^{\text{th}}$  stage of the project execution when the plot was made available for development during the  $k^{\text{th}}$  stage

$$(k = 1, m.) (l = k, m.)$$

As it is not possible to build flats on a given land plot in excess of its capacity, the following conditions are given:

$$\sum_{l=k}^m x_{jl}^k \leq q_j \quad (k = \overline{1, m.}) (j = \overline{1, n}) \quad (4)$$

Each land plot can be made available for development no more than once (at one specific time) and built up in only one way, therefore particular dates of making this land plot available exclude each other. To account for this, each possible time moment of making each of the land plots available is designated binary variables  $y_j^k$  : defined as follows:

$$y_j^k = \begin{cases} 1, & \text{when the land plot is prepared for development at stage } k \\ & \text{and consecutive stages} \\ 0, & \text{when it is not} \end{cases}$$

The condition stating that different dates of making the same land plot available are mutually exclusive can be formulated as:

$$\sum_{l=k}^m y_j^k \leq 1 \quad (j = \overline{1, n}) \quad (5)$$

The problem consists in defining particular dates when the land plots are to be made available and precise determination of the development rate in such a way as to minimize the outlays, brought down to comparable values, on the preparation and development of the land plots.

### The mathematical model of the problem

The above problem can be expressed in the form of the following mathematical programming (KORZAN 1978):

Determine the values of the variables  $y_j^k$  and  $x_{jl}^k$ , for which the objective function is:

$$\sum_{j=1}^n \sum_{k=1}^m y_j^k d_{jk} + \sum_{j=1}^n \sum_{k=1}^m c_j^k x_{jl}^k \rightarrow \min \quad (6)$$

and the set of restrictions is met:

$$\sum_{j=1}^n \sum_{k=1}^m x_{jl}^k \geq Q_l \quad (l = \overline{1, m.}) \quad (7)$$

$$\sum_{k=1}^m x_{jl}^k \leq q_j \quad (k = \overline{1, m.}) \quad (j = \overline{1, n}) \quad (8)$$

$$\sum_{k=1}^m y_j^k \leq 1 \quad (j = \overline{1, n}) \quad (9)$$

$$x_{jl}^k \geq 0 \quad (j = \overline{1, n}) \quad (l = \overline{1, m}) \quad (k = \overline{1, m.}) \quad (10)$$

$$y_j^k \in \{0, 1\} \quad (j = \overline{1, n}) \quad (k = \overline{1, m.}) \quad (11)$$

The conditions (7) guarantee the necessary supply of land for construction at each stage; conditions (8) guarantee that the capacity of any of the land plots will not be exceeded in terms of the number of flats built, and the remaining conditions describe logical requirements.

Model (6) – (11) will be large in size even for a relatively small number of potential land plots and number of stages in an investment project, and that means that any attempt at solving the model with mathematical programming methods will be cumbersome. Finding a solution will be much easier if network theory based methods are applied.

Figure 1 illustrates a network which comprises a set of all possible variants for the development of land plot number  $j$  ( $j = \overline{1, n}$ ). In this paradigm all possible dates of making the land plot available for development are treated as a potential store. All stores are identified by numbers corresponding to the stages in the project execution. The number of a potential source is identical to the lowest number of the stage in the project execution when this plot can be used for construction. All the stores are preceded by a common source

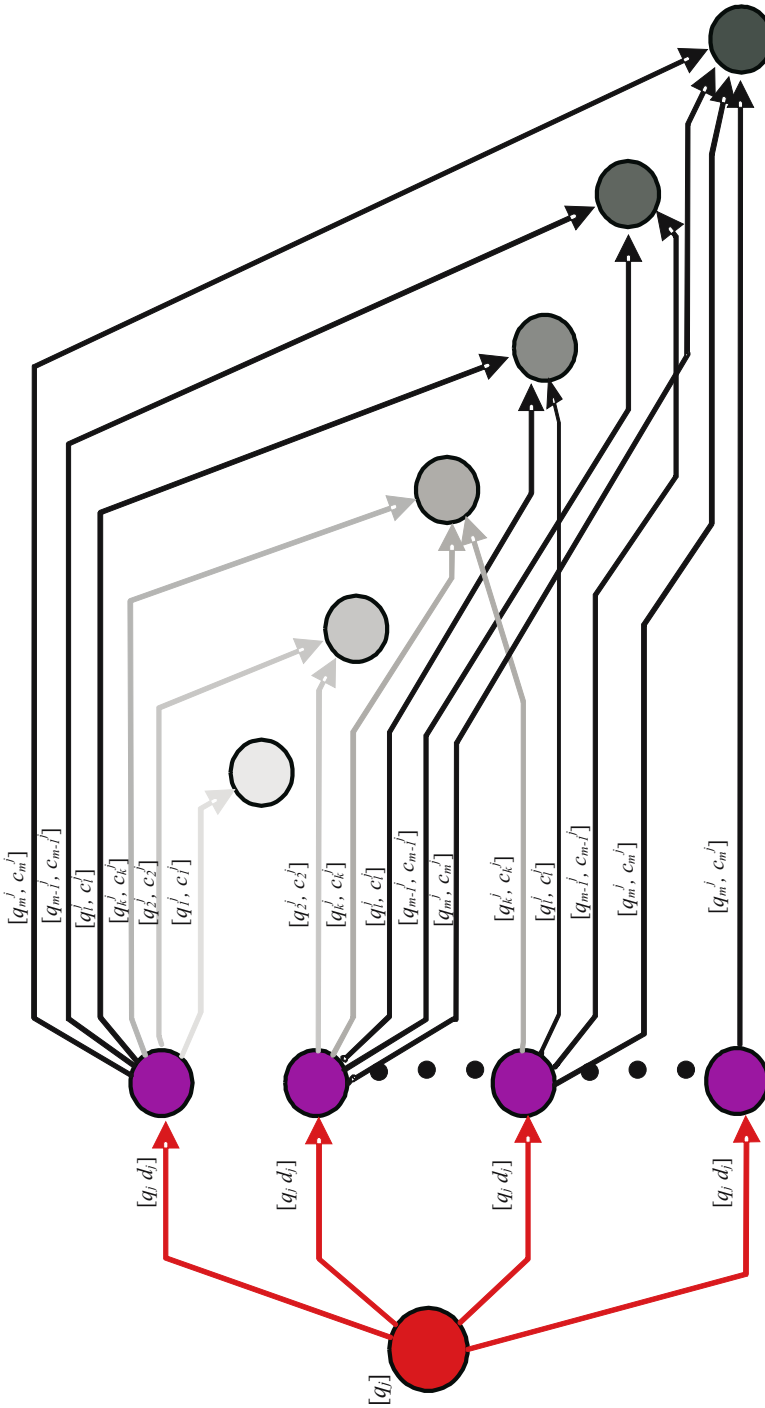


Fig. 1. A set of variants for development of a land plot in the investment project execution



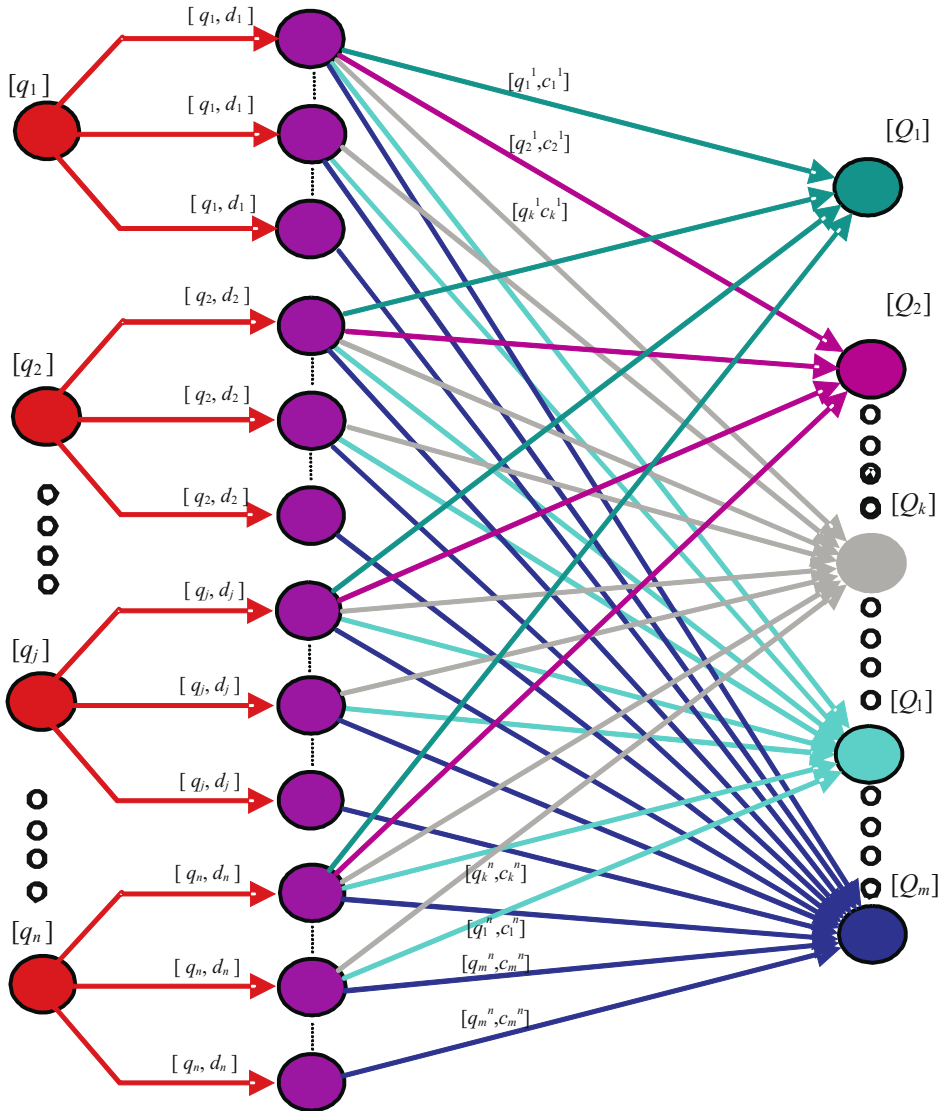


Fig. 2. The decision network including time factor

of efficiency marked as  $q_j$ . The arcs joining the source with the stores are designated bandwidth  $q_j$  and discounted value of making the land plot available  $d_j^k$ . The demand for flats at each stage is interpreted as outflows in the network of expenditures  $Q_k$ . Each store is connected by arcs with all the outflows of the same and higher order numbers. The bandwidth of each arc is

$q_j$  and a unit transport cost equals  $c_j^k$ , i.e. the discounted value of unit variable costs of developing the land plot.

The network illustrated in Figure 1 and expanded by inclusion of possible variants of using the other land plots can be seen in Figure 2. The determination of a satisfying flow at the minimum cost in this network will correspond to the determination of the time when these land plots should be developed and the extent of the construction works.

## Conclusions

As a formal concept, a network can serve as a natural and illustrative model for many problems. By presenting an investment project in the form of a graph it is possible to analyze the inner structure of a problem we need to deal with. In addition, in this case a solution to the problem can involve algorithms based on the theory of graphs, which are usually more effective than algorithms based on other mathematical theories. Besides, they simplify and accelerate the calculations. Proper formulation of the data enables decision makers to answer many questions. Most importantly, network-based paradigms can determine the sequence in which land plots should be developed so as to respond to the demand for housing space. Besides, they facilitate ongoing control of the size and structure of expenditures incurred by a given project. By describing arcs in a network with a cost function, it is possible to account for all the expenses which relate to the land utility criteria and economic conditions.

To recapitulate, it needs to be stated that methods based on the theory of networks and graphs can be an efficient and convenient tool, which will support decision makers while planning an investment project.

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