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ASSESSMENT OF ACCURACY OF EGM08 MODEL OVER THE AREA OF POLAND

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K e y w o r d s: EGM08, quasigeoid heights, ellipsoidal heights, normal heights.

A b s t r a c t

The paper presents the evaluation results for the new Earth Gravitational Model (EGM08) that was recently released by the US National Geospatial Intelligence Agency, using GPS and normal heights from precise levelling in the area of Poland. Detailed comparisons of quasigeoid heights obtained from the EGM08 model and other combined global geopotential models with GPS/levelling data have been performed in both absolute and relative sense. The test network covers the entire part of the Poland territory and consists of 360 sites which belong to the Polish national primary triangulation network, with direct levelling ties to the Polish vertical reference frame. The spatial positions of these sites have been determined at cm-level accuracy with respect to ETRF89 during a nation-wide GPS campaign that was organized in the frame of the EUREF activity. Additionally for relative accuracy evaluations of EGM08 model precise GPS/levelling traverse was used. Our results reveal that EGM08 offers a major improvement (more than 80%) in the agreement level among quasigeoidal, ellipsoidal and normal heights over the area of Poland, compared to the performance of previous combined geopotential models for the same area.

OCENA DOKŁADNOŚCI MODELU EGM08 NA OBSZARZE POLSKI

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S ł o w a k l u c z o w e: EGM08, odstepy, odstępy quasi-geoidy, wysokości elipsoidalne, wysokości normalne.

A b s t r a k t

W pracy podano wyniki oceny jakości nowego modelu geopotencjału Ziemi jaki ostatnio udostępnił US National Geospatial Intelligence Agency. Ocenę bezwzględnej i względnej dokładności modelu przeprowadzono przez porównanie odstępów quasi-geoidy wyliczonych z modelu z odstępami uzyskanymi z satelitarnych pomiarów GPS i niwelacji precyzyjnej. Bezwzględna dokładność modelu była testowana na satelitarnej sieci GPS, która pokrywa równomiernie obszar kraju. Sieć ta składa się z 360 punktów należących do krajowej podstawowej sieci triangulacyjnej. Trójwymiarowe współrzędne kartezjańskie punktów tej sieci wyznaczono w układzie ETRF89 z pomiarów satelitarnych GPS podczas kampanii zorganizowanej w ramach Podkomisji EUREF. Względna dokładność modelu była testowana na precyzyjnym trawersie pomierzonym techniką GPS i dowiązanym do sieci niwelacji precyzyjnej. Uzyskane wyniki pokazują, że model EGM08 daje znacznie lepszą zgodność (ponad 80%) między odstępami quasi-geoidy, wysokościami elipsoidalnymi i wysokościami normalnymi niż poprzednie modele.

Introduction

The earth gravity model EGM08 developed and released by the National Geospatial Intelligent Agency is a significant achievement in global field mapping. For the first time in modern geodetic history, a spherical harmonic model complete to degree and order 2159, with additional spherical harmonic coefficients extending up to degree 2190 and order 2159, is available for the representation of the earth's gravitational potential. This new model make possible with high spatial sampling resolution computation of mean gravity anomalies, geoid ellipsoid separations and other characteristic of gravity field for the entire globe. This model released in April 2008 for the earth science community is attracting interest of geodesists to assess its actual accuracy with different validation techniques and data sets from different part of globe e.g. (KOTSAKIS et al. 2008).

Similar study were conducted for the previous EGM96 model for the territory of Poland. In (ŁYSZKOWICZ 2003) vertical deflections computed from EGM96 model were compared with "observed" astro-geodetic deflections and the agreement on the level 0.5" was achieved. In (KRYNSKI, ŁYSZKOWICZ 2006) six different GGMs: EGM96, EIGEN-CH03S, GGM01S, GGM02S, GGM02, GGM02S/EGM96 were considered. Three kinds of numerical tests with the use of terrestrial gravity data, GPS/levelling height and quasigeoid models obtained from gravity were conducted. It was found that the best suited GGM model for the area of Poland is GGM02S/EGM96 model.

The main purpose of this paper is to present in details the EGM08 evaluation results that has been carried out for the territory of Poland using GPS and normal heights from levelling. In addition to the evaluation test which was carried out in previous studies (ŁYSZKOWICZ 2003), (KRYNSKI, ŁYSZKOWICZ 2006) another series of numerical tests are presented here for the first time.

Data sets

All the evaluation tests and their corresponding results that are presented in the following sections refer to a network of 360 GPS/levelling sites which covers the entire territory of Poland with a relatively uniform spatial distribution and precise GPS/levelling traverse (see Fig. 1).

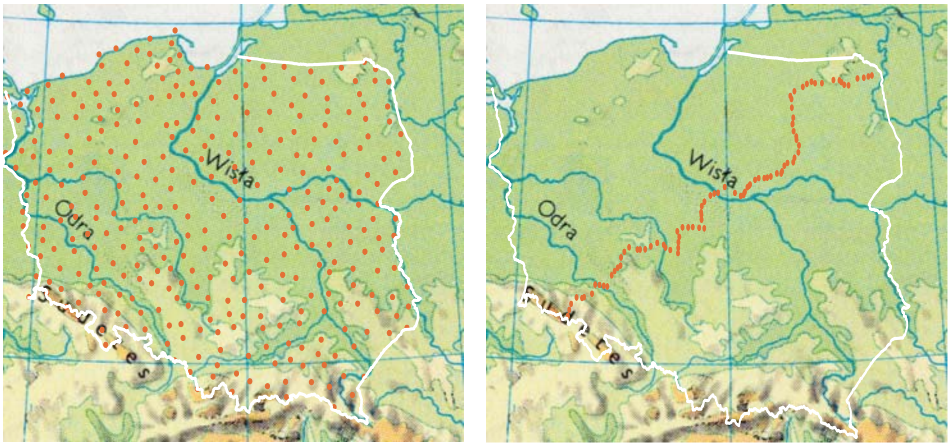

Fig. 1. Spatial distribution of the POLREF (left) and traverse (right) GPS/levelling points over the area of Poland

Ellipsoidal heights. The POLREF network that is a densification of EUREF-POL92 network (11 Polish stations linked in 1993 to ETRF89) consists of 360 sites surveyed in two 4h sessions each, in three campaigns from July 1994 to May 1995 (ZIELIŃSKI et al. 1997). Stations of the POLREF network were located at the sites of primary horizontal control network of Poland and were linked to the national vertical control by spirit levelling (Kronstadt86 datum), with standard deviation of normal height equal to 1.0–1.5 cm (GELO 1994). Standard deviation of ellipsoidal height (GRS80 ellipsoid) is 1.0–1.5 cm (ZIELIŃSKI et al. 1997).

For verification of quasigeoid models developed, as well as for estimation of their relative accuracy, a 868 km long control GPS/levelling traverse across the country has been established (KRYNSKI et al. 2005). The traverse surveyed in 2003 and 2004 in 5 campaigns consists of 190 stations of precisely determined ellipsoidal and normal heights. The stations were located at the benchmarks of the 1st or 2nd order vertical control network, or in their close vicinity. The 49 first order stations of the traverse were surveyed in one or two 24h sessions and remaining 141 stations (as densification points) were surveyed in 4h sessions. The coordinates of 49 the 1st order control stations were determined using the EPN strategy with the Bernese v.4.2 program. Accuracy of the coordinates determined in that way is at the level of single millimeters for majority of stations. The coordinates of densification points were calculated using the

Pinnacle program with the 1st order control stations as reference (KRYNSKI et al. 2005, CISAK, FIGURSKI 2005).

Normal heights. In Poland levelling network was measured three times. Finally levelling network measured in 1974–1982 consists of 135 loops with the average perimeter of about 221 km and total length of levelling lines 17 015 km. The network was connected with neighboring countries and with seven Polish tide gauges.

The levelling lines were measured by automatic levels: Opton Ni 1, Zeiss Ni 002. The following corrections were implemented to the raw data: 1) rod scale corrections, 2) rod temperature corrections, 3) tidal corrections, 4) normal Molodensky corrections. The final adjustment of the entire network was carried out in few versions in 1985. Accepted solution was obtained as a least square approach with stations constrains. Heights of 23 bench marks with their estimated accuracy (from new UPLN solution) was incorporated to the adjustment. After adjustment the standard deviation of height difference is equal ±0.844 mm $\sqrt{d_{\text{km}}}$ and standard deviation of adjusted heights is between ± 6.5 mm and ± 11 mm (WYRZYKOWSKI 1988).

GPS-based quasigeoid height. Based on the known ellipsoidal and normal heights, GPS-based geoid undulations have been computed at the 360 test network POLREF and additionally at 190 points of traverse according to the equation.

$$\zeta^{\text{GPS}} = h - H^n \tag{1}$$

where h is ellipsoidal height from satellite observations and H^n is normal height from levelling. The standard deviation of quasigeoid height ζ^{GPS} computed from GPS observations and normal heights in the case of POLREF network is ±1.4 – 2.1 cm, and in the case of precise traverse is ± 1.3 cm.

GGM-based quasigeoid heights. Quasigeoid heights have also been computed at the 360 POLREF GPS/levelling sites and 190 points of traverse using three different geopotential models. For the evaluation results presented herein we consider only the most recent geopotential models, which have been compiled from the combined contribution of various types of satellite data (CHAMP, GRACE, SLR), terrestrial gravity data, and altimetry data; see Table 1.

Table 1

Geopotential models used for the tests at the POLREF and at the traverse GPS/levelling sites

Model	$n_{\max}$	References
EGM08	2190	PAVLIS et al. 2008
EIGEN-5C	360	FÖRSTE et al. 2008
EGM96	360	LEMOINE et al. 1998

The quasigeoid heights were computed from the general formula.

$$\zeta(r,\phi,\lambda) = \zeta_0 + \frac{GM}{r\gamma} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)\, P_{nm} (\sin\phi) \quad (2)$$

where C_{nm}, S_{nm} are fully normalized spherical harmonic coefficients of degree n and order m, $nmax$ is the maximum degree of geopotential model, GM is product of the Newtonian gravitational constant and mass of the geopotential model, r, ϕ, λ are spherical polar coordinates, a is the equatorial radius of geopotential model and P_{nm} are the fully normalized associated Legendre'a functions.

The term ζ_0 is the zero term due to the difference in the mass of the Earth used in IERS Convention and GRS80 ellipsoid. It is computed according to the well known formula.

$$\zeta_0 = \frac{GM - GM_0}{R\gamma} - \frac{W_0 - U_0}{\gamma} \quad (3)$$

where the parameters GM_0 and U_0 correspond to the normal gravity field on the surface of the normal ellipsoid. For the GRS80 ellipsoid we have $GM_0 = 398\,600.5000 \times 10^9$ m^{3}s^{-2} and $U_0 = 62\,636\,860.85$ m^{2}s^{-2}. The Earth's parameter GM used in quasigeoid computation from geopotential models and the constant gravity potential W_0 on the quasigeoid according to IERS Conventions have been set to the following values: $GM = 398\,600.4415 \times 10^9$m^{3}s^{-2}, $W_0 = 62636856.00$ m^{2}s^{-2}. The mean Earth radius R and the mean normal gravity γ on the reference ellipsoid are taken equal to 6 371 008.771 m and 9.798 m s^{-2} respectively (GRS80 values). Based on the above conventional choices, the zero degree term from equation (3) yields the value $\zeta_0 = -0.442$ m, which has been added to the quasigeoid heights obtained from the corresponding spherical harmonic coefficients series expansions of all geopotential models.

The numerical computations for the spherical harmonic values of ζ from the various GGMs have been performed with the *geocol* software program that was kindly provided by dr Gabriel Strykowski from Danish National Space Center. The final GGM quasigeoid heights computed from equation (2) refer to the tide free system, with respect to a geometrically fixed reference ellipsoid (GRS80).

Height data statistics. The statistics of the individual height datasets that will be used in our evaluation tests are given in Table 2. Note that the statistics for the GGM quasigeoid heights refer to the values computed from equation (2) at the 360 points of POLREF network sites using full spectral resolution of each model.

Table 2

Statistics of the height datasets over the test network of POLREF GPS/levelling points (in meters)

	Mean	Std dev	Min.	Max
h	220.869	141.889	28.878	1645.515
H	186.763	140.059	-0.372	1601.822
N^{GPS} = h-H	34.106	4.435	27.078	43.733
N_{EGM96}	34.144	4.490	26.841	43.882
N_{EIGEN5}	34.001	4.453	26.948	43.723
N_{EGM08}	33.982	4.435	26.975	43.673

From the following table, it is evident the existence of a discrepancy from -10 up to + 4 cm (second column) between the zero reference surface of the Kronsztad86 vertical datum (which is associated with an unknown W_0 value) and the equipotential surface of the Earth's gravity field that is specified by the conventional value W_0 = 62 636 856.00 m^2 s^{-2} and realized by the various GGMs over the area of Poland.

Evaluation tests after a simple bias fit

A series of geopotential models evaluation tests was performed based on the point values for the ellipsoidal and normal heights in the test network. The statistics of the differences between the GPS based and the geopotential models quasigeoid heights are given in Table 3. In all cases, the values shown in this table refer to the statistics of the original misclosures h-H-ζ at the 360 POLREF GPS/levelling sites.

Table 3

Statistics of the original residuals $\zeta^{GPS} - \zeta$ at the POLREF GPS/levelling sites (in meters)

	Mean value (bias)	Std dev	Min.	Max
EGM96 (n_{max} = 360)	-0.038	0.190	-0.542	0.572
EIGEN5 (n_{max} = 360)	0.105	0.113	-0.224	0.520
EGM08 (n_{max} = 2190)	0.125	0.036	0.035	0.260

The differences in the estimated bias obtained from each model (Tab. 3) indicate the existence of regional distortions among the various GGM quasigeoids that are likely caused by long and medium wavelength errors in their original spherical harmonic coefficients. Furthermore, the magnitude of the estimated bias between ζ^{GPS} and ζ suggests that there is a visible offset between

– the equipotential surface corresponding to the IERS conventional value W_0 = 62 636 856.00 m^{-2}s^{-2} and realized by the various geopotential models over the territory of Poland,
– the vertical datum zero height reference surface that is realized through the GPS quasigeoid heights ζ^{GPS}, appears to be located 12 cm (approximately) above the levelling quasigeoid.

From the results given in the Table 3, it is evident that EGM08 offers a remarkable improvement in the agreement among ellipsoidal, normal and quasigeoid heights throughout the territory of Poland. Compared to the two previous geopotential models, the standard deviation of the EGM08 residuals $\zeta^{GPS} - \zeta$ over the entire test network decreases by a factor of 5 from ±19 cm to ±3.6 cm and the major contribution comes from the ultra-high frequency band 360 < n < 2190 of EGM08 model. Obtained results are remarkably good, because analogous investigations in the area of Greece show the only threefold decrease of the standard error (KOTSAKIS at al. 2008).

Table 4

Percentage of the 360 test points whose absolute values of their adjusted residuals $\zeta^{GPS} - \zeta$ (after a least squares constant bias fit) are smaller than some typical geoid accuracy levels

	< 2 cm	< 5 cm	< 10 cm	< 15 cm	< 20 cm
EGM96 (n_{max} = 360)	8.8%	23.3%	43.7%	60.8%	73.5%
EIGEN5 (n_{max} = 360)	16.5%	35.1%	64.3%	81.7%	91.7%
EGM08 (n_{max} = 2190)	43.7%	89.1%	97.9%	100%	–

In Table 4, we can see the percentage of the GPS/levelling sites in the test network whose adjusted residuals h-H-ζ (after the constant bias fit) fall within some standard quasigeoid accuracy level. The agreement between EGM08 and GPS quasigeoid heights is better than 2 cm for more than 40% of the total 360 test points, whereas for the other GGMs the same consistency level is only reached at 16% or less of the test points. Furthermore, almost 98% of the test points give an agreement between the EGM08 geoid and the GPS/levelling data that is better than 10 cm, compared to 64% (or less) in the case of two other global models that were tested.

Additionally evaluation tests of geopotential models was performed on the sites of precise traverse. The statistics of the differences between the GPS-based and the geopotential models quasigeoid heights are given in Table 5. Investigations were conducted for all (190) points and separately for 44 points being characterizing very high accuracy. The results of calculations are presented in Table 5.

Table 5
Statistics of the original residuals ζGPS – ζ at the 190 and 44 GPS/levelling sites of traverse (all values in meters)

	190 sites				44 sites			
	bias	std dev	Min.	max	bias	std dev	Min.	max
EGM96 (n_{max} = 360)	-0.025	0.115	-0.383	0.213	-0.015	0.115	-0.312	0.213
EIGEN5 (n_{max} = 360)	0.078	0.105	-0.270	0.275	0.086	0.106	-0.202	0.274
EGM08 (n_{max} = 2190)	0.075	0.022	0.027	0.126	0.087	0.019	0.043	0.126

From Table 5 appears that there are no essentials differences in the statistic of the evaluated EGM08 model on the base of all 190 and only 44 sites. Compared to the EGM96 geopotential model, the standard deviation of the EGM08 residuals $\zeta^{GPS} - \zeta$ over the traverse decreases by a factor of 6 from ±11.5 cm to ±1.9 cm. It means, that the existing traverse is the most exact combination of measurements GPS and levelling at present and should recommended to the evaluation of new quasigeoid models in Poland.

The horizontal spatial variations of the EGM08 residuals $\zeta^{GPS} - \zeta$ tested on POLREF network do not reveal systematic pattern within the test network. Both in latitude dependent and longitude dependent scatter plots, as shown in Figure 2 and Figure 3 are free of any sizeable north/south or east/west tilts over the area of Poland. In EGM96, however, some strong localized tilts and oscillations can be identified in their $\zeta^{GPS} - \zeta$ residuals, mainly due to larger systematic errors associated with their spherical harmonic coefficients and significant omission errors involved in the quasigeoid computation.

Evaluation results have also confirmed that EGM08 performs exceedingly better than the other models over the mountainous parts of the Poland test network. A strong indication can be seen in the scatter plots of the residuals $\zeta^{GPS} - \zeta$ (after the least squares constant bias fit) with respect to the normal heights of the corresponding GPS/levelling sites, see Figure 4. These plots reveal a height dependent bias between the GGM and the GPS quasigeoid heights, which is considerably reduced in the case of EGM08. Apparently, the higher frequency content of the new model gives a better approximation for the terrain dependent gravity field features over the area of Poland, a fact that is visible from the comparative analysis of the scatter plots in Figure 4.

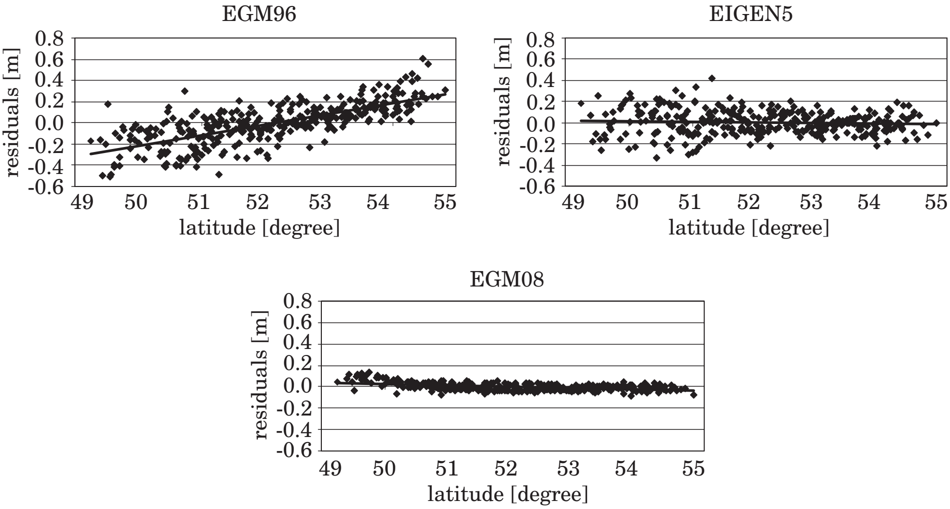

Fig. 2. Latitude dependent variation of the residuals $\zeta^{GPS} - \zeta$ (after a least squares constant bias fit) at 360 points of POLREF network

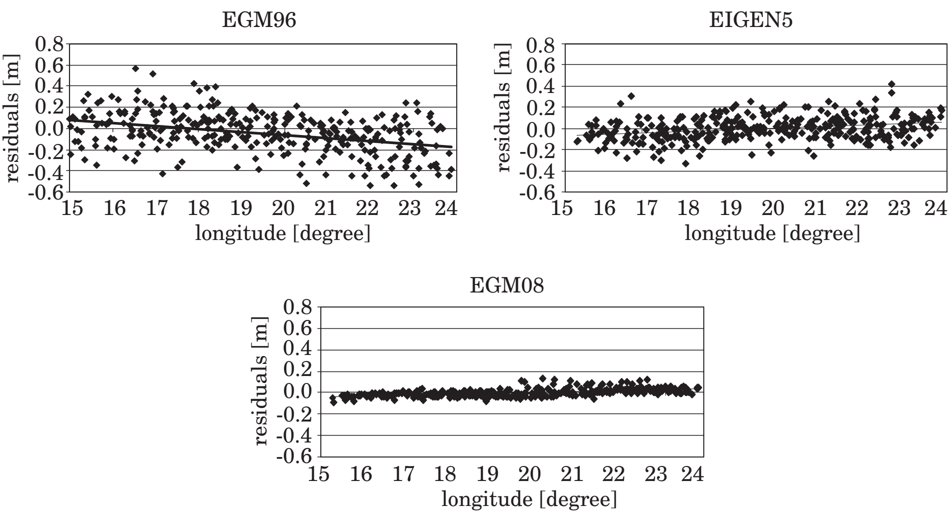

Fig. 3. Longitude dependent variation of the residuals $\zeta^{GPS} - \zeta$ (after a least squares constant bias fit) at 360 points of POLREF network

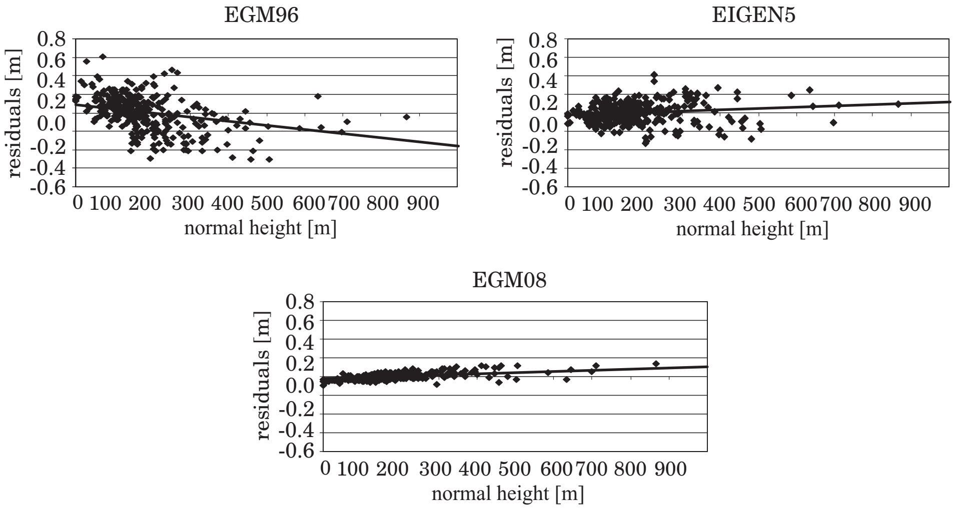

Fig. 4. Height dependent variation of the residuals $\zeta^{GPS} - \zeta$ (after a least squares constant bias fit) at 360 points of POLREF network

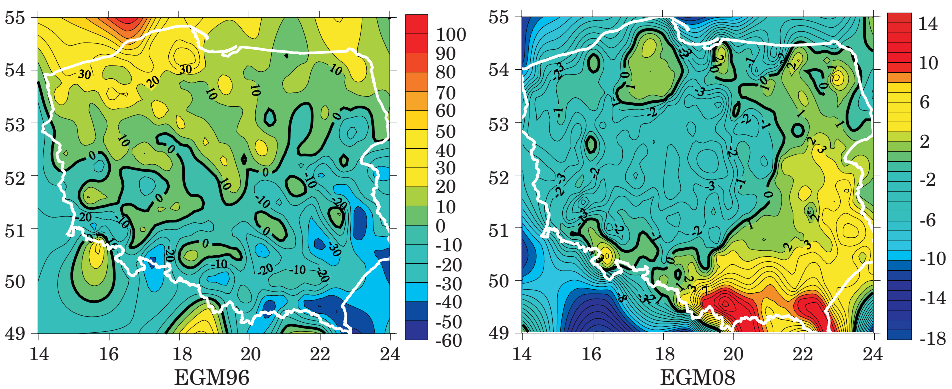

Fig. 5. Geographical distribution of the differences $\zeta^{GPS} - \zeta$ (after a least squares constant bias fit) at 360 sites of POLREF network (in cm)

The spatial distribution of the quasigeoid height residuals for the full resolution model EGM96 and EGM08 is depicted in two separate figures, each with a different color scaling scheme (Fig. 5). From the comparison of the left and right plots, we can verify the overall improvement in the quasigeoid representation over the area of Poland that is achieved with EGM08, compared to the EGM96 model. The achieved improvement is significant i.e. from decimeters to centimeters.

The Figure 5 (right) reveals the remaining inconsistencies with the GPS/leveling data, which are caused by the commission/omission errors of the new model and the systematic local distortions in the normal heights at the test points.

Evaluation tests with different parametric models

The various methods of the fit of gravimetric quasigeoid in Poland to Kronsztadt 86 height datum were study in (KRYNSKI, ŁYSZKOWICZ 2006). The parameters of the model were determined using least squares collocation. In practice, a trend was modelled by a plane function, and the residuals were modelled by a Hirvonen covariance function.

In this paper another series of numerical experiments has been carried out using a number of different parametric models for the least squares adjustment of the differences $\zeta^{GPS} - \zeta$. The motivation for these additional tests was to investigate the fitting performance of some known linear models that are frequently used in geoid/quasigeoid evaluation studies with heterogeneous height data, and to assess their feasibility in modeling the systematic discrepancies between the geopotential models and the GPS based quasigeoid surfaces over the Poland area. These tests were implemented with all three geopotential models that were initially selected for this study.

The various parametric models that have been fitted to the original misclosures h-H-ζ are given in Table 6. Model 1 uses a single constant bias parametric term and it is actually the same model that was employed for all tests of the previous section. Model 2 incorporates two additional parametric terms which correspond to an average north-south and east-west tilt between the geopotential model and GPS quasigeoid surfaces. Model 3 is the usual four parameter model which geometrically corresponds to a 3D spatial shift and an approximate uniform scale change of the geopotential model reference frame with respect to the underlying reference frame of the GPS heights (or vice versa). Finally, models 4, 5 and 6 represent height-dependent linear corrector surfaces that constrain the relation among ellipsoidal, normal and quasigeoidal heights in terms of the generalized equation.

$$h - (1 + \delta_{S_H})H - (1 + \delta_{S_H}) = a_0 \quad (4)$$

The above equation takes into consideration the fact that the spatial scale of the GPS heights does not necessarily conform with the spatial scale induced by the geopotential model quasigeoid undulations and/or the inherent scale of the normal heights obtained from spirit levelling. Moreover, the geopotential model of quasigeoid heights and/or the local normal heights are often affected by errors that are correlated, to a certain degree, with the Earth's topography (see the results in Figures 4 and 5), a fact that can additionally justify the use of model 4 or 6 for the optimal fitting between ζ^{GPS} and ζ.

Table 6

Various parametric models, (Kotsakis 2008)

Number of model	Model
1	$h_i - H_i - N_i = a_0 + v_i$
2	$h_i - H_i - N_i = a_0 + a_1 (\varphi_i - \varphi_0) + a_2 (\lambda_i - \lambda_0) \cos\varphi_i + v_i$
3	$h_i - H_i - N_i = a_0 + a_1 \cos\varphi_i \cos \lambda_i + a_2 \cos\varphi_i \sin \lambda_i + a_3 \sin\varphi_i + v_i$
4	$h_i - H_i - N_i = a_0 + \delta_{SH} H_i + v_i$
5	$h_i - H_i - N_i = a_0 + \delta_{SN} H_i + v_i$
6	$h_i - H_i - N_i = a_0 + \delta_{SH} H_i + \delta_{SN} + v_i$

The statistics of the adjusted residuals v_i in the test network of 360 GPS/levelling points, after the least squares fitting of the previous parametric models, are given in Table 7 and Table 8 for the case of EGM96 and EGM08, respectively.

Table 7

Statistics of the differences $\zeta^{GPS} - \zeta$ for the EGM96 quasigeoid undulations, after the least squares fitting of various parametric models at the 360 GPS/levelling sites (all values in meters)

Model	Bias	Std dev	Min.	Max
1	-0.038	0.190	-0.504	0.609
2	-0.038	0.117	-0.328	0.424
3	-41.713	0.117	-0.317	0.440
4	0.045	0.180	-0.487	0.923
5	0.406	0.181	-0.546	0.587
6	0.329	0.176	-0.525	0.839

Table 8

Statistics of the differences $\zeta^{GPS} - \zeta$ for the EGM08 quasigeoid heights, after the least squares fitting of various parametric models at the 360 GPS/levelling sites (all values in meters)

Model	Bias	Std dev	Min.	Max
1	0.124	0.036	-0.090	0.136
2	0.125	0.026	-0.068	0.106
3	25.279	0.024	-0.081	0.100
4	0.045	0.180	-0.487	0.923
5	0.119	0.036	-0.090	0.134
6	0.154	0.031	-0.240	0.092

From the above results, it can be concluded that the low order parametric models 2 and 3 which are commonly used in the combined adjustment of GPS, geoid/quasigeoid and leveled heights data offer slight visible improvement in the case of EGM08 model for the territory of Poland. Model 3 gives the best results reducing the standard deviation up to ± 2.4 cm in case of EGM08 model.

Table 9

Statistics of the differences $\zeta^{GPS} - \zeta$ for 44 points of traverse (all values in meters)

	Bias	Std dev	Min.	Max
EGM08 (n_{max} = 2190)	0.000	0.015	-0.035	0.035

In order to check the quality of fitting obtained at POLREF sites the quasigeoid computed from EGM08 was fitted to 44 quasigeoid heights at traverse bechmarks using model 3 from Table 6. The results of the fitting are presented in the Table 9. From the results given in the Table 9 appears, that if we have precise GPS measurements on benchmarks the standard deviation of fitting decrease from ±2.4 cm in the case of the POLREF network to ±1.5 cm in the case of the traverse.

Table 10

Percentage of the 44 test points whose absolute values of their adjusted residuals $\zeta^{GPS} - \zeta$ (after a least-squares constant bias fit) are smaller than some typical geoid accuracy levels

	< 1 cm	< 2 cm	< 3 cm	< 4 cm
EGM08 (n_{max} = 2190)	55%	84%	93%	100%

In Table 10, is given the percentage of the GPS/levelling sites in the test traverse whose adjusted residuals h-H-ζ after the model 3 fit fall within some standard quasigeoid accuracy level. The agreement between EGM08 and GPS quasigeoid heights is better than 1 cm for more than 55% of the total 44 test points, whereas for the POLREF network is 40% only. Furthermore, almost 93% of the test points give an agreement between the EGM08 geoid and the traverse GPS/levelling data that is better than 3 cm and 100% test points do not exceeded 4 cm.

All models which are tested in this section include a common parametric term in the form of a single constant bias. However, the various estimates of the common bias parameter a_0 as obtained from the least squares adjustment of each model, exhibit significant variations among each other (see first column in Table 7 and Table 8). Specifically, the estimated bias between ζ^{GPS} and ζ which is computed from the usual four parameter model appears to be highly inconsistent with respect to the corresponding estimates from the other parametric models. This is not surprising since the intrinsic role of the bias a_0 in model 3 is not to represent the average spatial offset between the geopotential models and the GPS quasigeoid surfaces, as it happens for example in the case of model 1. In fact, the three additional parametric terms in model 3 are the ones that absorb the systematic part of the differences ζ^{GPS} - ζ in the form of a three-dimensional spatial shift leaving to the fourth bias parameter a_0 the role of a scale-change effect (KOTSAKIS 2008).

Although less inconsistent with each other, the estimates of the bias parameter a_0 from the other parametric models show dm level fluctuations in their values. It should be noted though that the inclusion of additional spatial tilts (model 2) for the fitting between ζ^{GPS} and ζ does not distort the initial estimate of a_0 that was obtained from model 1 over the Poland territory. On the other hand, the use of height dependent scaling terms (models 4, 5 and 6) affects considerably the final estimates of the bias parameter a_0, as it can be easily verified from the results in Table 7 and Table 8.

The realistic estimate for the average spatial offset between a local vertical datum and a quasigeoid from geopotential models seems to have a strong dependence on the parametric model that is used for the adjustment of heterogeneous height data over a test network of GPS/levelling sites. There is strong theoretical and practical arguments that can be stated in favor of the generalized constraint in model 4, the use of the simple model 1 is not necessarily the safest choice for estimating the average spatial offset between a GGM based and a GPS based geoid over a regional network. In view of the frequent absence (or even ignorance) of a complete and reliable stochastic error model for the properly weighted adjustment of the differences $\zeta^{GPS} - \zeta$, a clear geometrical interpretation of the estimated bias a_0 is not always a straightforward task in GGM evaluation studies, (KOTSAKIS 2008).

Baseline evaluation tests

Besides the absolute evaluation tests that were previously presented, an additional set of evaluation results was also obtained through the comparison quasigeoid slopes from geopotential models with GPS/levelling quasigeoid slopes over the Poland territory along the all 190 GPS/levelling benchmarks of traverse (see Fig. 1). For all baselines formed within this traverse, the following differences of relative quasigeoid undulations were computed.

$$\Delta\zeta_{ij}^{GPS} - \Delta\zeta_{ij} = (h_j - H_j - h_i + H_i) - (\zeta_j - \zeta_i) \quad (5)$$

The computation of the above differences took place after the implementation of a least squares constant bias fit between the point values of the EGM08 geopotential model and GPS quasigeoid heights.

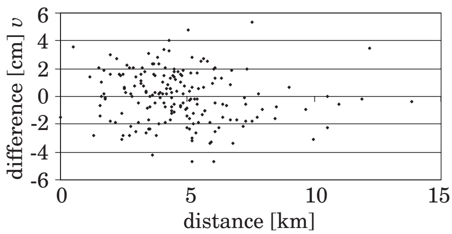

Fig. 6 Differences $\Delta\zeta^{GPS} - \Delta\zeta$ in the test traverse of GPS/levelling 190 benchmarks as a function of baseline length

The residual value computed from equation (5) were sorted in respect to the baseline length and plotted on Figure 6. In our test the baseline length from 0.5 up to 13 km were present for this quasigeoid evaluation scheme. Because of lack appropriate data, differences $\Delta\zeta^{GPS} - \Delta\zeta$ for baselines longer than 10 km were not consider here. The mean value of 190 differences is zero and its standard deviation is ±2 cm. Finally we can assume that for base line up to 10 km standard deviation of quasigeoid slope computed from EGM08 model is constant an equal ±2 cm. It gives the relative accuracy of the order 5×10^{-5}.

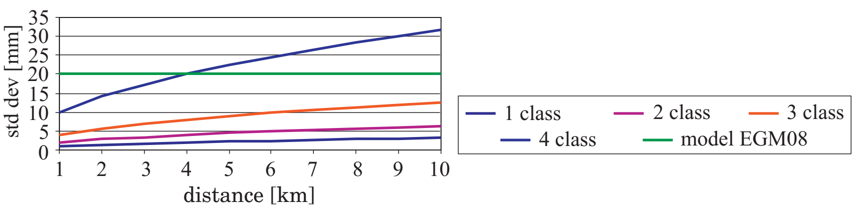

Fig. 7 Expected standard deviation of height differences for first, second, third and fourth class of levelling in Poland and estimated standard deviation (green line) of height differences computed from GPS/EGM08 data

Focusing on the quasigeoid slope evaluation results over short baseline distances up to 10 km can give us an indication for the expected accuracy in GPS/leveling projects when using an EGM08 reference quasigeoid for area of Poland. The preliminary analysis in the test network showed that the agreement between the height differences ΔH_{ij} computed directly from the known normal heights at the GPS/levelling sites and indirectly from the GPS/EGM08 ellipsoidal and quasigeoid heights, can be approximated by the mean error ± 2 cm. Although such a performance cannot satisfy mm level accuracy requirements for first, second and third class of precise levelling it can be satisfied for fourth class (Fig. 7) and provides a major step forward that can successfully accommodate a variety of engineering and surveying applications.

Conclusions

The results of evaluation tests have revealed that the EGM08 model is very accurate over all existing geopotential models for the area of Poland. The accuracy of EGM08 model is comparable with present gravimetric solution which is slightly beneath ± 2 cm. The average inconsistency level between ellipsoidal, normal and EGM08 quasigeoid heights is at the level ± 2.0 cm (Tab. 5) reflecting mainly the regional effects of the remaining commission errors in the models spherical harmonic coefficients, as well as other local systematic errors coming from vertical datum and normal heights. Fitting EGM08 quasigeoid heights to the precise traverse using model 3 slight improvement of accuracy is seen (see Table 9).

In terms of relative geoid accuracy, the EGM08 model evaluated up to 10 km gives value ±2 cm for the standard deviation of the slope residuals $\Delta\zeta^{GPS} - \Delta\zeta$ over all baseline lengths that were considered here and can be satisfied for the fourth levelling class (Fig. 7). It provides a major step forward in variety of engineering and surveying applications.

The results presented here provide a promising evidence for the successful use of EGM08 in future geodetic applications over the area of Poland. However, in view of its possible in the near future using in GPS based leveling projects in Poland, a more detailed analysis with additional interpolation models and "spatial corrector surfaces" for modeling the differences $\Delta\zeta^{GPS} - \Delta\zeta$ is required to achieve cm level consistency for the transformation between GPS/EGM08 and normal heights.

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