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NUMERICAL ANALYSIS OF FLOW BIFURCATIONS IN A CLOSED-OFF CHANNEL

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K e y w o r d s: bifurcation, fluid flow, CFD.

A b s t r a c t

This article describes the results of numerical simulations of fluid flow in a closed-off channel with a plain nozzle. Simulations were performed with the application of the Finite Volume Procedure in the Multi Flower 2D non-commercial application. The main objective of the study was to determine the effect of channel geometry on flow structure. A total of 27 system configurations were analyzed for nine geometry variants and three pressure variants in the feed nozzle. Based on the obtained results, five main flow types were determined, including Hopf bifurcation structures. This paper presents simulation results and it analyzes the observed flow types in view of general bifurcation conditions. The final section of this paper describes the attempts at repeating numerical simulations with the use of the FLUENT commercial application. This simulation makes a reference to an earlier experiment carried out by DYBAN et al. (1971), and it is a continuation of previous research described in detail by BADUR, SOBIESKI (2001).

NUMERYCZNA ANALIZA ZJAWISKA BIFURKACJI W PRZEPŁYWIE PŁYNU PRZEZ ŚLEPO ZAKOŃCZONY KANAŁ

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A b s t r a k t

Artykuł przedstawia badania numeryczne przepływu płynu przez ślepo zakończony kanał z umieszczoną wewnątrz prostą dyszą zasilającą. Symulacje wykonano metodą objętości skończonych z zastosowaniem niekomerycjnego pakietu Multi Flower 2D. Podstawowym celem badań było określenie wpływu geometrii kanału na strukturę przepływu. Badania wykonano dla 27 konfiguracji układu – dziewięciu wariantów geometrii i trzech wariantów ciśnień w dyszy zasilającej. Analiza wyników doprowadziła do wytypowania pięciu głównych struktur przepływowych, w tym również struktur o charakterze bifurkacji Hopfa. W pracy zestawiono wyniki symulacji w formie zbiorczej, a także przeanalizowano zaobserwowane typy przepływów pod kątem ogólnych uwarunkowań bifurkacji. W końcowej części opisano próby powtórzenia symulacji numerycznych za pomocą komercyjnego pakietu FLUENT. Badania symulacyjne nawiązują do eksperymentu (DYBAN et al. 1971) i są kontynuacją wcześniejszych prac, opisanych szczegółowo w artykule (BADUR, SOBIESKI 2001).

Introduction

There are various examples of devices and sites with suddenly expanded parts (pipelines, channels, orifices, valves, gates, faults and many others) which perturb fluid flow and obstruct the bifurcation process. The new, post-bifurcation flow types may be divergent or oscillating in nature (BADUR, SOBIESKI 2001, KURNIK 1997). The flow type observed after the active bifurcation parameter has been exceeded (in most cases, the Reynolds number or the Mach number) is firstly determined by geometry and secondly – by velocity (DYBAN et al. 1971, CHIANG et al. 2001).

Various published sources describe experimental studies and numerical bifurcation simulations in suddenly expanded channels or ducts (Fig. 1). The first group is inclusive of a study carried out by DYBAN et al. (1971) which inspired the simulation analyses presented in this article. The experiment described in the cited study will be discussed in subsequent sections of this paper. The experiments and simulations carried out by CHIANG et al. (2001) are also a highly valuable source of data. They present detailed measurements of basic flow types which were modeled with the use of simulation techniques at a very high level of consistency. Similar experimental and numerical simulation studies have been described by MULINA et al. (2003). The most noteworthy papers in the second group include the work of BATTAGLIA et al. (1997) which delivers a detailed account of a numerical analysis of an expanded channel at various geometry and Reynolds number configurations. A similar study was carried out on a smaller scale by MANICA, DE BORTOLI (2003) to investigate an analogous flow system with the use of a non-Newtonian fluid model. Numerical bifurcation analyses were also performed by Polish researchers, including BADUR et al. (1999).

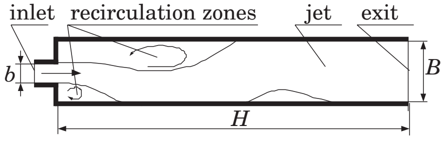


Fig. 1. Bifurcation in a two-dimensional planar channel
Source: BATTAGLIA (1997)

Author investigating numerical modeling of flow structures in suddenly expanded channels or ducts focus mainly on the determination of the critical Reynolds number at which the system is destabilized or bifurcation is observed. DYBAN et al. (1971) is the only researcher to have discussed in detail the forms and structures observed in a flow system following changes in geometry and flow parameters. The absence of in-depth studies investigating the above inspired the author to conduct the experiment presented in this paper, whose main objective was to classify the possible range of bifurcation forms in closed-off channels with a plain feed nozzle. This geometry was applied to compare simulation results with the above cited experiment. Basic numerical analyses of the same experiment were discussed by BADUR and SOBIESKI (2001), including an extensive introduction to the bifurcation theory, a description of the mathematical simulation model and the resulting software application. In the conclusions section, the author compared the results of the experiments and simulations, noting the reported similarities and differences. For this reason, the above aspects were intentionally omitted or only briefly indicated in this article. Detailed information can be found in the above cited study by BADUR and SOBIESKI (2001).

Laboratory experiment

The test stand, as described in the study by DYBAN et al. (1971), for the simulation model developed in this paper is presented in Figure 2. It comprises a closed-off channel with a moving rear wall (*4*) and a feed nozzle through which the fluid is supplied. The nozzle inlet b was adjustable in a range of 5 to 27 [mm]. The maximum channel length *H*, controlled with the movement of the rear wall, was 191 [mm]. Fluid was evacuated from the device via two symmetrical exits (*2*). The casing (*3*) was fixed, and the authors set up two variants of the test stand with the width of the main channel chamber equal to 25 and 51 [mm]. The working medium was air and water with aluminum particles. Flow was observed through the channel's top, transparent lid. Geometry was determined with the use of geometric indicators defined by the *b*/*B* and *H*/*B* ratios.

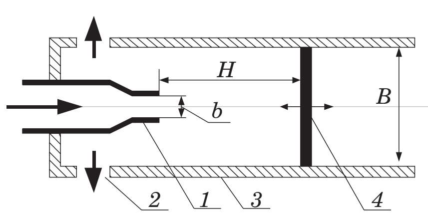


Fig. 2. Test stand: *1* – nozzle (inlet), *2* – exits, *3* – fixed casing, *4* – moving wall
Source: DYBAN et al. (1971)

The authors observed two main types of flow reversal: stationary symmetric flow and non-stationary, asymmetric flow occurring in several variants (Fig. 3). Flow characteristics were determined based on the geometric dimensions of the channel chamber and on the width of the nozzle inlet. The experiment has been discussed in detail and compared with simulation results by BADUR and SOBIESKI (2001).

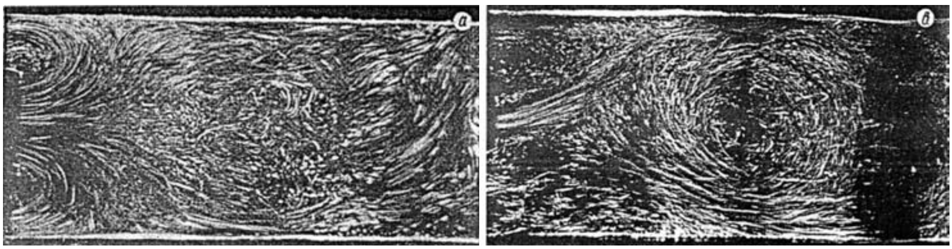


Fig. 3. Symmetric flow (on the left) and asymmetric flow (on the right)
Source: DYBAN et al. (1971)

Computer modeling

Computer-aided simulations involved a series of numerical computations for characteristic cases of channel geometry. The following assumptions were made prior to modeling:

- numerical computations will be performed for three channel lengths and three nozzle widths in a dimensionless system (for H/B = 1.4, 2.8, 4.4; b/B = 0.16, 0.24 and 0.32),
- all numerical computations will be performed for three sets of input parameters (p_c = 0.12, 0.15 and 0.2 [MPa]). The authors of the experiment identified only the range of parameters, not their values,
- the discussed phenomena can be mathematically described with the use of simplified, two-dimensional equations. This assumption was made in view of the experiment's clearly two-dimensional character,
- the numerical experiment will account for only one channel width of B = 25 [mm] to limit the number of simulations,
- the effect of viscosity will be emphasized. The effect of viscosity was computed with the use of the Fluent commercial CFD code.

Three different pressure levels were set at the nozzle inlet for every geometric case: 120000 [Pa], 150 000 [Pa] and 200 000 [Pa]. Pressure at the exits was roughly equivalent to atmospheric pressure at 100000 [Pa]. System temperature was constant at 293 [K]. The working medium was air.

Numerical simulations were carried out with the use of the Multi Flower 2D package (SOBIESKI 2008) developed based on the Finite Volume Procedure and used previously in modeling flows with bifurcations (CUDAKIEWICZ 2005, PUCHALSKI 2008). The following balance equations for mass, momentum, energy and component proportions (BADUR and SOBIESKI 2001, SOBIESKI 2006) were solved during the calculations:

– mass balance: $\frac{\partial}{\partial t}\rho + \nabla(\rho\vec{v}) = 0,$ (1)

– momentum balance: $\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla(\rho\vec{v}\otimes\vec{v}) = \nabla(-p\overleftrightarrow{I} + \overleftrightarrow{\tau})\,\rho\vec{s}_b,$ (2)

– energy balance: $\frac{\partial}{\partial t}(\rho e) + \nabla(\rho e\vec{v}) = \nabla((-p\overleftrightarrow{I} + \overleftrightarrow{\tau})\,\vec{v} + \vec{q}) + \rho s_e,$ (3)

– component balance: $\frac{\partial}{\partial t}(\rho Y_k) + \nabla(\rho Y_k\vec{v}) = \nabla(\vec{J_k}) + \rho s_k,$ (4)

where: ρ – density of mixture [kg m^{-3}], $\vec{v}$ – average velocity of mixture [m s^{-1}], $\overleftrightarrow{\tau}$ – total stress tensor [Pa], $\vec{s_b}$ – source of mass forces [N m^{-3}], e – total energy [J], p – pressure of mixture [Pa], $\overleftrightarrow{I}$ – unit tensor [-], $\vec{q}$ – total heat flux [J/(m^{2}s^{-1})], s_e – source of energy [J/(m^{3}s^{-1})], Y_k – mass share of the kth component [-], $\vec{J_k}$ – total diffusion flux [mol/(m^{2}s^{-1})], s_k – mass source [kg/m^{3}s^{-1}], n_s – number of mixture components. The subscript in the equation (4) may be from 1 to n_s.

Second-order Godunov-type scheme was applied to reconstruct convective fluxes on balance cell walls. Time discretization was based on an explicit scheme at CFL equal to 5. The time step was fixed for all finite volumes. The number of grid cells reached, subject to case, from 43 300 to 85 200. 50 000 iterations were performed for each case. The time required to compute each case was determined by grid size, and it ranged from several to more than ten hours. Balance equations were solved by the relaxation method. The above parameters were selected experimentally at the stage of initial simulations which are not discussed in this paper.

Simulation results

Five principal flow types (Fig. 4) were determined based on an analysis of numerical results obtained with the use of the Multi Flower 2D application:

1) TYPE *A* (e.g. for b/B = 0.16, H/B = 2.8 and p_c = 0.2 [MPa]) – the jet oscillates along the entire channel, it is bifurcated and returns symmetrically in the direction of the side channels. This type of bifurcation is encountered in relatively long channels and at low nozzle width.

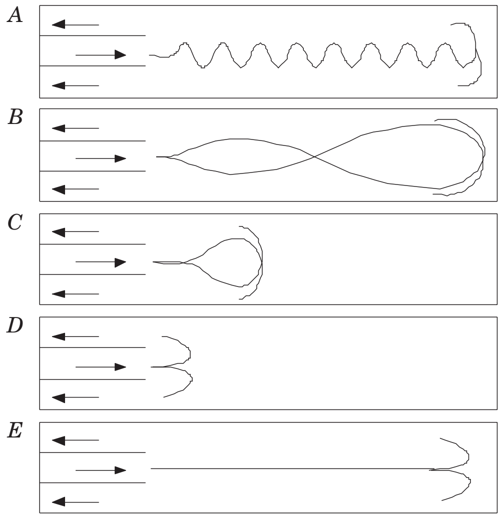


Fig. 4. Types of flow structures observed in numerical simulations

2) TYPE B (e.g. for b/B = 0.16, H/B = 4.4 and p_c = 0.12 [MPa]) – in the downstream end of the channel, the jet returns non-symmetrically in the direction of the side channels, alternating between the channels during flow. This type of bifurcation is encountered in very long channels and at low nozzle width.

3) TYPE C (e.g. for b/B = 0.24 and H/B = 4.4 and p_c = 0.15 [MPa]) – non-symmetrical jet recovery in the channel. The main jet impinges on a channel wall and, consequently, the attachment point moves along the channel to the opposite side. The flow from one side to the other is irregular. There are no disturbances in the downstream end of the channel which leads to the formation of a dead zone. This type of bifurcation is encountered in relatively long channels and at higher nozzle widths. This flow structure proved to be most sensitive to the value of inlet pressure. At higher pressure (0.2 [MPa] in this experiment), bifurcations were noted along a wide range of H/B and b/B indicators. As inlet pressure decreased (for p_c = 0.15 and 0.12 [MPa]), bifurcations disappeared, and a stable flow, classified as type D, was noted.

4) TYPE D (e.g. for b/B = 0.32 and H/B = 1.2 and p_c = 0.15 [MPa]) – symmetrical jet recovery, the main jet is dispersed and bifurcated at a distance of around 1 – 1.5 nozzle width b from the nozzle exit. Vortices were not reported in the remaining parts of the channel. The observed flow was highly stable. This type of flow occurs at a high b/B ratio regardless of channel length.

5) TYPE E (e.g. for b/B = 0.16 and H/B = 1.2 and p_c = 0.12 [MPa]) – the main jet is observed all the way down to the cut-off wall after which it recirculates symmetrically. The flow is symmetrical or quasi-symmetrical, subject to the pressure applied at the nozzle inlet. This type of bifurcation is encountered in short channels.

Tables 1, 2 and 3 present the flow structures for the three investigated pressure values and 50 000 iterations. It was found very difficult to interpret the results and classify them into particular flow types. Stationary flow types (E or D) were the easiest to classify, although transitory forms were evidently present. An example of the above is the solution for b/B = 0.16, H/B = 1.2 and p_c = 0.2 [MPa] which displays a visible transition trend from type E to type A flow. Other transition trends in a stationary system are noted in cases where b/B = 0.32. The system is stationary (type D) at low and medium pressure, but when pressure is increased at the inlet, a transition to another stationary form (type E) or oscillating form (type C) is reported, subject to the H/B indicator. Non-stationary cases are even more difficult to classify, especially as regards type B and type C flows where all efforts to ascertain whether the nozzle jet reaches the rear wall are highly subjective.

In the analyzed geometric system, there are at least three parameters responsible for bifurcation points: two geometric indicators and the inlet/outlet pressure ratio. Those points were not captured due to low geometry resolution and initial parameters of the system. The above could have been achieved if it were possible to model wall movements in the simulation model (self-adopting grids would have to be applied for such a broad range of changes) and to set initial conditions in the form of functions. The current version of the Multi Flower 2D model does not provide the above options.

Despite certain interpretation difficulties, Tables 1, 2 and 3 support the determination of the bifurcation type based on the system's geometric indicators. To validate the usefulness of table data, an additional simulation was carried out for b/B = 0.16, H/B = 2.0 and total pressure at the inlet p_c = 130 000 [Pa]. It was assumed that type A or type E flow or a combination of the two flow types would be observed. Simulation results supported the above assumption (Fig. 5). A transitory type was noted. The flow was stable after passing through the nozzle, but oscillations with a growing amplitude appeared as the flow approached the cut-off wall. The loss of flow stability, propagating from the cut-off wall to the nozzle exit, is probably a transitory form between flow types A and E. Transitory forms between other flow types can be deduced in a similar manner without additional simulations. This problem is analyzed below in reference to the general bifurcation conditions.

Table 1

Flow structures for p_c = 0.2 [MPa]

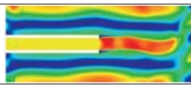
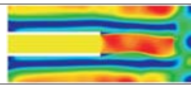
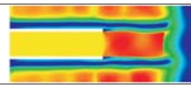
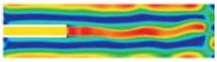
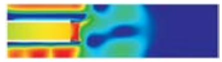
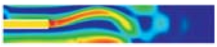
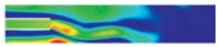
	b/B = 0.16	b/B = 0.24	b/B = 0.32
H/B = 1.2			
H/B = 2.8			
H/B = 4.4			

Table 2

Flow structures for pc = 0.15 [MPa]

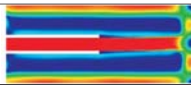
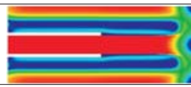
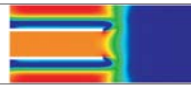
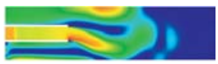
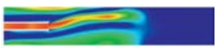
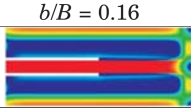
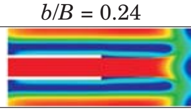
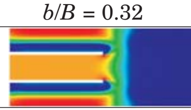
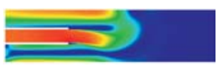
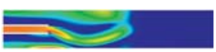
	b/B = 0.16	b/B = 0.24	b/B = 0.32
H/B = 1.2			
H/B = 2.8			
H/B = 4.4			

Table 3

Flow structures for pc = 0.12 [MPa]

	b/B = 0.16	b/B = 0.24	b/B = 0.32
H/B = 1.2			
H/B = 2.8			
H/B = 4.4			

Bifurcation theory defines two basic requirements which must be fulfilled for bifurcation to take place (BADUR and SOBIESKI 2001). The first requirement is that the system has to be relatively "lean" with a dominant parameter working in one direction. In line with the second requirement, there must be a "free area" perpendicularly to the dominant direction. The present research validates both requirements. For example, if b/B = 0.32, and H/B = 1.2, the flow emerging from the nozzle is wide, and the channel is short. The resulting system is not "lean", bifurcations do not take place and the entire flow

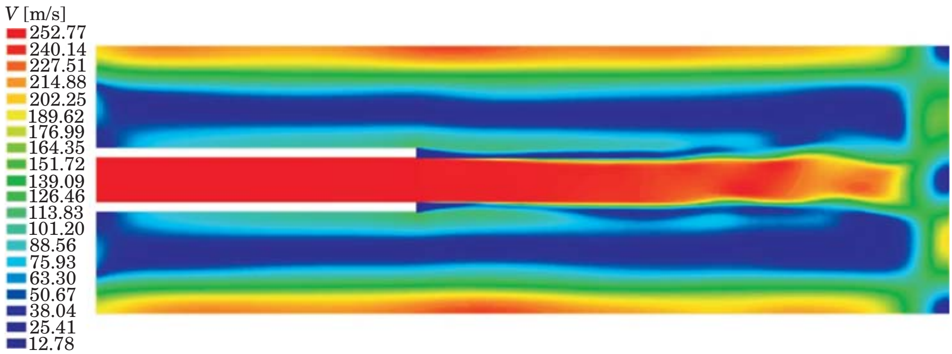


Fig. 5. Total velocity distribution for b/B = 0.16, H/B = 2.0 and p_c = 130000 [Pa]

is stationary. There is clearly no "free area" perpendicular to the dominant direction. With an improvement in the "leanness" criterion (e.g. for $b/B = 0.16$, $H/B = 1.2$ and $p_c = 0.2$ [MPa]) through a decrease in fluid jet width, the system displays a bifurcation trend (shown in Figures as a minor flow asymmetry). A further narrowing of the width of nozzle b would probably result in type B flow. A similar situation is observed as regards flow types C and B. The narrowing of the width of nozzle b improves the "leanness" criterion and clearly supports the development of an oscillating flow form.

When the investigated system is analyzed in view of bifurcation requirements, it can be generally concluded that as indicator H/B approaches unity, bifurcations do not take place because the "leanness" criterion is not met, and as the b/B ratio increases, the "free area" criterion deteriorates, leading to jet "rigidity" and loss of bifurcation. Flow velocity (in this case, modified by total pressure at nozzle inlet) is also an important consideration. The higher the flow velocity, the more likely the transition from a divergent to an oscillating regime, which supports the findings of other authors.

Bifurcation-sensitive parameters

Two complementary methods were employed to determine whether a given flow belongs to a divergent or oscillating bifurcation category. The first method involved an observation of changes in fluid parameters (mostly pressure) in a selected cell of the numerical grid (Fig. 6). The cell was positioned minimally below the lower nozzle edge, at a distance of approximately 1b from the exit cross-section. Pressure changes as a function of time were monitored to determine which of the observed cases could be classified as Hopf bifurcations.

Classical Hopf bifurcations with uniform and periodic changes in flow structure were noted only in types *A* and *B*. Type *D* was completely stationary, and the remaining flow types, although non-stationary, were not marked by clear periodicity.

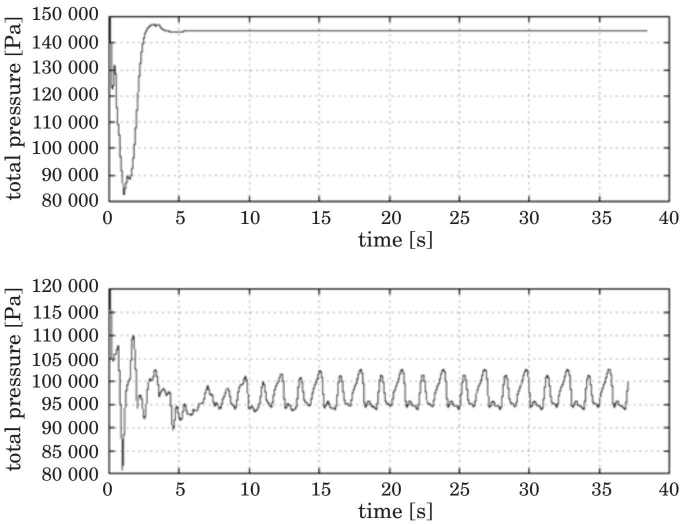


Fig. 6. Total pressure observations in a selected cell: type *D*: b/B = 0.32 and H/B = 1.2, p_c= 150 000 [Pa] (top); type *B*: b/B = 0.16 and H/B = 4.4, p_c = 120 000 [Pa] (bottom)

The second method applied to determine bifurcation type relies on a diagram of convergence of calculations. As previously noted, the solution to linear equations (the ultimate objective of the computation problem) is produced in the Multi Flower 2D application by the relaxation method. In stationary cases, the solution (values of the simple variable vector) is constant over time and an asymptotic result is established at a given level of accuracy. The above is not possible in stationary cases because the searched unknown vector values are subject to change regardless of the degree of flow nonstationarity. Since Hopf bifurcation nonstationarities are cyclical, this periodicity should also be manifested in the equation solving process. This is the case, as demonstrated by the convergence process in Figure 7 (which also illustrates the nonstationary case presented in Fig. 6). The time of one cycle determined in the discussed case was 1.4804 [s]. This method seems to deliver a higher degree of accuracy than the first method due to more pronounced changes in parameter values. The diagram of changes in parameter values in a selected computational cell has a local character in space (which could give rise to doubts as to the method's effectiveness), while the residue process represents flow on a global scale. It should also be noted that flow type cannot be determined based on the distribution of scalar and vector fields due to difficulties in interpreting their similarities and differences.

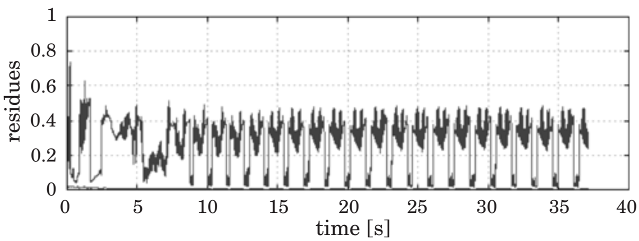


Fig. 7. Convergence process for b/B = 0.16; H/B = 4.4; p_c = 120 000 [Pa] (type B)

In subsequent parts of the experiment, the FLUENT commercial application was used to determine the effect of viscosity on changes in bifurcation type. Although various efforts were made to ensure that both simulation models are identical (as regards the mathematical model and the applied numerical techniques), satisfactory results were not achieved in the FLUENT application with respect to non-stationary flows. Identical results were reported only in stationary cases. Significant discrepancies in the flow structure as well as in the computed ranges of physical quantities were observed even in the case of small bifurcations. In view of the above, it was concluded that the FLUENT application lacks an appropriate solver for modifying flows with Hopf bifurcations. Perhaps, better results could have been achieved with a different configuration of the simulation model, however, it was not the objective and further investigations were not performed. It should be noted that the FLUENT application supports the modeling of other types of bifurcation, such as the Taylor-Couette bifurcation (DOMAŃSKI 2006).

Conclusions

The following conclusions can be drawn from the experiment:

– computer-aided simulations proved to be consistent with experimental results at the qualitative and, partly, at the quantitative level. A detailed quantitative comparison could not have been carried out due to the absence of the required experimental data (distribution of flow scalar and vector fields). Similar flow structures with a corresponding range of b/B and H/B geometrical indicators were noted in numerical simulations;

– a new type of flow, not described by the authors of the experiment, was discovered during numerical simulations. Type E flow is observed in short channels (which are deficient in the "leanness" parameter) and at low values of the b/B ratio (high availability of "free area" perpendicularly to the dominant direction). Presumably, this is a theoretical flow type which is possible, but difficult to achieve in reality due to imperfections which are almost always encountered (disruption of the ideal flow state);

– estimated areas of oscillating bifurcation, with a division into types, were proposed based on numerical simulations. In view of the analyzed conditions of the bifurcation process, it could be assumed that the main flow types which are possible in the investigated model have been identified in the study. The performed simulations illustrate the transition process between various bifurcation points and the encountered flow structures;

– once again, this experiment confirmed the usefulness of the Multi Flower 2D application for modeling flows with bifurcations. The applied mathematical models and computational algorithms support the propagation of nonstationary bifurcations and the modeling of relevant systems. The mathematical model found in the Multi Flower 2D application does not comprise any options making a reference to the bifurcation theory – this is a characteristic feature of the application which reflects the method of implementing mathematical models and the applied numerical techniques and methods;

– a method for determining the oscillation frequency of Hopf bifurcations based on the calculation process was developed in the study. This method was applied in simulations performed in the Multi Flower 2D package.

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