

THEORETICAL MINIMUM VALUE OF PDOP DETERMINATION

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Key words: pseudolites, GPS, The Nelder-Mead Simplex Method.

Abstract

The paper presents the results of PDOP theoretical minimum value determination. The Nelder-Mead Simplex Method which was used to calculations is described. The general characteristic of this method and the algorithm description are given. Eleven cases with different number of visible satellites (or pseudolites) was considered. In order to increasing the results reliability the calculations for each case were many times repeated. In each repetition of the iteration process another starting vector of parameters was qualified.

WYZNACZENIE TEORETYCZNEJ MINIMALNEJ WARTOŚCI PDOP

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Słowa kluczowe: pseudolity, GPS, metoda sympleksu Neldera-Meada.

Streszczenie

Przedstawiono wyniki wyznaczenia teoretycznej minimalnej wartości PDOP. Do obliczeń zastosowano metodę sympleksu Neldera-Meada. Podano ogólną charakterystykę metody oraz opis algorytmu. Rozpatrzono jedenaście przypadków różniących się liczbą widocznych satelitów (lub pseudolitów). W celu zwiększenia wiarygodności wyników obliczenia dla każdego przypadku wielokrotnie powtarzano. W każdym powtórzeniu procesu iteracyjnego przyjmowano inną wartość początkową wektora parametrów.

Introduction

GPS-receivers are becoming useful tools for engineering structure monitoring. Deformation measurements require relatively high accuracy. However, GPS is restrained by the satellite constellation geometry. The satellite sky distribution is not always good enough to obtain accurate results. In such cases pseudolites can be used (BARNES et al. 2003, MENG et al. 2002). Pseudolites transmit GPS-like signals. These instruments can be used to augment GPS data.

It is important to know what maximal accuracy is possible to obtain with a determined number of visible satellites or pseudolites.

This knowledge is required while we decide if GPS technique is proper for performing geodetic task e.g. engineering structure monitoring. The purpose of research described in this paper is determination maximal accuracy possible to obtain with GPS technique. Only geometrical aspects are taken into consideration missing the point of environmental influences. The most popular values describing positioning accuracy are DOP factors (dilution of precision) (CZARNECKI 1994, HOFMAN-WELLENHOF et al. 1992, LEICK 1995, LAMPARSKI 2001, KRAUTER 1999).

The positioning accuracy can be estimated using matrix Q , consisting of DOP factors in the following form (CZARNECKI 1994):

$$Q = (G^T G)^{-1} = \begin{bmatrix} NDOP^2 & & other & terms \\ & EDOP^2 & & \\ & & VDOP^2 & \\ other & terms & & TDOP^2 \end{bmatrix} \quad (1)$$

where:

G – the design matrix in the following form:

$$G = \begin{bmatrix} \sin z_1 \cos \alpha_1 & \sin z_1 \sin \alpha_1 & \cos z_1 & 1 \\ \sin z_2 \cos \alpha_2 & \sin z_2 \sin \alpha_2 & \cos z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \sin z_n \cos \alpha_n & \sin z_n \sin \alpha_n & \cos z_n & 1 \end{bmatrix}$$

where:

z_i – zenith distance of the direction vector between the receiver site and i -th satellite,

α_i – azimuth of the direction vector between the receiver site and i -th satellite.

$NDOP$, $EDOP$, $VDOP$ – DOP values in the northern, eastern and vertical directions in a local coordinate system.

Positioning *DOP* (*PDOP*) is estimated as:

$$PDOP = \sqrt{NDOP^2 + EDOP^2 + VDOP^2} \quad (2)$$

In practice the elevation cutoff mask is set to range from 10° to 20° to avoid high values of the ranging errors (e.g. signal multipath or signal blockage by the passing vehicles).

KRAUTER (1999) has presented the results of the calculations of the *PDOP* values for some cases of satellite configurations. However, it is not known if the satellite configurations which were chosen for the calculations were really optimal. In order to calculate the theoretical minimum value of *PDOP* for an assumed number of satellites (and the elevation cutoff mask) a numerical solution can be applied. There are several methods for the minimization of a function of n variables. One of the most effective is the Nelder-Mead Simplex Method (NELDER, MEAD 1965).

The Nelder-Mead Simplex Method

This method starts with $M+1$ points defining an initial simplex. The initial point X_0 must be determined. Other points of simplex can be expressed by:

$$X_i = X_0 + \lambda \cdot e_i$$

$M \times 1$ $M \times 1$ $M \times 1$

where:

α is constant

e_i are M vectors in the following forms:

(3)

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \dots e_M = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The function value at X_i is expressed by Φ_i .
Moreover:

$$\Phi_h = \max_i (\Phi_i),$$

$$\Phi_l = \min_i (\Phi_i),$$

$$\bar{X} = \frac{1}{M} \sum_{i \neq l} X_i \quad (\text{centre of simplex}).$$

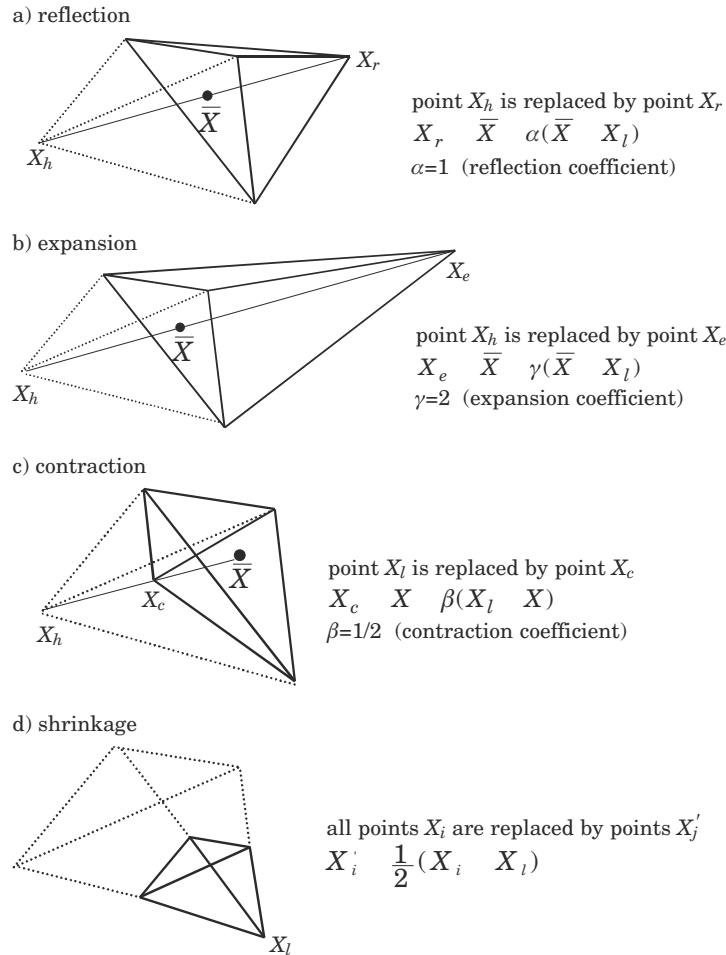


Fig. 1. Simplex transformation

The search for the maximum of the objective function is performed by simplex transformations. The simplex can be transformed by four operations:

The presented method is shown in the flow diagram.

The optimization process starts with calculating Φ_i at each point of simplex, calculating \bar{X} and determining X_l , Φ_l , X_h , Φ_h . In the first step of each iteration reflection is performed. If this operation produces a new minimum, expansion is performed. If expansion produces a new minimum, X_h is replaced by X_e , or X_h is replaced by X_r . If reflection produces a new maximum or the condition: $(\Phi_r > \Phi_i, i \neq h)$ is fulfilled, contraction is performed. If contraction produces good result $(\Phi_c < \Phi_h)$, X_h is replaced by X_c

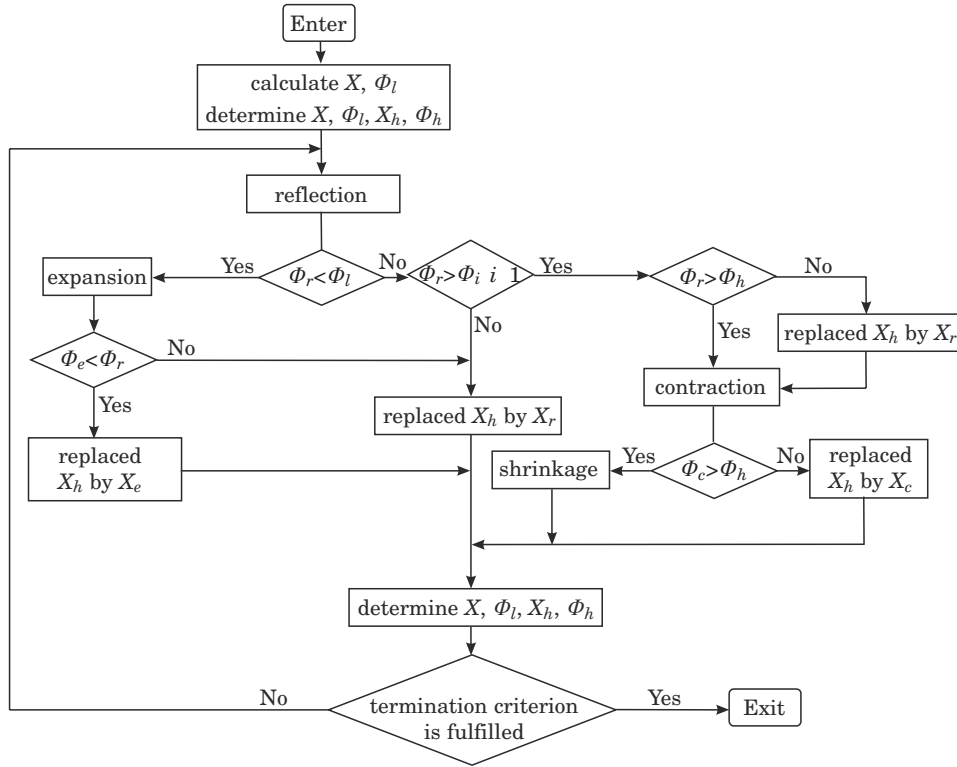


Fig. 2. Flow diagram

or shrinkage is carried out. Then X_l , Φ_l , X_h , Φ_h and \bar{X} are determined and the termination criterion is checked. The computation process is halted when the termination criterion is fulfilled.

Example

Elements of vector X_{M-1}^X are azimuths and zenith angles of satellites. PDOP is expressed by values of the function Φ . We considered different cases: when visible satellites were: 4, 5, 6, ... or 14 and when cutoff mask were: 0° , -90° , 10° , 20° . The values of the azimuths of the satellites were at random generated from the range $0-360^\circ$. We considered four cases for the ranges of the zenith angles values: $0^\circ-90^\circ$, $0^\circ-80^\circ$ (mask 10 degrees), $0^\circ-70^\circ$ (mask 20 degrees), and $0^\circ-180^\circ$. These values were at random generated separately from each range for each case with determined number of visible satellites. Thus the whole number of considered cases equals: $n=11 \times 4=44$ (11 cases for different number of visible satellites and 4 cases for

different values of the elevation cutoff mask). In order to obtain more reliable results the estimation (and data generation) was repeated 50 times for each case. The minimum for 50 repetitions was assumed as the final result. The following condition was assumed as the termination criterion: ($\Phi_h - \Phi_1 < 0.01$ and $d < 0.001$) (d -simplex diameter).

Results

The results of the calculations are given below:

The values presented in the third column of Table 1 were also reported by KRAUTER (1999), who obtained the same results.

Table 1

| Number of satellites | Minimum PDOP values | | | |
|----------------------|---------------------------------|------|------|------|
| | Min PDOP | | | |
| | elevation cutoff mask (degrees) | | | |
| | 0 | -90 | 10 | 20 |
| 4 | 1.63 | 1.50 | 1.82 | 2.14 |
| 5 | 1.47 | 1.35 | 1.61 | 1.85 |
| 6 | 1.32 | 1.22 | 1.47 | 1.70 |
| 7 | 1.25 | 1.13 | 1.38 | 1.58 |
| 8 | 1.17 | 1.06 | 1.30 | 1.48 |
| 9 | 1.09 | 1.00 | 1.23 | 1.41 |
| 10 | 1.05 | 0.95 | 1.15 | 1.33 |
| 11 | 1.01 | 0.90 | 1.11 | 1.30 |
| 12 | 0.96 | 0.87 | 1.08 | 1.22 |
| 13 | 0.93 | 0.83 | 1.03 | 1.19 |
| 14 | 0.88 | 0.80 | 1.00 | 1.14 |

Figure 3 contains diagrams showing relation between number of satellites and possible to obtain minimum PDOP values for elevation cutoff masks: 200 (green), 10° (blue), 0° (brown), -90° (red).

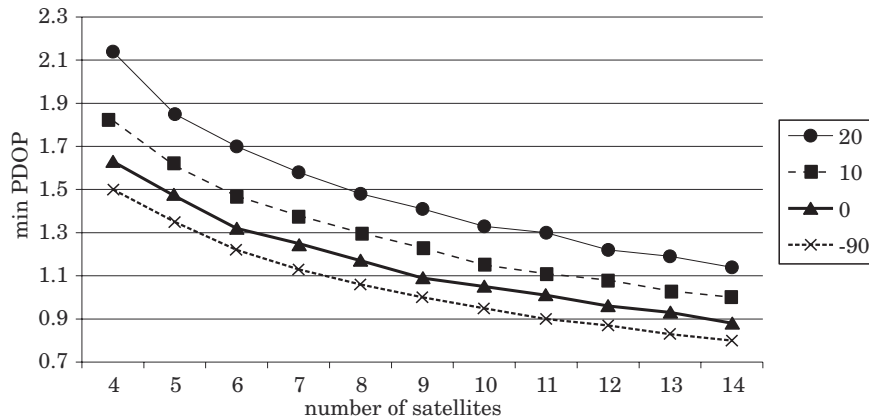


Fig. 3 Theoretical minimum PDOP value versus number of satellites

Conclusions

On the basis of obtained results the following conclusions can be derived:

1. The more satellites, the lower PDOP values are possible to obtain.
2. The significance of elevation cutoff mask value is the most in the case of four visible satellites
3. If the elevation cutoff mask is set to 10° , then minimum PDOP value amounts to 1.00 (for fourteen visible satellites or pseudolites). The lower PDOP values are possible to obtain if elevation cutoff mask amounts to 0° (for twelve visible satellites or pseudolites) or is lower than 0° (even for ten visible satellites or pseudolites). In this case pseudolites are placed lower than GPS-receiver antenna.

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Accepted for print 2004.11.05