M2. Ionizing radiation. Determination of linear and mass attenuation coefficients of γ radiation for various materials

Topics:

- The structure of atomic nucleus [1] – Chap. 42.
- Nuclear fission and fusion [1] – Chap. 43.
- The mass defect and binding energies [1] – Chap. 43.
- Types of ionizing radiation. Process of creation and properties of α and β particles [1] – Chaps. 42-4 through 42-6.
- Process of creation of γ radiation.
- The law of radioactive decay, half-life period [1] – Chaps. 42-4 and 42-7.
- The change of intensity of the matter-penetrating radiation beam. The attenuation coefficient.
- Units of ionizing radiation related to radioactive source activity, radiation dose and equivalent dose [1] – Chap. 42-8.
- The biological consequences of ionizing radiation.
- Phenomena related to interaction of ionizing radiation with matter.

At centre of atoms exists nucleus containing *Z* protons and *N* neutrons:

– total number of *nucleons* (either protons or neutrons) is denoted as *A*:

$$A = Z + N$$

– *A* is also known as the *atomic mass number* because for a mole of atoms, the molar mass is $M_M \approx A$ g/mol

for ex., for a mole of atomic oxygen with *A* = 16, *MM* = 15.99491 g/mol ≈ 16 g/mol.

Mass of proton, $M_p = 1.673\times10^{-27}$ kg

Mass of neutron, $M_n = 1.675\times10^{-27}$ kg

Mass of electron, $M_e = 9.109\times10^{-31}$ kg

$$M_A \approx ZM_H + NM_n - BE(A, Z, N)$$

Mass of atom:

where *BE* (*A,Z,N*) is the binding energy of the nucleus (function of *A*, *Z*, and *N*), and M_H is the mass of the hydrogen atom.

Binding energy is the energy released when protons and neutrons are brought together to form a nucleus

For some elements, we can have naturally occurring stable (i.e. non-radioactive) forms for different values of *N,* hence *A.*

Nuclei with the same *Z* but different *N* are called *isotopes,*

for ex., hydrogen (Z=1) has 2 stable isotopes with A = 1 (N = 0) and A = 2 (N = 1),

for ex., tin (Sn) (Z=50) has 10 stable isotopes with A=112, 114, 115, 116, 117, 118, 119, 120, 122, 124.

Nuclei are often identified uniquely via the form ^{A}Z**?** where the element name stands in for Z for ex., ^{112}Sn, ^{114}Sn, ^{115}Sn,...

A nucleus may decay when there is a combination of products that have a smaller total energy. Most common decay modes are:

α decay:

- Emission of a ^{4}He nucleus
- In a decay $A, Z, N \rightarrow A\text{-}4, Z\text{-}2, N\text{-}2 \Rightarrow$ nucleus loses 4 nucleons: 2 protons and 2 neutrons.

β decay:

- Emission of β particles (electrons or positrons) in the transition from one nucleon to another,
- β particles can have negative charge (electrons) or positive charge (positrons),
- Another particle always involved is the neutrino, n – a chargeless, nearly massless, extremely weakly interacting particle,
- Typical energies releases are on the order of a few MeV,
- In β^- decay $A, Z, N \rightarrow A, Z{+}1, N\text{-}1 \Rightarrow$ a neutron turns into a proton
- β^+ decay $A, Z, N \rightarrow A, Z\text{-}1, N{+}1 \Rightarrow$ a proton turns into a neutron,

γ decay:

Emission of high energy photons in the transition from an excited nucleus state to a lower state. A, Z, N of nucleus remains the same.

The phenomena related to the penetration of matter by radiation, in the first place depends on the type of radiation. The interaction of neutral particles (like neutrons) with matter is weak, while charged particles and electromagnetic radiation interact strongly with electronic shell of atom.

γ-Radiation can interact both with electrons and nuclei, but also with electric field generated by electrons and nuclei. These interactions might lead either to full absorption or scattering of γ radiation.

The process of γ-rays attenuation is ruled by three phenomena:

- Photoelectric effect – interaction of γ-rays with electrons leads to full absorption of γ quantum and ejection of electron from the electronic shell of atom.
- Compton's effect – scattering of γ-radiation on electrons, where photon change both its energy and direction of movement.
- The effect of electronic pair creation – photon is fully absorbed and electron-positron pair is created. This process required presence of any atom near the photon.

The phenomena described above cause decrease of the intensity of radiation beam with increasing thickness of beam-absorbing material. If the radiation beam with initial intensity I_o penetrates an absorbent of thickness x then, as a consequence of absorption and scattering of photons, its intensity drops to I. Increase of x by dx causes decrease of I by dI. The relative decreasing of the intensity of beam dI/I is proportional to the thickness increment dx:

$$\frac{dI}{I} = -\mu dx \qquad (1)$$

The minus sign in the Eq. (1) means that the radiation intensity decreases.

The factor of proportionality is called the linear attenuation coefficient:

$$\mu = \frac{dI}{I}\frac{1}{dx} \qquad (2)$$

The linear attenuation coefficient can be described as relative decrease of radiation intensity by an absorption material of the unit thickness. The unit of the linear attenuation coefficient is [m^{-1}]. A value of the coefficient depends on i.a. density of the absorbing material and the wavelength of radiation.

Integration of the equation (1) leads to the formula

$$I = I_0 e^{-\mu x} \qquad (3)$$

where:

I_0 – the intensity of incident beam,

I – the intensity of emergent beam, which passed through an absorbent of thickness x.

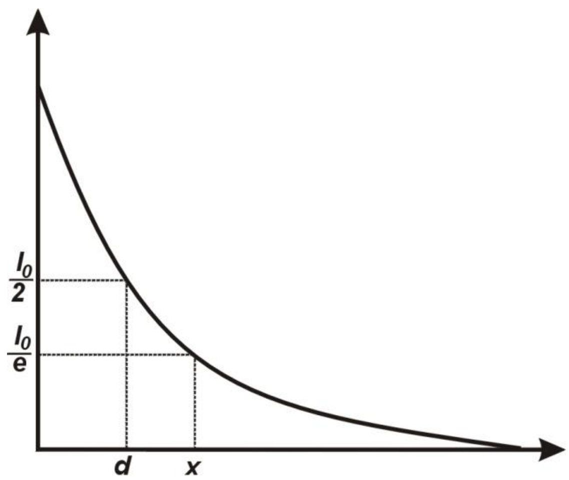


Fig. 1

Fig. 1 shows the dependence of beam intensity *I* on the thickness of the absorbent *x*. (Eq. 3).

The plot can be used to determine the linear attenuation coefficient. For this purpose, one should define the thickness x_e of an absorbent, for which the intensity of the beam decreases *e*-times, i.e. $I = \frac{I_0}{e}$. Application of this definition to the Eq. (3) leads to:

$$\frac{I_0}{e} = I_0 e^{-\mu x_e}$$

Transformation and simplification leads to:

$$\frac{1}{e} = e^{-\mu x_e}, e = e^{\mu x_e}$$

therefore

$$\mu x_e = 1$$

and

$$x_e = \frac{1}{\mu}$$

therefore

$$\mu = \frac{1}{x_e} \qquad (4)$$

A value of attenuation coefficient is equal to the converse of absorbent thickness x_e, which decreases intensity of incident beam *e*-times.

The Half Value Layer (HVL) *d* signifies the thickness of a material required to reduce the intensity of the emergent radiation to half of its incident magnitude (Fig. 1).

The relation between μ and *d* comes from Eq. (3), where according to HVL definition $I = \frac{I_0}{2}$ for $x = d$. Therefore:

$$\frac{I_0}{2} = I_0 e^{-\mu d}$$

Simplification, transformation and application of logarithm function to both sides of equation leads to:

$$d = \frac{0{,}693}{\mu}$$

Experiments show that for small atomic number materials, the linear attenuation coefficient is approximately proportional to the density of absorbent. Therefore the mass attenuation coefficient has more universal meaning

$$\mu_m = \frac{\mu}{\rho} \qquad (5)$$

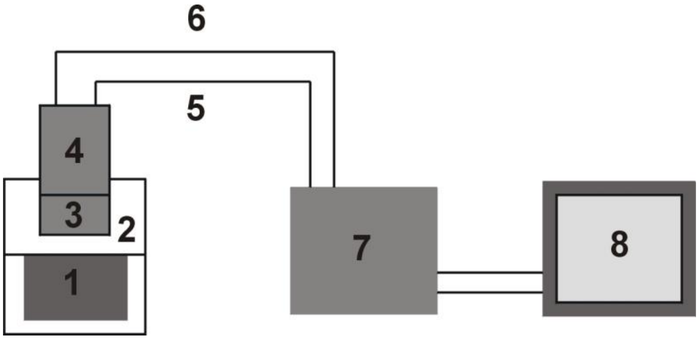
The mass attenuation coefficient allows direct comparison of attenuation of radiation for various materials. The mass attenuation coefficient does not depend on physical and chemical properties of absorbent, but it approximately depends on the atomic number of absorbent *Z* and radiation wavelength λ.

The goal of this experiment is determination of γ-ray linear and mass attenuation coefficients for various materials.

The γ radiation sources used in the experiment are encapsulated i.e. one cannot access the radioactive material.

WARNING! One must not touch the window in the source encapsulation, because the damage may cause a leakage of radioactive substance outside.

The measurement equipment is an assembly of the following components:



1 – the niche for encapsulated source and absorbent plates,

2 – lead protection screen,

3 – scintillator NaJ(Tl),

4 – photomultiplier,

5 – high voltage cable (computer $\rightarrow$ photomultiplier),

6 – signal cable (photomultiplier $\rightarrow$ computer),

7 – computer,

8 – computer screen.

The intensity of γ radiation beam decreases with depth of the absorbent penetration according to the Eq. (3). The γ quantum causes luminescent flashes, which are registered by photomultiplier. The number of flashes *N* is proportional to the intensity of γ beam, which enters the scintillator. In the absence of the absorbent material, the number of scintillation flashes N_o is proportional to the initial intensity I_0:

$$N_0 \propto I_0$$

The number of scintillation flashes *N* is proportional to the intensity *I* of emergent beam which passed through the absorbent of thickness *x*

$$N \propto I$$

Application of these relations to the Eq. (3) leads to the formula, which describes the number of scintillation flashes *N* as a function of the absorbent thickness *N = f(x)*.

$$N = N_o e^{-\mu x}$$

The plot of *n = f(x)* can be used for determination of the linear attenuation coefficient μ (see. Fig. 2), which is given by

$$\mu = \frac{1}{x_e}$$

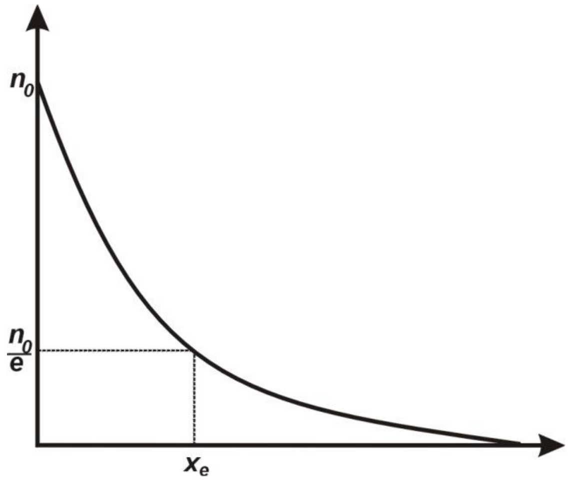


Fig. 2

The linear attenuation coefficient can also be determined from linear relation $ln\frac{n_0}{n} = f(x)$, which is given by the slope of the fitted straight line:

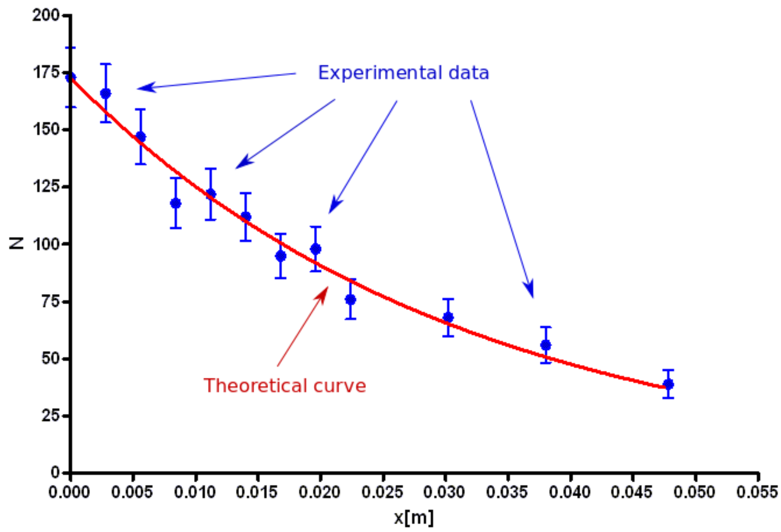
$$\mu = \frac{\Delta ln\frac{n_0}{n}}{\Delta x}$$

Overview and scope of the experiment

The theory predicts the following dependence of number of quanta (N) which enters the detector on the thickness of absorbent (x)

$$N(x) = \boldsymbol{N_0} e^{-\boldsymbol{\mu} x} \quad (1)$$

where: $\boldsymbol{N_0}$ is number of quanta which enters detector without and absorbent plates and $\boldsymbol{\mu}$ is <u>attenuation coefficient</u> of investigated material. The <u>scope</u> of this experiment is <u>determination of the attenuation coefficient (μ).</u> It is achieved by fitting of the theoretical curve (1) to the experimental data as shown in figure below.



The experiment is divided into two parts:

- **Determination of attenuation coefficient** ($\boldsymbol{\mu}$). In this part number of quanta entering the detector (N) is measured as a function of increasing thickness of stack of absorbent plates. Then the exponential decay of the form (1) is fitted to the experimental data. As a result best estimates of two parameters N_0 and μ are obtained.
- **Determination of mass attenuation coefficient** ($\boldsymbol{\mu_m}$)**.** The coefficient does not depend on chemical and physical state of an investigated substance and is more appropriate for comparison of different absorbents. Numerically it is attenuation coefficient (μ) divided by density of the absorbent (ρ).

Detailed instruction

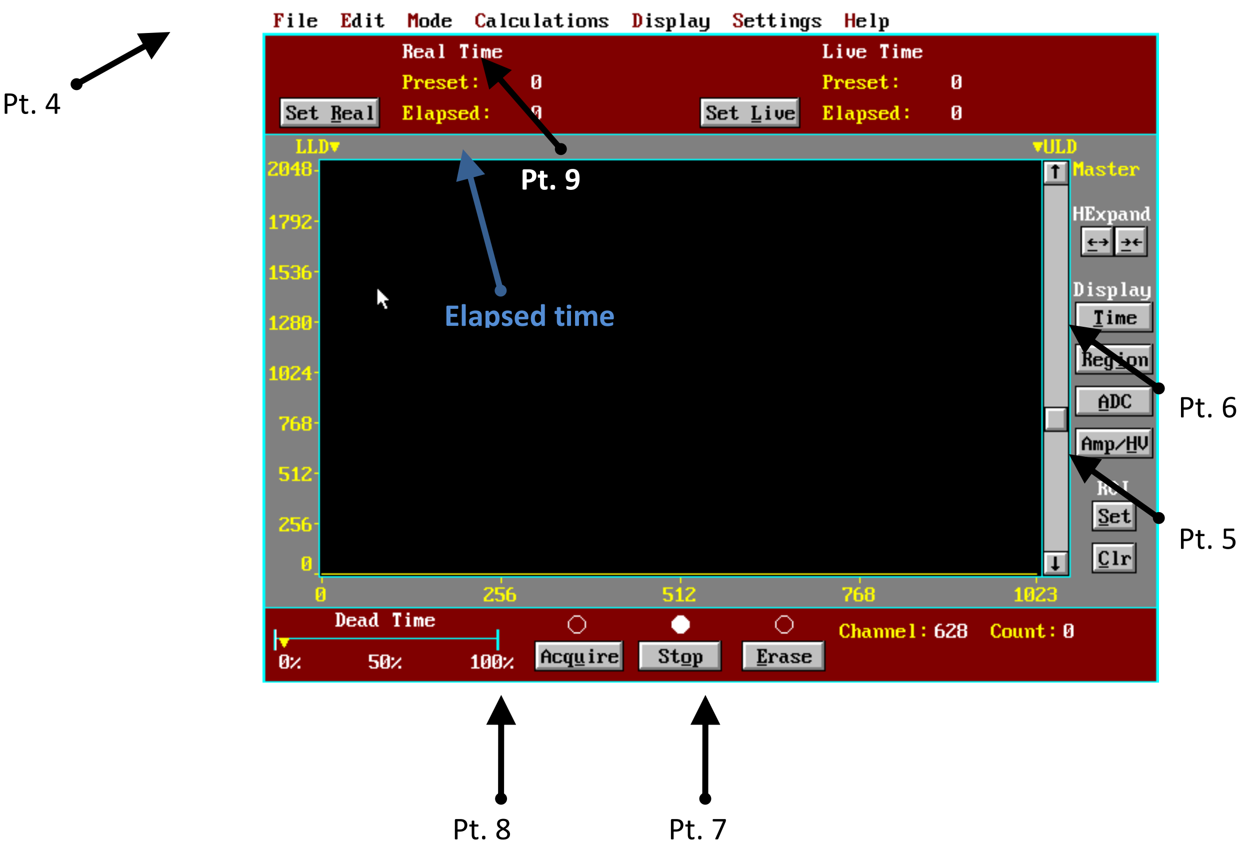
1. Prepare the following Table of Results containing: d – thickness of single plate, x – total thickness of absorbent (stack of plates), N – number of quanta entering the detector for given thickness, $u(N)$ – uncertainty (error) of number of counts N.

$d(m)$	$x(m)$	N	$u(N) = \sqrt{N}$
0	0	…	…
…	…	…	…
⋮	⋮	⋮	⋮

2. Use separate instruction to learn how to use Vernier's caliper. Set Vernier's caliper to random value and write down the reading. Ask a teacher if it is correct.
3. Switch on the computer
4. Choose: File→Load Setup→ M2.stp

Usually the computer is already switch on and the appropriate program is already running.

The conditions of experiment (type of isotope, type of absorbent, number of absorbent plates, the measurement channel) are defined by teacher.



5. Check if the power is switched on. Click Amp/HV, it should be set to ON.
6. Click (Time).
7. Clear the screen (Erase).
8. Make a measurement of radiation intensity $N_0 = N(x = 0)$ for a given isotope. Start the measurement (Acquire). It lasts 100 s.
9. Smooth the measurement: choose Calculation→Smooth Data; repeat it 3 times.
10. Use cursor to set the measurement channel at the maximum of spectra (the channel is defined by teacher)
11. Read the value of N_0 for the selected channel and put it in *Table of Results*.
12. Clear the screen (Erase).
13. Measure a thickness of the thin absorbent plate (d) and place it between the source of radiation and the scintillator (measurement chamber).
14. Start the measurement (Acquire) to determine the intensity of radiation N for given absorbent thickness. It lasts 100 s.
15. Smooth the data 3 times (Calculation→Smooth Data) and read value of *N* (**IMPORTANT**: do not change the measurement channel). Place d, x and *N* in *Table of Results*.
16. Repeat the measurement for increasing number of absorbent plates. Plates should be added to the stack starting from thinnest through two mid-thickness and finally the thickest one. The stack thickness should be continuously increased. **Never remove plates from the measurement chamber!** Put d, x and *N* in *Table of Results*.
17. Start *GraphPad Prism* program in a mode with uncertainties (standard error) on y-axis and enter x, N and $\sqrt{N}$ data into the first three columns. Then perform fit of *one phase exponential decay* (eq. 1) to your experimental data. The program will find the best possible N_0 (called *SPAN* in the program) and μ (called K in the program) coefficients. The last parameter is the ***attenuation coefficient*** determined in your experiment.
18. Calculate the mass attenuation coefficient $\mu_m = \frac{\mu}{\rho}$. Compare calculated value with table values for investigated material.

Density of absorbent (aluminum): $\rho = 2700\ kg/m^3$

[1] Walker J., Halliday and Resnick, *Principles of physics : international student version*, 9 th ed., extended, Hoboken : John Wiley & Sons, Inc., 2011. , ISBN 978-0-470-56158-4